Modern Public Key Cryptography

Public Key Encryption

Daniel Kales based on slides by Sebastian Ramacher and David Derler

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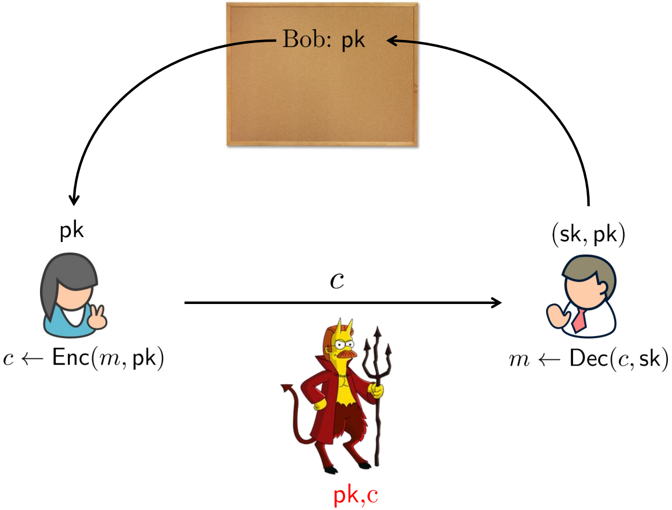
SCIENCE
PASSION
TECHNOLOGY

Public Key Encryption - The Setting



Formal Definition

Public Key Encryption

A PKE scheme is a tuple of PPT algorithms:

- *KeyGen*(1^κ): This probabilistic algorithm takes a security parameter κ and outputs a pair of keys $(\mathrm{sk}, \mathrm{pk})$ (pk fixes plaintext space $\mathcal{M}$ and ciphertext space $\mathcal{C}$).
- *Enc*(m, pk): This (probabilistic) algorithm takes a message $m \in \mathcal{M}$ and a public key pk and outputs a ciphertext $c \leftarrow \mathrm{Enc}(m, \mathrm{pk}) \in \mathcal{C}$.
- *Dec*(c, sk): This deterministic algorithm takes a ciphertext $c \in \mathcal{C}$ and a private key sk and outputs $m \leftarrow \mathrm{Dec}(c, \mathrm{sk}) \in M \cup \{\perp\}$.

Security

Correctness

$$\forall(\mathrm{sk}, \mathrm{pk}) \leftarrow \mathit{KeyGen}(1^{\kappa}) : \Pr\left[\mathrm{Dec}(\mathrm{Enc}(m, \mathrm{pk}), \mathrm{sk}) = m\right] = 1 - \epsilon(\kappa)$$

How to define when a scheme is secure?

- Given c and pk it should be hard to find m?
  - Very weak guarantees …
- We will gradually develop the idea of security for PKE

Overview of Target and Attacks

Targets (hardest to easiest)

- One-wayness (OW): hard to invert
- Semantically secure (Indistinguishable - (IND)): no information about the message in $\text{Enc}(m, pk)$ is leaked
- Non-malleable (NM): for any non-trivial relation R it is hard to compute $\text{Enc}(R(m), \text{pk})$ from $\text{Enc}(m, \text{pk})$

Overview of Target and Attacks

Attacks (weak to strong)

- Passive attacks: Chosen plaintext attack (CPA)
- Active attacks: Chosen ciphertext attacks (CCA)

Highest security guarantees if strongest attacker can not even achieve the weakest target: NM-CCA2 (IND-CCA2)

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Textbook RSA Encryption

Use a trapdoor one-way function for encryption (e.g., RSA, Rabin)

KeyGen(1^κ):	Pick two random κ-bit primes p, q, set $N = pq$, pick e s.t. $\gcd(e, \varphi(N)) = 1$, compute $d \leftarrow e^{-1} \bmod \varphi(N)$ output $(\mathsf{sk}, \mathsf{pk}) \leftarrow ((d, N), (e, N))$
Enc(m, pk):	On input $m \in \mathbb{Z}_N^*$ and $\mathsf{pk} = (e, N)$, compute and output $c \leftarrow m^e \pmod N$
Dec(c, sk):	On input c and $\mathsf{sk} = (d, N)$, compute and output $m \leftarrow c^d \bmod N$

