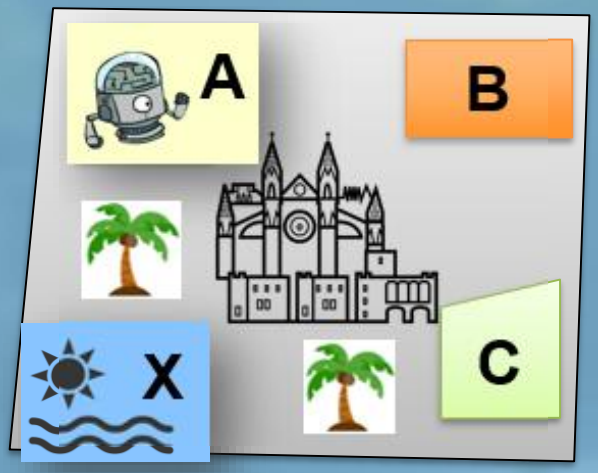
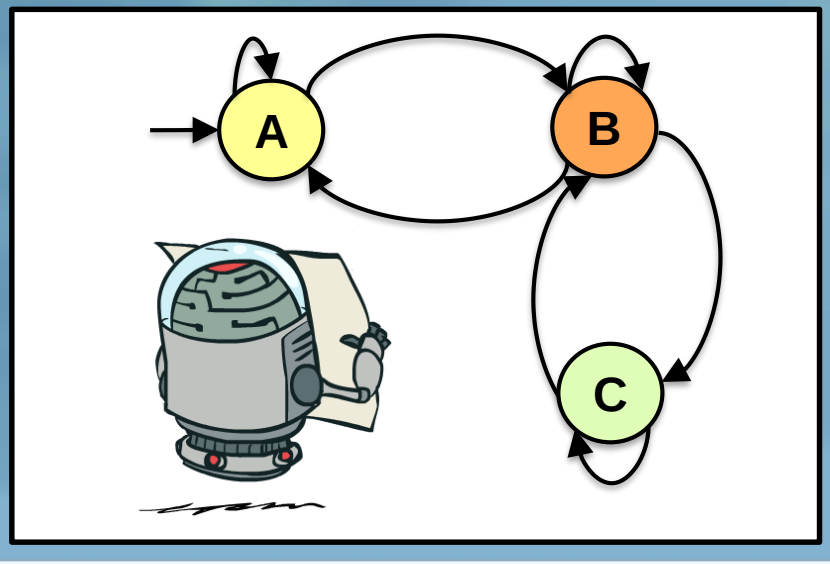
Graz University of Technology
Institute for Applied Information
Processing and Communications

Automata and LTL Model Checking

Bettina Könighofer





Model Checking SS24

May 06th 2024

Model Checking of LTL

given an LTL property φ and a Kripke structure M

check whether $M \models \varphi$

Model Checking of LTL

given an LTL property φ and a Kripke structure M
check whether M $\models \varphi$

1. Construct $\neg\varphi$
2. Construct a **Büchi** automaton $\mathcal{S}_{\neg\varphi}$
3. Translate M to an automaton $\mathcal{A}$.
4. Construct the automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\mathcal{S}_{\neg\varphi})$
5. If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies φ
6. Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
   - a computation in M that does not satisfy φ

Plan for Today

- LTL-Model Checking via Automata-Theoretic Approach
  - Basic facts of automata theory
  - Represent system models and specifications via automata
  - Model-checking algorithm

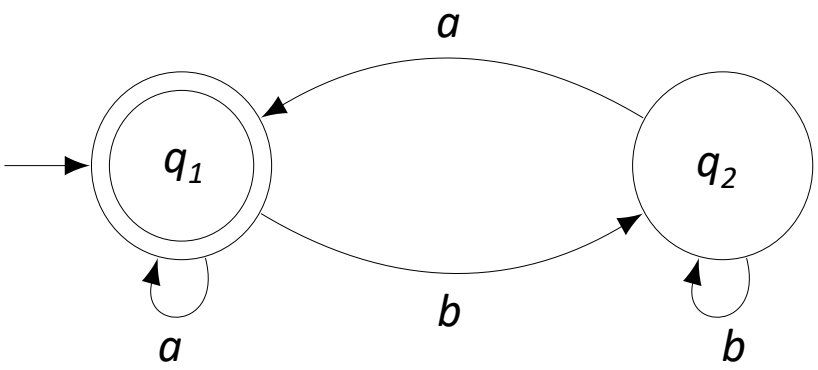
Outline

- Finite automata on finite words
- Automata on infinite words (Büchi automata)
- Deterministic vs non-deterministic Büchi automata
- Intersection of Büchi automata
- Checking emptiness of Büchi automata
- Automata and Kripke Structures
- Model checking using automata
- Generalized Büchi automata
- Translation of LTL to Büchi automata

