Modern Public Key Cryptography

Efficient Zero-Knowledge

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based on slides by David Derler

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SCIENCE
PASSION
TECHNOLOGY

Outline

Efficient ZK Proofs of Knowledge

Efficient NIZK with Random Oracles

Efficient ZK for General Circuits

Recall: Zero Knowledge Proofs

NP-language L w.r.t. relation R

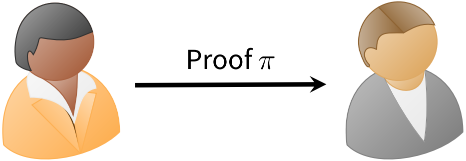
- $x \in L \iff \exists\, w : (x, w) \in R$

Non-interactive proof system

$(x, w) \in R$

$\pi \leftarrow \mathrm{Proof}(x, w)$

$x \overset{?}{\in} L$



$\checkmark / \times \leftarrow \mathrm{Verify}(x, \pi)$

Recall: Zero Knowledge Proofs contd'

Completeness

- Honestly computed proof for $(x, w) \in R$ will always verify

Soundness

- Infeasible to produce valid proof for $x \notin L$

Extractability

- Stronger variant of soundness
- Extract witness from valid proof (using trapdoor)

Recall: Zero Knowledge Proofs contd'

Witness Indistinguishability (WI)

- Distinguish proofs for same x w.r.t. different w, w'

Zero-Knowledge (ZK)

- Stronger variant of witness indistinguishability
- Simulate proofs without knowing w (using trapdoor)

Honest-Verifier Zero Knowledge

complete: honestly computed proofs must always verify

special-sound: dishonest proofs can only verify with negligible probability

(special) honest-verifier zero-knowledge: verifier learns nothing beyond validity of the proof

We consider Σ-protocols

- 3-move public coin HVZKPoK

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Σ-protocols for DLOG (Schnorr proof)

Prove knowledge of dlog $k \in \mathbb{Z}_p$ in DL commitment $h = g^k$ of p-order group $G = \langle g \rangle$:

$\mathcal{P}(g, k)$		$\mathcal{V}(g, h)$
pick $r \xleftarrow{R} \mathbb{Z}_p$, $q \leftarrow g^r$	$\xrightarrow{q}$	
	$\xleftarrow{c}$	pick challenge $c \xleftarrow{R} \mathbb{Z}_p$
$z \leftarrow r + ck$	$\xrightarrow{z}$	$g^z \stackrel{?}{=} q \cdot h^c$

We write PoK$\{(\alpha) : h = g^\alpha\}$ to denote such a proof

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Completeness?

Σ-protocols (ctd.)

How is special soundness formalized?

- $\mathcal{P}^*$ can only answer correctly if c guessed!
  - If challenge space chosen large enough,

  $\Rightarrow$ soundness error negligible with one round
- Otherwise, we can extract secret ($\Rightarrow \mathcal{P}$ knows secret)!

Extraction for Schnorr protocol:

- After first showing, rewind $\mathcal{P}$ to step 2
- Two valid showings $(q, c, z), (q, c', z')$: $g^z = q \cdot h^c$ and $g^{z'} = q \cdot h^{c'}$

$\Rightarrow$ $g^{(z-kc)} = g^{(z'-kc')}$, i.e., $k = (z - z')(c - c')^{-1}$

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How to show (special) honest-verifier ZK?