Security of Textbook RSA

- How hard is it to recover m given c and $\mathrm{pk} = (e, N)$
  - This has been formalized as the RSA problem and is assumed to be hard
  - Assumes that c (and thus m) is a random element of $\mathbb{Z}_N$
  - Very strong assumption for a secure PKE
- Some of the problems of textbook RSA
  - RSA encryption is deterministic: Small message space $\mathcal{M}' \subseteq \mathcal{M}$, just test any $m \in \mathcal{M}'$
  - RSA encryption function is a homomorphism:

$$\mathrm{Enc}(m_0, \mathrm{pk}) \cdot \mathrm{Enc}(m_1, \mathrm{pk}) = m_0^e \cdot m_1^e = (m_0 \cdot m_1)^e = \mathrm{Enc}(m_0 \cdot m_1, \mathrm{pk})$$

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One-Wayness

One-Wayness

For all PPT adversaries $\mathcal{A}$ and security parameters κ there is a negligible function ϵ such that:

$$\Pr\left[\begin{array}{c}(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(1^\kappa), m \xleftarrow{R} \mathcal{M}, \\ m^* \leftarrow \mathcal{A}(\mathsf{pk}, \mathsf{Enc}(m, \mathsf{pk})) : m^* = m\end{array}\right] \leq \epsilon(\kappa).$$

- Not meaningful for most applications of PKE (but okay for RSA-KEM)
- $\mathcal{A}$ may still compute some information about m
- How to formalize "does not leak any information"?

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More Powerful Passive Adversaries (CPA)

Experiment $\mathbf{Exp}^{\mathsf{IND-CPA}}_{\Pi,\mathcal{A}}(\cdot)$:

Adversary $\mathcal{A}$		Challenger $\mathcal{C}$
	$\xleftarrow{\mathsf{pk}}$	$(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(1^\kappa)$
	$\xrightarrow{(m_0, m_1)}$	$b \xleftarrow{R} \{0, 1\}$
	$\xleftarrow{c^*}$	$c^* \leftarrow \mathsf{Enc}(m_b, \mathsf{pk})$ 
	$\xrightarrow{b^*}$	if $b^* = b$ return 1; else return 0;

IND-CPA Security

- We can define the advantage of adversary $\mathcal{A}$ for the IND-CPA experiment for scheme Π as

$$\mathbf{Adv}^{\mathsf{IND-CPA}}_{\Pi,\mathcal{A}}(\kappa) = \left| \Pr[\mathbf{Exp}^{\mathsf{IND-CPA}}_{\Pi,\mathcal{A}}(\kappa) = 1] - \frac{1}{2} \right|.$$

- For a secure scheme, the advantage is negligible as a function of κ for any PPT $\mathcal{A}$

IND-CPA

For all PPT adversaries $\mathcal{A}$ and security parameters κ there is a negligible function ϵ such that:

$$\Pr\left[\begin{array}{r}(\mathsf{pk},\mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\kappa), ((m_0,m_1),\mathsf{state}) \leftarrow \mathcal{A}(\mathsf{pk}),\\ b \leftarrow \{0,1\}, c \leftarrow \mathsf{Enc}(m_b,\mathsf{pk}), b^* \leftarrow \mathcal{A}(\mathsf{state},c) : b^* = b\end{array}\right] \leq \frac{1}{2} + \epsilon(\kappa).$$

IND-CPA Security With RSA

- Textbook RSA is obviously not IND-CPA secure
  - It is deterministic: $\mathcal{A}$ simply computes $c' \leftarrow \mathrm{Enc}(m_0, \mathrm{pk})$ and outputs 0 if $c' = c$ and 1 otherwise
  - Leaks Jacobi symbol
  - If $e = 3$ and $m < N^{\frac{1}{3}}$, then $m = c^{\frac{1}{3}}$ (in the integers)
- No deterministic PKE scheme can be IND-CPA secure: encryption has to be randomized