Finite Automata on **Finite** Words

Regular Automata

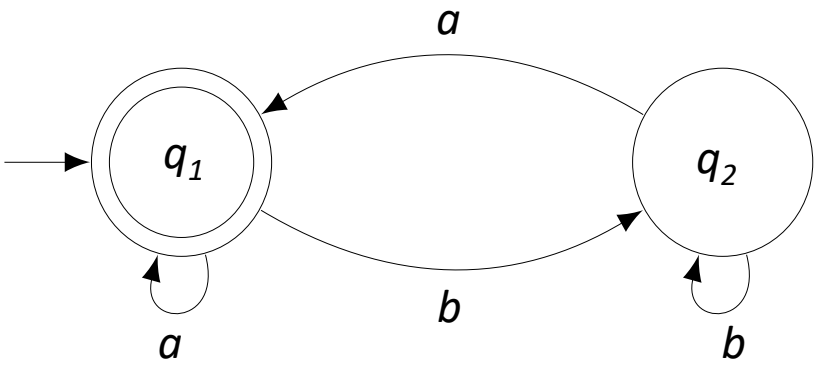
- $\mathcal{A} = (\Sigma, Q, \Delta, Q^0, F)$
- Σ is the finite alphabet
- Q is the finite set of states
- $\Delta \subseteq Q \times \Sigma \times Q$ is the transition relation
- Q^0 is the set of initial states
- F is the set of accepting states



Finite Automata on Finite Words

Regular Automata

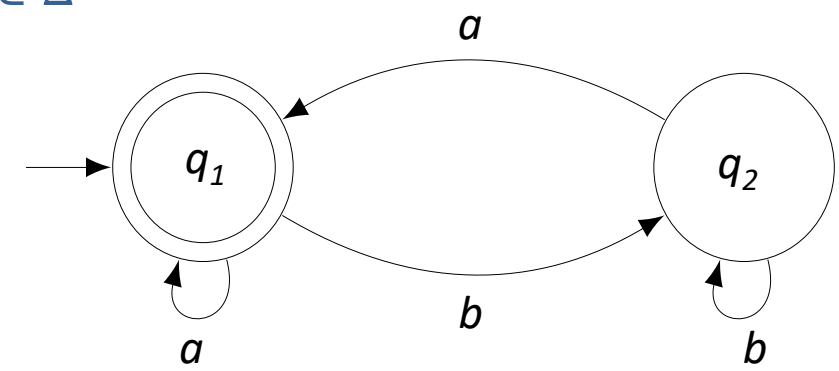
- Example: $\mathcal{A} = (\mathbf{\Sigma}, \mathbf{Q}, \mathbf{\Delta}, \mathbf{Q}^0, \mathbf{F})$
- $\mathbf{\Sigma} = \{a, b\}$
- $\mathbf{Q} = \{q_1, q_2\}$
- $\mathbf{\Delta} = \{ (q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_1), (q_2, b, q_2)\},$
- $\mathbf{Q}^0 = \{q_1\}$
- $\mathbf{F} = \{q_1\}$



Words and Runs on Finite Automata

- A **word** v is a string (sequence) in $\Sigma^\star$ of length $|v|$
- A **run** ρ is a path in the graph of $\mathcal{A}$.

- Given a **word** $v = a_1, a_2, \ldots, a_n$ and automaton $\mathcal{A}$
- A **run** $\rho = q_0, q_1, \ldots q_n$ of $\mathcal{A}$ over v is a sequence of states s.t.
  - $q_0 \in \mathbf{Q}^0$
  - for all $0 \leq i \leq n-1$, $(q_i, a_{i+1} q_{i+1}) \in \Delta$

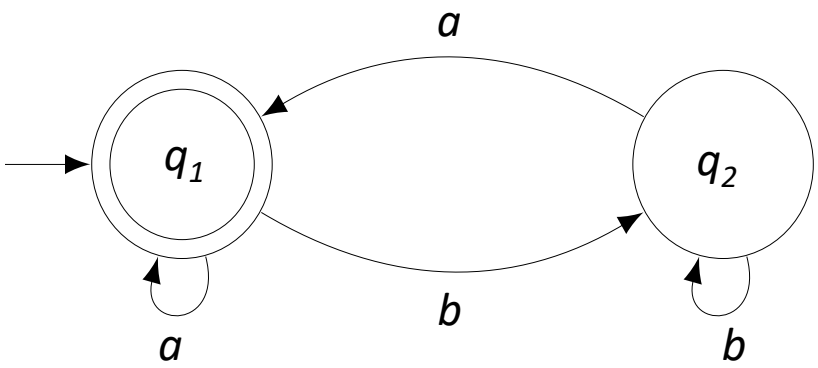


Accepting Words and Runs

- A run $\boldsymbol{\rho} = q_0, q_1, \ldots q_n$ is accepting $\Leftrightarrow q_n \in \mathbf{F}$
- $\mathcal{A}$ accepts a word $v = a_1, a_2, \ldots, a_n \Leftrightarrow$
  if there is a corresponding accepting run $\boldsymbol{\rho}$ (i.e., $q_n \in \mathbf{F}$)

What words does $\mathcal{A}$ accept?