- Interaction between $\mathcal{P}$ and $\mathcal{V}$ can be efficiently simulated (HVZK $\rightarrow \mathcal{S}$ does not use $\mathcal{V}^*$)

Simulation of Schnorr protocol

- Pick $c, z \xleftarrow{R} \mathbb{Z}_p$ and set $q \leftarrow g^z / g^c$
- (q, c, z) valid: $g^z = q \cdot g^c$
- (q, c, z) distributed like real interaction

- For special HVZK, $\mathcal{S}$ also gets c as input

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Composition of Σ-protocols:

- Possible to prove more general relations by combining several protocol instances
- E.g. possible to prove relations:
  - AND,
  - OR,
  - EQ,
  - NEQ,
  - Interval, …
- Combination is again Σ-protocol (3-move structure)

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Σ-protocols (Schnorr AND Proof)

Two values: $h_1 = g^{k_1}, h_2 = g^{k_2}$

$\mathcal{P}(g, k_1, k_2)$		$\mathcal{V}(g, h_1, h_2)$
pick $r_1, r_2 \xleftarrow{R} \mathbb{Z}_p$		
$q_1 \leftarrow g^{r_1},\ q_2 \leftarrow g^{r_2}$	$\xrightarrow{q_1, q_2}$	
	$\xleftarrow{c}$	pick $c \xleftarrow{R} \mathbb{Z}_p$
$z_1 \leftarrow r_1 + ck_1,\ z_2 \leftarrow r_2 + ck_2$	$\xrightarrow{z_1, z_2}$	$g^{z_1} \stackrel{?}{=} q_1 \cdot {h_1}^c$
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We write $\text{PoK}\left\{ (\alpha_1, \alpha_2) : h_1 = g^{\alpha_1} \wedge h_2 = g^{\alpha_2} \right\}$

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Two values: $h_1 = g^{k_1}, h_2 = g^{k_2}$,

- where $\mathcal{P}$ only knows (w.l.o.g.) k_1

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Σ-protocols (Pedersen Commitments)

Pedersen commitment $C = g^m \cdot h^r$ to $m \in \mathbb{Z}_p$

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Non-Interactive PoKs (Fiat-Shamir Heuristic)

Goal: Make interactive proofs non-interactive

$\Rightarrow$ Then anyone can verify!

Idea: Let prover compute challenge c on its own

- s.t. challenge unpredictable

How? Use hash function on initial commitment q

Applications:

- NIZKPoKs by itself an application!
- Signature schemes from identification schemes

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Schnorr Signature

Non-interactive Schnorr protocol

- + inclusion of message m into computation of challenge c!

$\Rightarrow$ Secure digital signature in ROM

Apply Fiat-Shamir:

- $q \leftarrow g^r$ as in Schnorr protocol
- Set challenge $c \leftarrow H(m \| q)$, where H hash function
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If H is random-oracle, value c not predictable!

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Schnorr Signature (ctd.)

Scheme

KeyGen(1^κ): Choose $\mathcal{G}^\kappa = (\mathbb{G}, p, g)$, $k \xleftarrow{R} \mathbb{Z}_p$, compute $h \leftarrow g^k$ and return $(sk, pk) \leftarrow (k, h)$

Sign(m, sk): Pick $r \xleftarrow{R} \mathbb{Z}_p^*$, compute $q \leftarrow g^r$, $c \leftarrow H(m \| q)$ and $z \leftarrow r + ck$ and output $\sigma \leftarrow (c, z)$

Verify(m, σ, pk): Return $[c = H(m \| g^z / h^c)]$

EUF-CMA secure in ROM based on DLP!

Notes

Is HVZK too weak in practice?

- Fiat-Shamir Heuristic
  - Verifier is forced to be honest
  - ZK in random oracle model
- Conversion for HVZK Σ-protocols to ZK ones [2]

Omega Protocols

- Online extractability instead of rewinding $\mathcal{P}$
- Compatible with the UC framework
- Tighter reductions

Notes

Is HVZK too weak in practice?

- Fiat-Shamir Heuristic
  - Verifier is forced to be honest
  - ZK in random oracle model
- Conversion for HVZK Σ-protocols to ZK ones [2]

Omega Protocols

- Online extractability instead of rewinding $\mathcal{P}$
- Compatible with the UC framework
- Tighter reductions

ZK for General Circuits

So far we have seen practically efficient proofs for statements regarding discrete logarithms.

