

Deduction Rules

Propositional Logic

	Introduction	Elimination
\wedge	$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \wedge_i$	$\frac{\varphi \wedge \psi}{\varphi} \wedge_{e1} \quad \frac{\varphi \wedge \psi}{\psi} \wedge_{e2}$
\vee	$\frac{\varphi}{\varphi \vee \psi} \vee_{i1} \quad \frac{\psi}{\psi \vee \varphi} \vee_{i2}$	$\frac{\varphi \vee \psi \quad \begin{array}{ c } \hline \varphi \text{ ass.} \\ \vdots \\ \chi \\ \hline \end{array} \quad \begin{array}{ c } \hline \psi \text{ ass.} \\ \vdots \\ \chi \\ \hline \end{array}}{\chi} \vee_e$
\rightarrow	$\frac{\begin{array}{ c } \hline \varphi \text{ ass.} \\ \vdots \\ \psi \\ \hline \end{array}}{\varphi \rightarrow \psi} \rightarrow_i$	$\frac{\varphi \quad \varphi \rightarrow \psi}{\psi} \rightarrow_e$
\neg	$\frac{\begin{array}{ c } \hline \varphi \text{ ass.} \\ \vdots \\ \perp \\ \hline \end{array}}{\neg \varphi} \neg_i$	$\frac{\varphi \quad \neg \varphi}{\perp} \neg_e$
\perp	no rule	$\frac{\perp}{\varphi} \perp_e$
$\neg\neg$	$\frac{\varphi}{\neg\neg\varphi} \neg\neg_i$	$\frac{\neg\neg\varphi}{\varphi} \neg\neg_e$

Derived Rules

$\frac{\varphi \vee \neg \varphi}{\text{LEM}}$	$\frac{\begin{array}{ c } \hline \neg \varphi \text{ ass.} \\ \vdots \\ \perp \\ \hline \end{array}}{\varphi} \text{PBC}$	$\frac{\varphi \rightarrow \psi \quad \neg \psi}{\neg \varphi} \text{MT}$
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Predicate Logic

	Introduction	Elimination
\forall	$\frac{\begin{array}{ l} x_0 \\ \vdots \\ \varphi[x_0/x] \end{array} \quad x_0 \text{ fresh}}{\forall x \varphi} \quad \forall_i$	$\frac{\forall x \varphi}{\varphi[t/x]} \quad \forall_e$
\exists	$\frac{\varphi[t/x]}{\exists x \varphi} \quad \exists_i$	$\frac{\begin{array}{ l} x_0 \\ \varphi[x_0/x] \text{ ass.} \\ \vdots \\ \chi \end{array} \quad x_0 \text{ fresh}}{\chi} \quad \exists_e$