$\mathcal{L}(\mathcal{A})$ = {the empty word} ∪
{all words that end with a}
= $\{\varepsilon\} \cup \{a, b\}^* a$

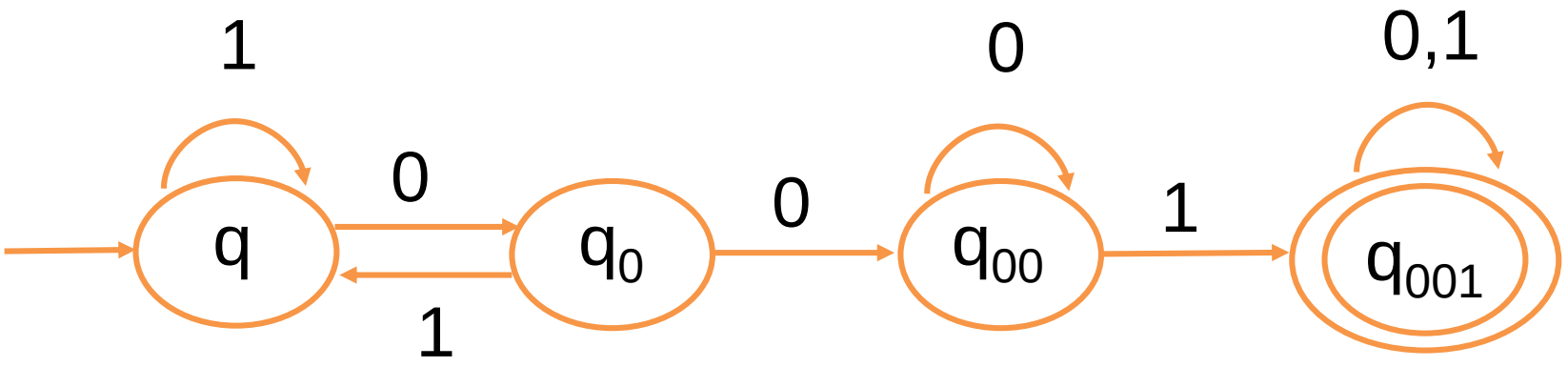


Languages on Finite Automata

- Language of $\mathcal{A}$
  - $\mathcal{L}(\mathcal{A}) \subseteq \Sigma^*$, is the set of words that $\mathcal{A}$ accepts.
- Languages accepted by finite automata are ***regular languages***.

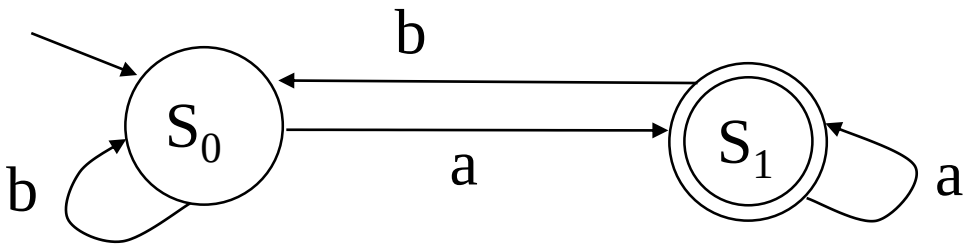
Finite Automata on Finite Words
Regular Automata

Build an automaton that accepts all and only those strings that contain 001.

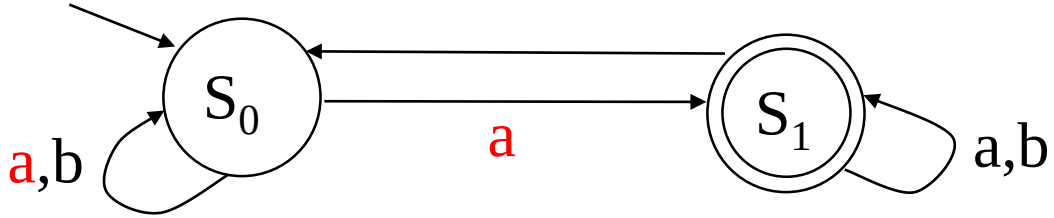


Deterministic & Non-Deterministic Automata

- $\mathcal{A}$ is deterministic if Δ is a function (one output for each input).
  - $\forall q \in Q, \forall a \in \Sigma: |\Delta(q,a)| \leq 1$, and
  - $|Q^0| = 1$
- Det. automata have exactly one run for each word.