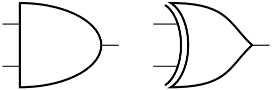
- Very useful in practice
- Building block in many useful protocols
  - secure voting schemes
  - anonymous transactions
  - anonymous credentials

What about arbitrary statements?

Interlude (Completeness of boolean circuits)

Any function computable in finite time can be expressed using a boolean circuit using 2-input gates.

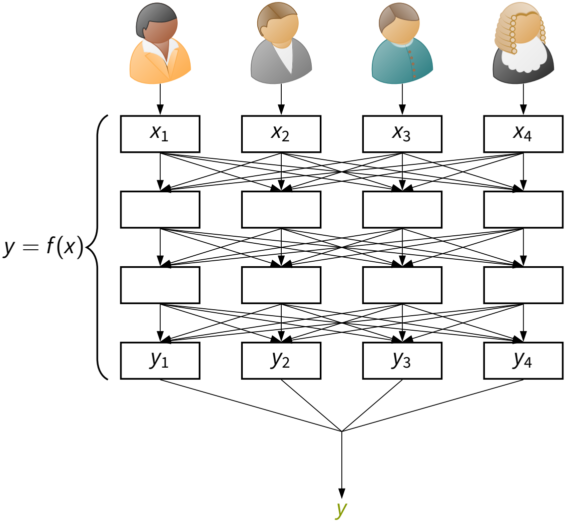
- You may have heard that the `NAND` gate is complete
- So is a combination of `AND` and `XOR` gates
  - This is nice because it maps to fundamental mathematical operations
  - Addition mod 2 $\equiv$ Binary XOR gate
  - Multiplication mod 2 $\equiv$ Binary AND gate



Multiparty Computation

A method to securely evaluate a public function between a number of parties, who hold private inputs.

- Many different protocols exists
  - Many work on a circuit representation of the function
  - Each gate corresponds to a “step” in the MPC protocol
  - Parties may need to communicate to evaluate a gate together
- $(n-1)$-privacy: even if all but one party collude, they cannot learn any information about the true values