IND-CPA Security With RSA

- Hard-core bit for RSA and IND-CPA security
  - Modify RSA assumption to output z such that z is least significant bit (LSB) of m
  - If you can compute LSB, then you can invert RSA
    - LSB is hardest bit to compute in RSA (a hard-core bit)
    - Can be used for encryption, but inefficient (bitwise)

$$\mathrm{Enc}(m, \mathrm{pk}) := (LSB(x) \oplus m, x^e \bmod N) \text{ for } m \in \{0,1\} \text{ and } x \xleftarrow{R} \mathbb{Z}_N$$

$$\mathrm{Dec}((c_1, c_2), \mathrm{sk}) := LSB(c_2^d \bmod N) \oplus c_1$$

IND-CPA Security With RSA

- More efficiency with random oracles (RSA-CPA)
  - Let $H : \mathbb{Z}_N \to \{0,1\}^\ell$ be a random oracle

$$\mathrm{Enc}(m, \mathrm{pk}) := (H(x) \oplus m, x^e \bmod N) \text{ for } m \in \{0,1\}^\ell \text{ and } x \xleftarrow{R} \mathbb{Z}_N$$

$$\mathrm{Dec}((c_1, c_2), \mathrm{sk}) := H(c_2^d \bmod N) \oplus c_1$$

- IND-CPA secure in the random oracle model

RSA-CPA IND-CPA Proof Idea

- To obtain information about m from $(H(x) \oplus m, x^e \pmod N)$, one has to learn some information about $H(x)$
- As H is a random oracle, the only way to learn any information about $H(x)$ is to evaluate H at x
- An adversary who learns anything about m thus knows x
- The adversary thus can break the RSA assumption
- If adversary does not query $H(x)$, then challenge ciphertext c is independent from m_b

Concrete vs. Asymptotic Security

- Asymptotic security does not care about the runtime of the reduction (as long as polynomial time)
- Concrete security relates runtime t and success probability ϵ of adversary to t' and ϵ' of reduction
- Reduction is tight if $\epsilon \approx \epsilon'$ and $t \approx t'$ ($\frac{t'}{\epsilon'} \approx \frac{t}{\epsilon}$)
- Non-tight if $t \ll t'$ or if $\epsilon \gg \epsilon'$ (tightness gap is $\frac{t'\epsilon}{t\epsilon'}$)
  - $\frac{t'}{\epsilon'} \geq q_{\mathcal{O}} \cdot \frac{t}{\epsilon}$, where $\mathcal{O}$ is some oracle (RO, signing, etc.)
- Tightness relates security of the scheme to the problem

RSA-CPA IND-CPA Proof

Theorem

If there exists an (t, q_H, ϵ) IND-CPA adversary against RSA-CPA, then there is a (t', ϵ') solver for the RSA assumption with $\epsilon' \geq 2\epsilon$ and $t' \leq t + (q_H^2 + q_H \cdot t_{exp})$.

Proof (by Reduction):

- Reduction $\mathcal{B}$ obtains RSA challenge (e, y, N) (want to find x s.t. $y \equiv x^e \pmod N$)
- $\mathcal{B}$ runs $\mathcal{A}$ on pk $= (e, N)$ and obtains challenge (m_0, m_1)
- $\mathcal{B}$ gives ciphertext (r, y) to $\mathcal{A}$ for $r \xleftarrow{R} \{0,1\}^\ell$ and RSA challenge y (as long as x s.t. $y \equiv x^e \pmod N$) not queried to H, challenge ciphertext information theoretically hidden)

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RSA-CPA IND-CPA Proof (ctd.)

Simulation of the random oracle H (maintainig a list Q of tuples (x_i, h_i) - initially empty)

$\underline{H(x_j)}$

```
z ← x_j^e (mod N)
if z = y then
output x_j and B aborts // solved RSA challenge
else
    if x_j in Q then
    return h_j
    else
    h_j ←R {0,1}^ℓ
    store (x_j, h_j) in Q
    return h_j
    end if
end if
```

RSA-CPA IND-CPA Proof (ctd.)