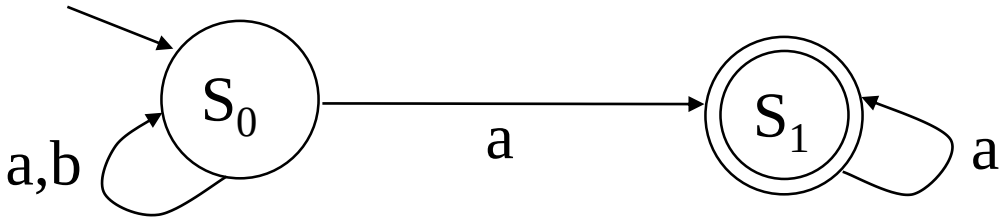
- Non-det. automata
  - Can have transitions $(q,a,q'),(q,a,q'') \in \Delta$ and $q'' \neq q'$
  - Can have ε-transitions (transitions without a letter)



Language of an NFA

- NFA (Nondeterministic Finite Automata) accepts all words that have a run that ends in an accepting state
- What is the language of this automaton?

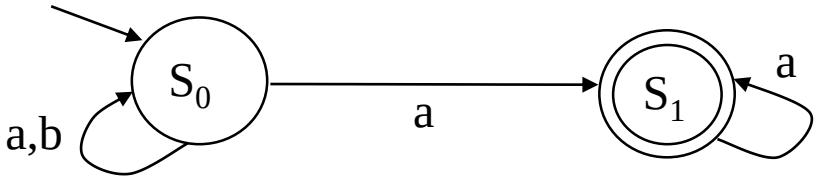
$\mathcal{L}(\mathcal{A})$ = {all words that end with a}



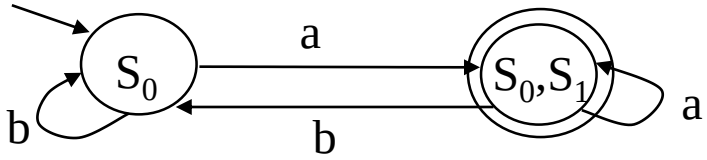
NFA on finite words to DFA

- Any non-deterministic finite automata on *finite words* can be translated into an equivalent deterministic automaton.

Non-deterministic automaton $\mathcal{A}$



Equivalent Det. automaton $\mathcal{A}$'



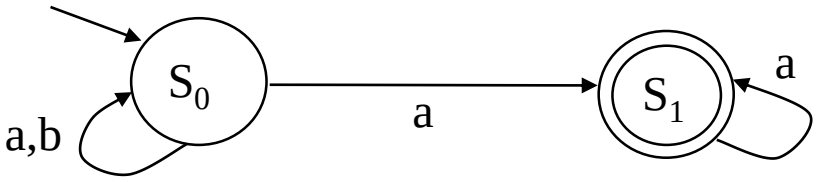
Equivalent Deterministic Automaton

- Algorithm: **Subset-Construction (exponential blow-up)**
  - NFA: $\mathcal{A} = (\Sigma, Q, \Delta, Q^0, F)$
  - DFA: $\mathcal{A}' = (\Sigma, P(Q), \Delta', \{Q^0\}, F')$ such that

P … Powerset

Each state in $\mathcal{A}'$ corresponds to a set of states that $\mathcal{A}$ after reading some input sequence

Non-deterministic automaton $\mathcal{A}$



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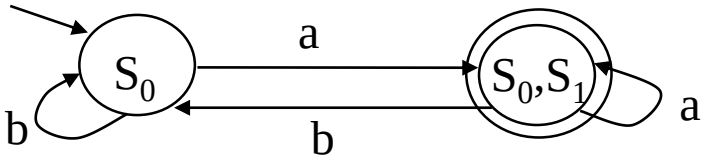

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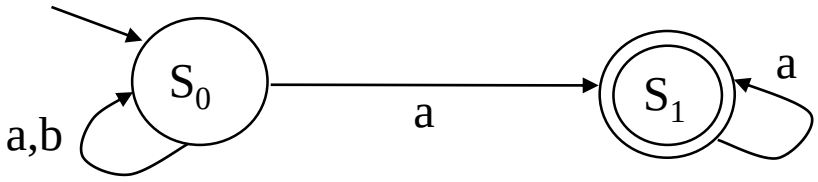
  - DFA: $\mathcal{A}' = (\mathbf{\Sigma}, \mathrm{P}(\mathbf{Q}), \mathbf{\Delta}', \{\mathbf{Q}^0\}, \mathbf{F}')$ such that
    - $\mathbf{\Delta}': P(Q) \times \Sigma \rightarrow P(Q)$ where $(Q_1, a, Q_2) \in \Delta'$ if

$$Q_2 = \bigcup_{q \in Q_1} \{q' | (q, a, q') \in \Delta\}$$

Example: $(\{s_0, s_1\}, a, \{s_0, s_1\}) \in \Delta'$ since
- $(s_0, a, s_0) \in \Delta$ and $(s_0, a, s_1) \in \Delta$
- $(s_1, a, s_0) \in \Delta$