MPC-in-the-Head Proof Systems

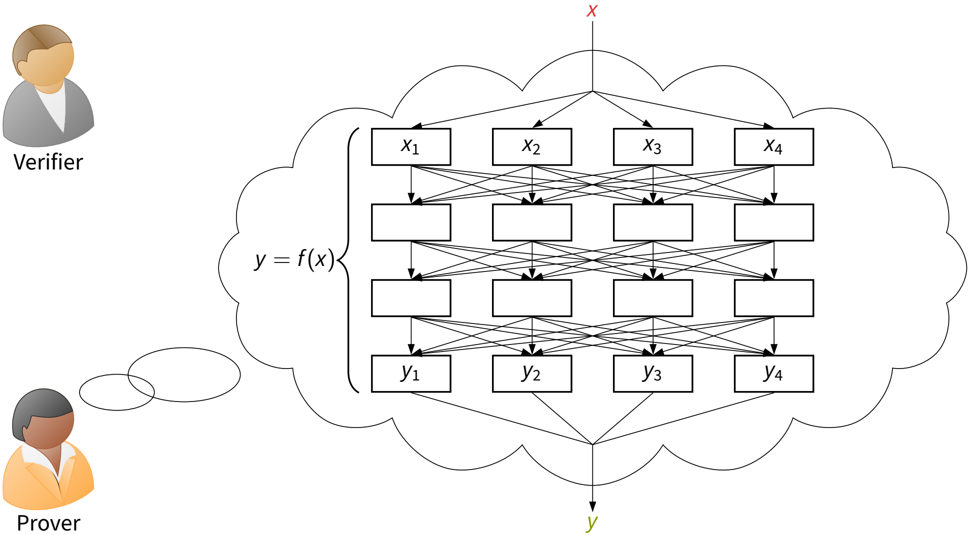
Thinking about Computations

MPC-in-the-Head Paradigm

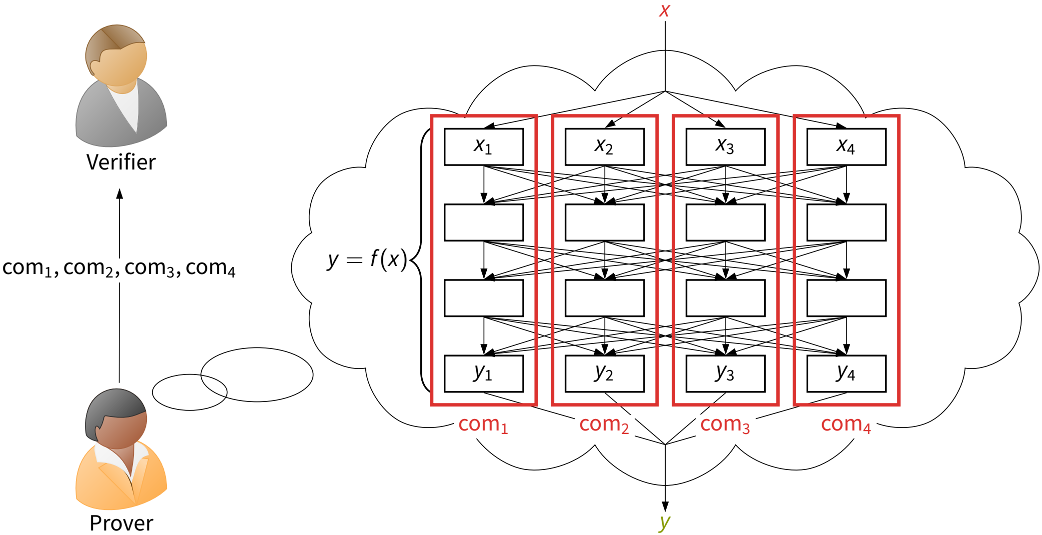
Technique by Ishai et al. (2008) to build a zero-knowledge proof system:

- Take a Multiparty Computation Protocol
- Simulate the evaluation of the function with N players
- Commit to the internal state and messages sent by the players
- Reveal a fraction of the internal states based on a random challenge
  - Not enough to leak any information about the real values
  - Enough that the consistency between the revealed parties can be verified
  - Gain some assurance that the remaining states are also ok

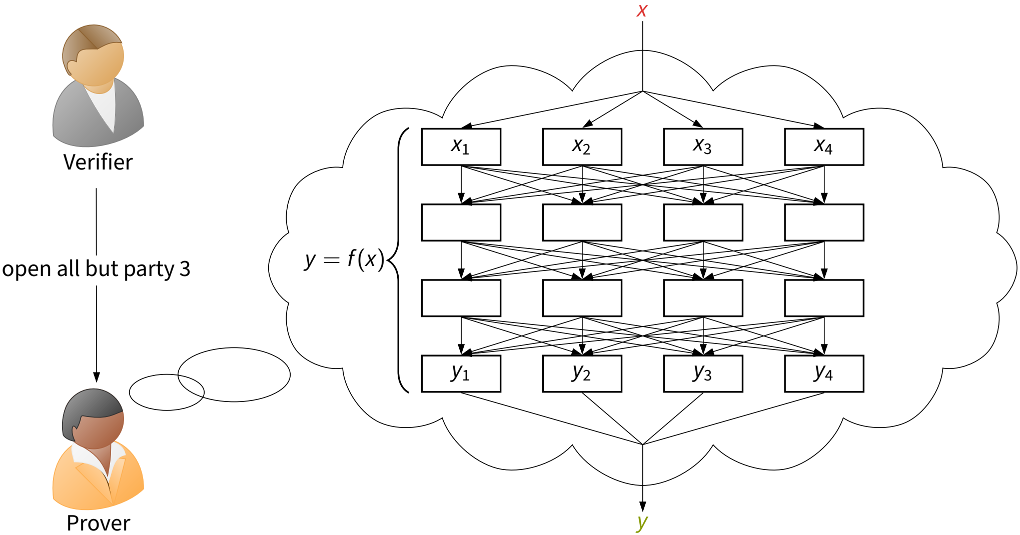
MPC-in-the-Head Paradigm (cont.)