- W event that $\mathcal{A}$ wins the IND-CPA game (with prob. $\frac{1}{2} + \epsilon$); Q event that $\mathcal{A}$ queries $H(x)$ s.t. $y = x^e \pmod{N}$.

$$\begin{aligned}
\Pr[W] &= \Pr[W|Q] \cdot \Pr[Q] + \Pr[W|\neg Q] \cdot \Pr[\neg Q] \\
&\leq \Pr[Q] + \frac{1}{2} \cdot \Pr[\neg Q] \\
&= \Pr[Q] + \frac{1}{2}(1 - \Pr[Q]) \\
&= \frac{1}{2} + \frac{1}{2} \cdot \Pr[Q] \\
\frac{1}{2} + \epsilon &\leq \frac{1}{2} + \frac{1}{2} \cdot \Pr[Q] \\
2\epsilon &\leq \underbrace{\Pr[Q]}_{\leq \epsilon'}.
\end{aligned}$$

- If ϵ non-negl., so is ϵ'; contradicting RSA assumption. $\square$
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Tightness of the Reduction

- RSA-CPA has a tight reduction (additive factor)
- RSA-FDH has no tight reduction (multiplicative factor)
  - Bellare et al.'s proof looses a factor of q_H
  - Coron's proof only looses a factor of q_S
  - It is often assumed that $q_H \leq 2^{60}$ and $q_S \leq 2^{30}$
  - Assume RSA with 80 bit security: 1248 bit modulus (ECRYPT II)
  - To obtain this security level w.r.t. Bellare et al.'s analysis we at least require 4000 bit RSA!

Encrypting With Diffie-Hellman

- Let $\mathbb{G}$ be a group of prime order p and g a generator
- No trapdoor known to invert discrete exponentiation function
- How to encrypt?

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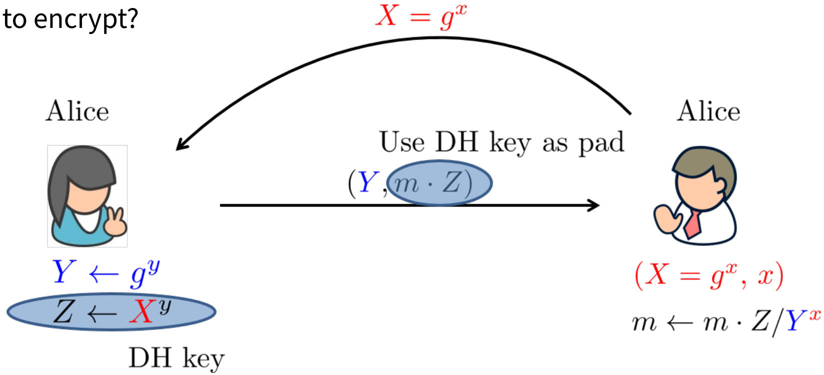


ElGamal Encryption Scheme

ElGamal

KeyGen(1^κ): Pick group $\mathbb{G} = \langle g \rangle$ with $|\mathbb{G}| = p \approx 2^\kappa$ prime, pick $x \xleftarrow{R} \mathbb{Z}_p$ and output $(\mathrm{sk}, \mathrm{pk}) \leftarrow (x, X = g^x)$

Enc(m, pk): Let $m \in \mathbb{G}$, pick $y \xleftarrow{R} \mathbb{Z}_p$ and output $(c_1, c_2) \leftarrow (g^y, m \cdot X^y)$

Dec(c, sk): Let $c = (c_1, c_2)$, compute and output $m \leftarrow c_2 / c_1^x$

ElGamal Encryption Scheme (ctd.)

Recall: DDH Assumption

Let $\mathbb{G} = \langle g \rangle$ with $|\mathbb{G}| = p$ prime, $\log_2 p = \kappa$, then $\forall$ PPT $\mathcal{A}$

$$|\Pr[x, y \xleftarrow{R} \mathbb{Z}_p : \mathcal{A}(g^x, g^y, g^{xy}) = 1] -$$
$$\Pr[x, y, z \xleftarrow{R} \mathbb{Z}_p : \mathcal{A}(g^x, g^y, g^z) = 1]| \leq \epsilon(\kappa)$$

and let us denote this probability as $\mathbf{Adv}^{\mathsf{DDH}}_{\mathbb{G},g,p}(\mathcal{A})$.