Non-deterministic automaton $\mathcal{A}$



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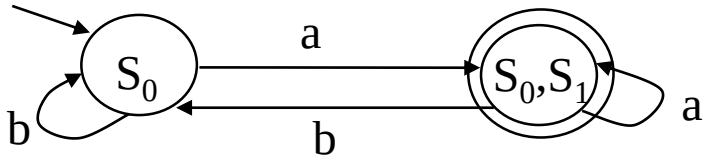

Equivalent Deterministic Automaton

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Example: $(\{s_0, s_1\}, b, \{s_0\}) \in \Delta'$ since
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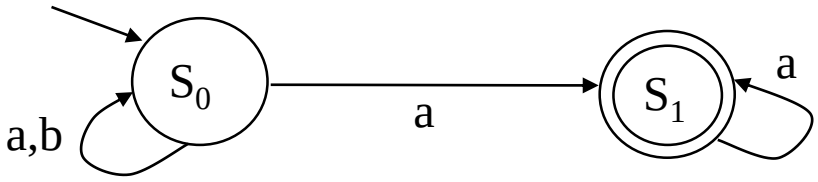
Non-deterministic automaton $\mathcal{A}$



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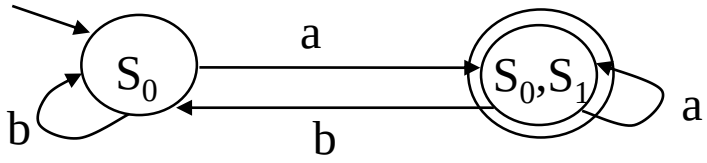

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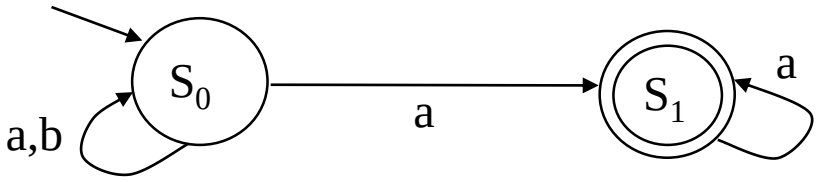
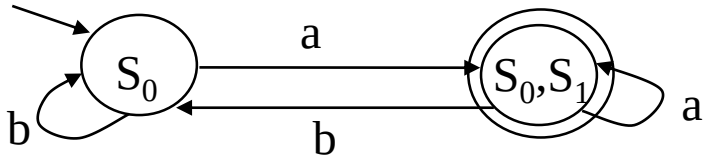
$$Q_2 = \bigcup_{q \in Q_1} \{q' | (q, a, q') \in \Delta\}$$

    - $F' = \{Q' | Q' \cap F \neq \emptyset\}$

Non-deterministic automaton $\mathcal{A}$



Equivalent Det. automaton $\mathcal{A}'$



Equivalent Deterministic Automaton

- Compute the equivalent DFA
  - $\mathcal{A}$' = (Σ, P($\mathbf{Q}$),Δ',{$\mathbf{Q}^0$},$\mathbf{F}$') such that
    - Δ': $P(Q) \times \Sigma \rightarrow P(Q)$ where $(Q_1, a\ , Q_2) \in \Delta'$ if

$$Q_2 = \bigcup_{q \in Q_1} \{q' | (q, a, q') \in \Delta\}$$

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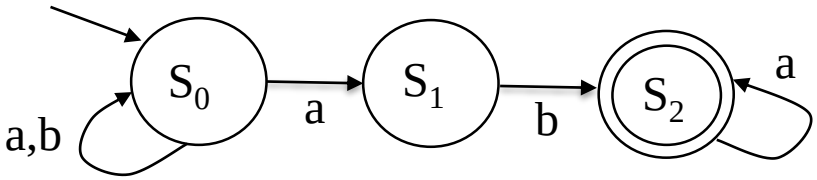
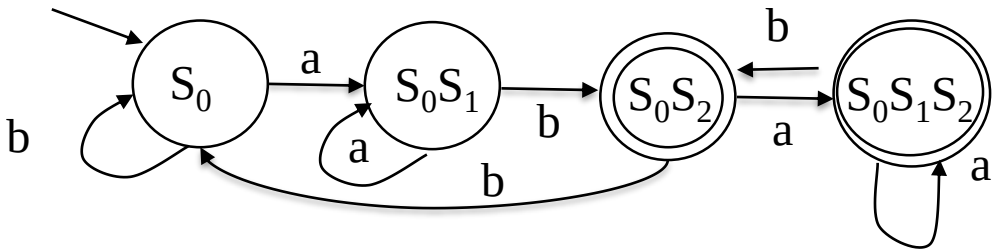
Non-deterministic automaton $\mathcal{A}$