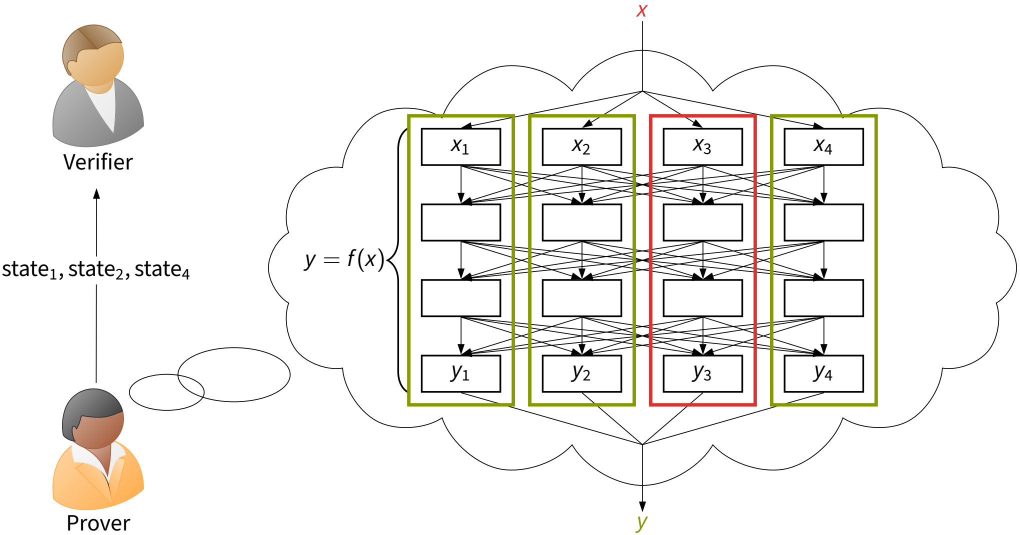
MPC-in-the-Head Paradigm (cont.)



MPC-in-the-Head Paradigm (cont.)



MPC-in-the-Head Paradigm (cont.)



MPCitH as a Sigma Protocol

Can view MPCitH protocol as a Σ-protocol:

- $\mathcal{P}_0$:
  - Prover simulates the MPC execution
  - Commits to state of all players
- $\mathcal{P}_1$:
  - Prover reveals all messages and internal states (except party ch)
- $\mathcal{V}$:
  - Verifier repeats execution with revealed parties
  - Verify consistency of revealed parties

Prover		Verifier
$\mathsf{com} \leftarrow \mathcal{P}_0(x)$	$\xrightarrow{\mathsf{com}}$	
		$\mathsf{ch} \xleftarrow{\$} \mathsf{ChS}(1^k)$
	$\xleftarrow{\mathsf{ch}}$	
$\mathsf{resp} \leftarrow \mathcal{P}_1(x, \mathsf{com}, \mathsf{ch})$		
	$\xrightarrow{\mathsf{resp}}$	
		$b \leftarrow \mathcal{V}(y, \mathsf{com}, \mathsf{ch}, \mathsf{resp})$

Non-Interactive MPCitH proofs

- Fiat-Shamir transformation
  - As seen above
  - Prover calculates challenge
  - Set challenge $c \leftarrow \mathcal{H}(\mathsf{com})$