Theorem

If DDH assumption holds, ElGamal is IND-CPA.

ElGamal IND-CPA Proof

Proof (by Game Hopping):

In future Lecture!

Need to first look at constructing proofs via Game Hopping!

- Basic Idea:
  - Write down scheme as algorithm
  - Transform algorithm by changing it slightly
    - Argue that Adversary cannot distinguish between old and new algorithm
  - Repeat until we arrive at one algorithm that cannot be broken
    - e.g., it does not have access to the secret key at all

ElGamal and DDH

- Careful with choice of groups
  - In $\mathbb{Z}_p^*$ for p prime DDH is not hard (take prime order q subgroup, e.g., $p = 2q + 1$)
  - In symmetric pairings no DDH; in the XDH setting DDH is not hard in $\mathbb{G}_2$
- Can switch to Linear ElGamal (DLIN)

DLIN Assumption

Let $\mathbb{G}$ with $|\mathbb{G}| = p$, $\log_2 p = \kappa$ and $u, v, h \in \mathbb{G}$, then $\forall$ PPT $\mathcal{A}$

$$|\Pr[x, y \xleftarrow{R} \mathbb{Z}_p : \mathcal{A}(u^x, v^y, h^{x+y}) = 1] -$$
$$\Pr[x, y, z \xleftarrow{R} \mathbb{Z}_p : \mathcal{A}(u^x, v^y, h^z) = 1]| \leq \epsilon(\kappa)$$

Linear ElGamal

Linear ElGamal

KeyGen(1^κ): Pick group $\mathbb{G} = \langle g \rangle$ with $|\mathbb{G}| = p \approx 2^\kappa$ prime, pick $u, v \xleftarrow{R} \mathbb{Z}_p$, $h \xleftarrow{R} \mathbb{G}$, set $(U, V, h) \leftarrow (h^{1/u}, h^{1/v}, h)$ and output $(\mathrm{sk}, \mathrm{pk}) \leftarrow ((u, v), (U, V, h))$

Enc(m, pk): Let $m \in \mathbb{G}$, pick $y, z \xleftarrow{R} \mathbb{Z}_p$ and output $(c_1, c_2, c_3) \leftarrow (U^y, V^z, m \cdot h^{y+z})$

Dec(c, sk): Let $c = (c_1, c_2, c_3)$, compute and output $m \leftarrow c_3/(c_1^u \cdot c_2^v)$

Theorem

If DLIN assumption holds, Linear ElGamal is IND-CPA.

Problems With IND-CPA Security

Malleability: Adversary may change a ciphertext such that plaintexts are related

- RSA-CPA:

$$(H(x) \oplus m \oplus m', x^e \bmod N)$$

- ElGamal:

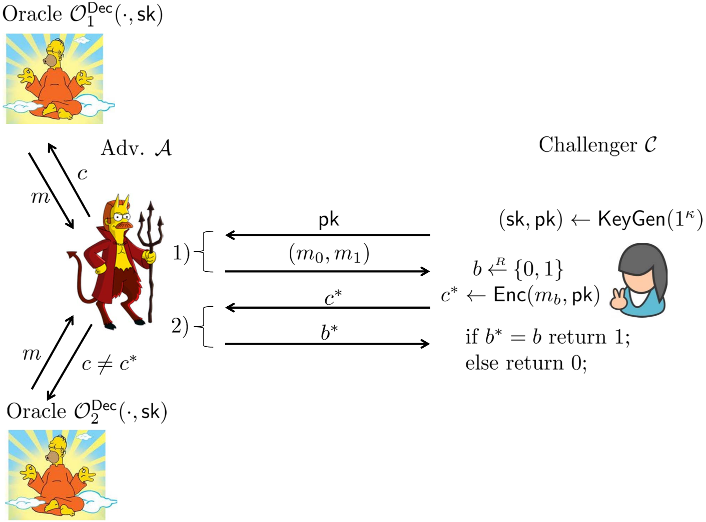
$$(g^{y_0}, m_0 X^{y_0}) \star (g^{y_1}, m_1 X^{y_1}) = (g^{y_0+y_1}, (m_0 \cdot m_1) X^{y_0+y_1})$$