Equivalent Det. automaton $\mathcal{A}$'



Equivalent deterministic automaton

- Compute the equivalent DFA
  - $\mathcal{A}$' = (Σ, P($\mathbf{Q}$),Δ',{$\mathbf{Q}^0$},$\mathbf{F}$') such that
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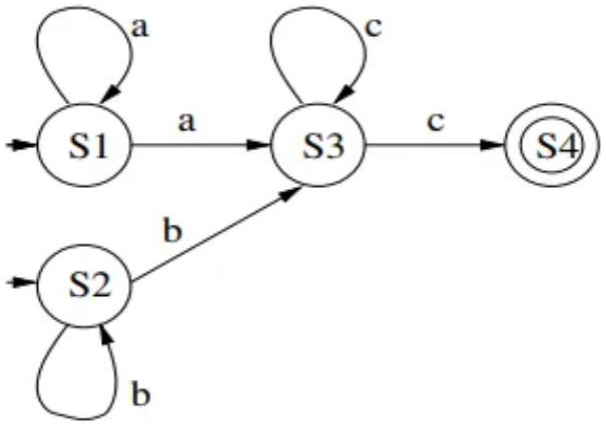
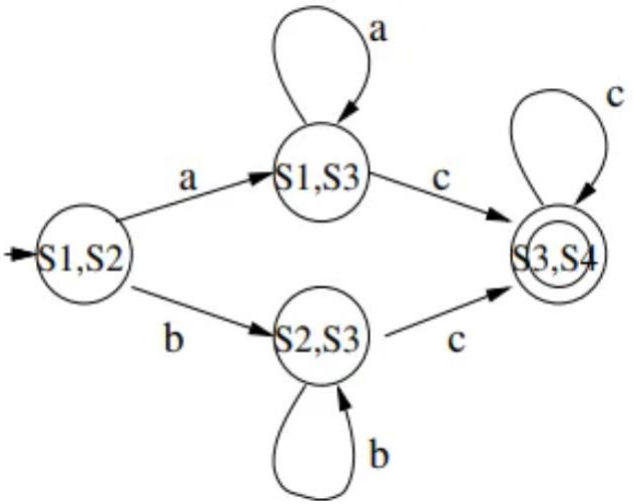
$$Q_2 = \bigcup_{q \in Q_1} \{q' | (q, a, q') \in \Delta\}$$

    - $F' = \{Q' | Q' \cap F \neq \emptyset\}$

Non-deterministic automaton $\mathcal{A}$



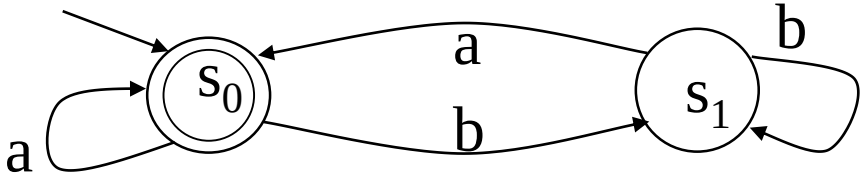
Equivalent Det. automaton $\mathcal{A}$'



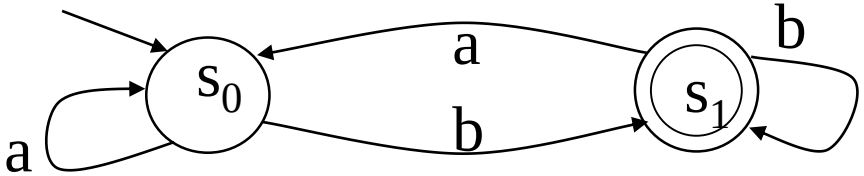
Complement of DFA

- The complement automaton $\overline{\mathcal{A}}$ accepts exactly those words that are rejected by $\mathcal{A}$
- Construction of $\overline{\mathcal{A}}$
  - Substitution of accepting and non-accepting states