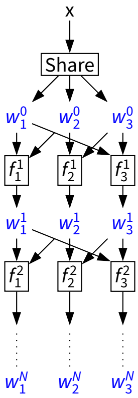
Prover		Verifier
$\mathsf{com} \leftarrow \mathcal{P}_0(x)$		
	$\xrightarrow{\mathsf{com}}$	
		$\mathsf{ch} \xleftarrow{\$} \mathsf{ChS}(1^k)$
	$\xleftarrow{\mathsf{ch}}$	
$\mathsf{resp} \leftarrow \mathcal{P}_1(x, \mathsf{com}, \mathsf{ch})$		
	$\xrightarrow{\mathsf{resp}}$	
		$b \leftarrow \mathcal{V}(y, \mathsf{com}, \mathsf{ch}, \mathsf{resp})$

Prover		Verifier
$\mathsf{com} \leftarrow \mathcal{P}_0(x)$		
	$\xrightarrow{\mathsf{com}}$	
$\mathsf{ch} \leftarrow \mathcal{H}(\mathsf{com})$		
$\mathsf{resp} \leftarrow \mathcal{P}_1(x, \mathsf{com}, \mathsf{ch})$		
	$\xrightarrow{\mathsf{resp}}$	
		$\mathsf{ch} \leftarrow \mathcal{H}(\mathsf{com})$
		$b \leftarrow \mathcal{V}(y, \mathsf{com}, \mathsf{ch}, \mathsf{resp})$

ZK for General Circuits [8, 5]

Instantiation of MPC-in-the-Head approach

1. (2,3)-decompose circuit into three shares
2. Revealing 2 parts reveals no information
3. Evaluate decomposed circuit per share
4. Commit to each evaluation
5. Challenger requests to open 2 of 3
6. Verifies consistency



Proof for $y = \text{SHA-256}(x)$: 13ms to create, 5ms to verify, $\approx$ 220 kilobytes

What you should know...

- Interactive Proof Systems
- Concept of Interactive ZK Proofs (Security Properties)
- Proofs of Knowledge:
  - Security Properties
  - Σ-protocols (Schnorr, compositions, ...)
  - Fiat-Shamir Transform
- Schnorr Signature Scheme
- Idea of ZK for General Circuits
  - MPC-in-the-Head

Questions?

Further Reading I

[1] Mihir Bellare and Oded Goldreich.

On defining proofs of knowledge.

In *Advances in Cryptology - CRYPTO '92, 12th Annual International Cryptology Conference, Santa Barbara, California, USA, August 16-20, 1992, Proceedings*, pages 390–420, 1992.

[2] Ronald Cramer, Ivan Damgård, and Philip D. MacKenzie.

Efficient zero-knowledge proofs of knowledge without intractability assumptions.

In *Public Key Cryptography, Third International Workshop on Practice and Theory in Public Key Cryptography, PKC 2000, Melbourne, Victoria, Australia, January 18-20, 2000, Proceedings*, pages 354–373, 2000.

[3] Ivan Damgard.

On Σ-protocols.

http://cs.au.dk/~ivan/Sigma.pdf.

[4] Juan A. Garay, Philip D. MacKenzie, and Ke Yang.

Strengthening zero-knowledge protocols using signatures.

J. Cryptology, 19(2):169–209, 2006.

Further Reading II

[5] Irene Giacomelli, Jesper Madsen, and Claudio Orlandi.

Zkboo: Faster zero-knowledge for boolean circuits.

In *USENIX Security*, 2016.

[6] Oded Goldreich.

Computational Complexity - A Conceptual Perspective.

Cambridge University Press, 2008.

[7] Jens Groth and Amit Sahai.

Efficient non-interactive proof systems for bilinear groups.

Cryptology ePrint Archive, Report 2007/155, 2007.

http://eprint.iacr.org/2007/155.

[8] Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, and Amit Sahai.

Zero-knowledge from secure multiparty computation.

In *Proceedings of the 39th Annual ACM Symposium on Theory of Computing, San Diego, California, USA, June 11-13, 2007*, pages 21–30, 2007.