- Sometimes desired (computing on encrypted data)
- Sometimes problematic (e.g., Bleichenbacher)

Active Adversaries (CCA)

- How to formalize malleability (NM)? Dolev et al. have done this back in 1993 with a simulation-based notion
- Bellare et al. have shown that NM implies the IND notion
  - The strongest notion NM-CCA2 is equivalent to IND-CCA2
  - IND notion is more convenient to use
- Idea of a stronger IND notion
  - Give the adversary access to a decryption oracle
- IND-CCA2 automatically yields security in universal composabability (UC) framework

Active Adversaries (CCA)



Active Adversaries (CCA)

- IND-CCA1 (lunchtime attacks)
  - $\mathcal{A}$ only access to $\mathcal{O}_1^{\mathsf{Dec}}$ (before seeing the challenge ciphertext)
  - Best we can get for homomorphic schemes (within our notions)
- IND-CCA2
  - $\mathcal{A}$ also has access to $\mathcal{O}_2^{\mathsf{Dec}}$ (after seeing the challenge)
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IND-CCA2 Schemes?

- In the random oracle model
  - RSA-OAEP(+) (RSA)
  - Hybrid ElGamal (strong DH)
  - Twin Hybrid ElGamal (DH)
- Without random oracles
  - Hash proof system (e.g., Cramer-Shoup)
  - From CPA secure IBE schemes
  - Twin-encryption using non-interactive zero-knowledge
- Conversion from IND-CPA (e.g., Fujisaki-Okamoto)

Theory vs. Practice

- Many "pure" PKES not useful for content encryption
- Use of hybrid encryption (KEM/DEM) approach
  - Combine PKE with symmetric encryption (and MAC or RO)
  - Generic conversions follow this approach
- Plain PKE schemes still very useful
  - Homomorphic encryption
  - Threshold encryption
  - Zero-knowledge proofs of knowledge (plaintext equality, inequality)

Relation Among Notions

$$
\begin{tikzcd}
\text{NM-CPA} \arrow[d] \arrow[dr, shift left, "/" marking] & \text{NM-CCA1} \arrow[l] \arrow[d] \arrow[r, shift right, "/" marking] & \text{NM-CCA2} \arrow[l, shift right] \arrow[d, shift right] \\
\text{IND-CPA} & \text{IND-CCA1} \arrow[l] \arrow[ul, shift left, "/" marking] & \text{IND-CCA2} \arrow[l] \arrow[u, shift right]
\end{tikzcd}
$$

Additional Security Notions for PKE

- Replayable CCA (RCCA)
  - IND-CPA too weak and IND-CCA2 often too strong
  - Capture schemes that are CCA2-secure except for allowing re-randomization of ciphertexts
    - Altering ciphertext is "ok", if it decrypts to original message
- Circular Security
  - Encrypt a secret key under the corresponding public key (1-cycle)
  - Important for fully homomorphic encryption (bootstrapping)
    - Homomorphically evaluating decryption function on ciphertext (use encryption of secret key)

Additional Security Notions for PKE

- Key Dependent Message (KDM) security
  - Generalization of circular security
  - Encrypted messages might depend on arbitrary function of secret keys
- Security under leakage
  - Leak a bounded number of bits of secret key
  - Leak an adversarially chosen function of secret key
  - ...

What you should know...

- Security models for PKE
  - Active and passive adversaries
  - “Games” for different Adversaries
- Asymptotic vs. concrete security
- Basic Idea of (Random) Oracles

Questions?

Further Reading I

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Relations Among Notions of Security for Public-Key Encryption Schemes.

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Efficient identity-based encryption without random oracles.

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