$\mathcal{A}$

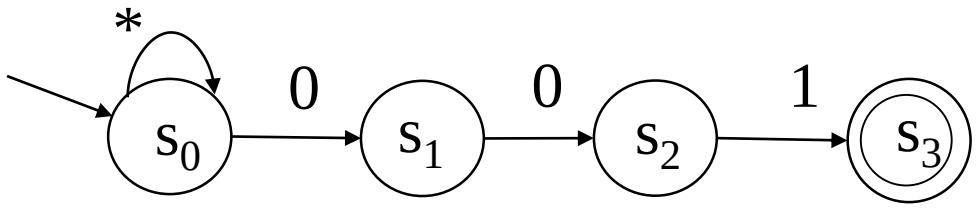


$\overline{\mathcal{A}}$



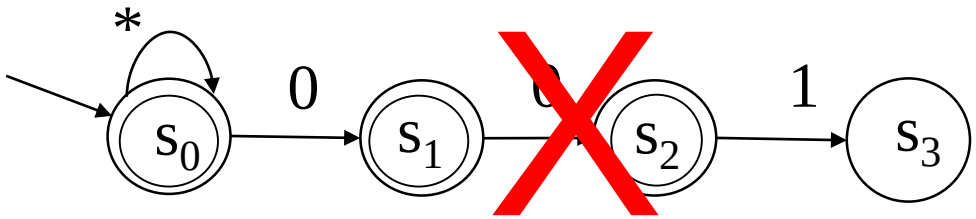
Consider NFA that accepts words that end with 001

$\mathcal{A}$



Let's try switching accepting and non-accepting states:

$\overline{\mathcal{A}}$



The language of this automaton is {0,1}* - this is wrong!

Complement of NFA

- The complement automaton $\overline{\mathcal{A}}$ accepts exactly those words that are rejected by $\mathcal{A}$
- Construction of $\overline{\mathcal{A}}$
  1. Determinization: Convert NFA to DFA
  2. Substitution of accepting and non-accepting states

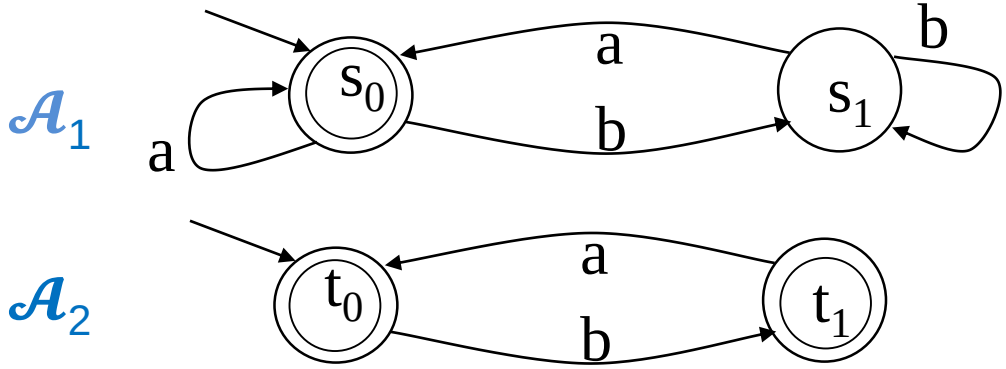
Intersections of NFAs

- Given two languages, L_1 and L_2, the intersection of L_1 and L_2 is

$$L_1 \cap L_2 = \{ w \mid w \in L_1 \; and \; w \in L_2 \}$$

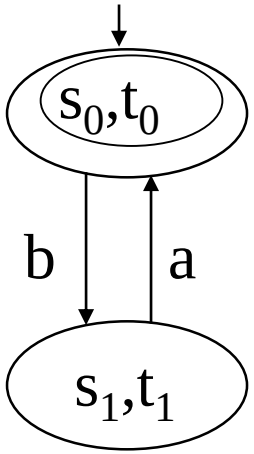
- Product automaton of $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2$ has $L(\mathcal{A}) = L(\mathcal{A}_1) \cap L(\mathcal{A}_2)$
  - $Q = Q_1 \times Q_2$ (Cartesian product),
  - $\Delta((q_1, q_2), a) = (\Delta_1(q_1, a), \Delta_2(q_2, a))$
  - $Q^0 = Q_1^0 \times Q_2^0$
  - $(q_1, q_2) \in F$ *iff* $q_1 \in F_1$ *and* $q_2 \in F_2$

Intersections of NFAs



$$\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2$$

1. States: (s_0,t_0), (s_0,t_1), (s_1,t_0), (s_1,t_1).
2. Initial state: (s_0,t_0).
3. Accepting states: (s_0,t_0), (s_0,t_1).



Outline

- Finite automata on finite words
- **Automata on infinite words (Büchi automata)**
- Deterministic vs non-deterministic Büchi automata
- Intersection of Büchi automata
- Checking emptiness of Büchi automata
- Automata and Kripke Structures
- Model checking using automata
- Generalized Büchi automata
- Translation of LTL to Büchi automata

Automata on **Infinite** Words

- We are interested in reactive systems
  - Designed to not hold during normal execution
- Computations are infinite sequences
  - Words are $v \in \Sigma^{\omega}$, where ω denotes infinitely many (i.e., $|v| = \infty$)
- Languages accepted by finite automata on infinite words are called ***ω-regular languages*.**
- Büchi Automata → Simplest automata over infinite words

Automata on **Infinite** Words (Büchi)

$$\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$$

- An infinite run ρ is accepting $\Leftrightarrow$ it visits an accepting state an infinite number of times.
  - $inf(\rho)$ … set of states in ρ that appear infinitely often
  - $\inf(\rho) \cap F \neq \emptyset$

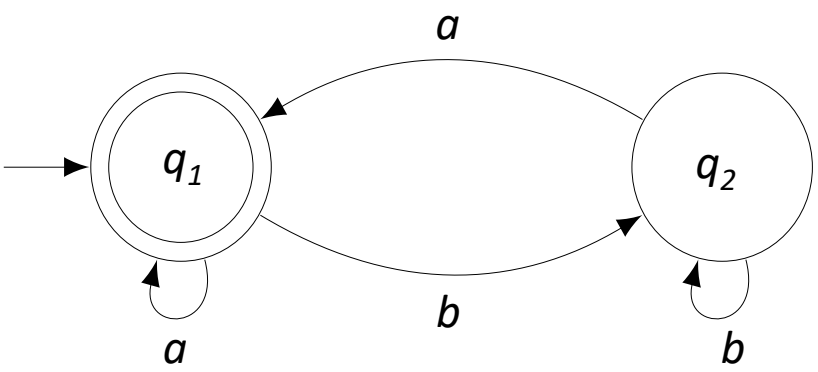
- $\mathcal{L}(\mathcal{B}) \subseteq \Sigma^\omega$ is the set of all infinite words that $\mathcal{B}$ accepts

Automata on **Infinite** Words (Büchi)

$$\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$$

- ρ is accepting $\Leftrightarrow \inf(\rho) \cap F \neq \emptyset$

- What is the language of this automaton?



$\mathcal{L}(\mathcal{B})$ = {words with an
Infinite number of a's}
or
$\mathcal{L}(\mathcal{B}) = (\{a,b\}^* a)^\omega$

In LTL: *GF(a)*

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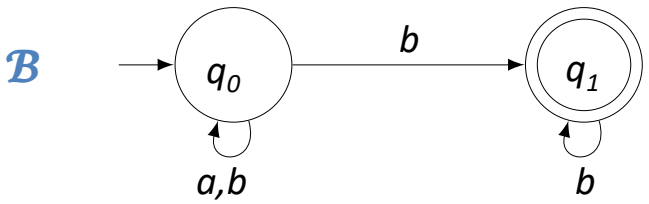
Det. and Non-det. Büchi Automata

- Deterministic Büchi automata are strictly less expressive than nondeterministic ones.
  - That is, not every nondeterministic Büchi automaton has an equivalent deterministic Büchi one.

Det. and Non-det. Büchi Automata

Theorem: There exists a non-deterministic Büchi automaton $\mathcal{B}$ for which there is no equivalent deterministic one.

Proof: The proof shows that there is no det. Büchi Automaton for "**finitely many**". Detailed proof see book.



$\mathcal{L}(\mathcal{B})$ = {words with a finite number of a's}
or
$\mathcal{L}(\mathcal{B}) = \{a,b\}^* b^\omega$

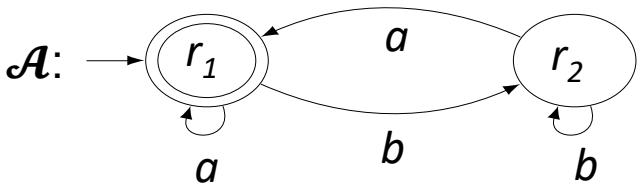
In LTL :
FG¬a or **FG**b

Det. and Non-det. Büchi Automata

Lemma 2: Deterministic Büchi automata are **not** closed under complementation.

Proof:

- Consider the language $\mathcal{L}$ = {words with infinitely many a's}.
- Construct a deterministic Büchi automaton $\mathcal{A}$ that accepts $\mathcal{L}$.
- Its complement is $\mathcal{L}$'={words with finitely many a's}, for which there is no deterministic Büchi automaton (see Theorem). □



Det. and Non-det. Büchi Automata

Theorem: Nondeterministic Büchi automata are closed under complementation.

- The construction is very complicated. We will not see it here.
- Büchi showed an algorithm for complementation that is double exponential in the size n of the automaton
- Mooly Safra proved that it can be done by

$$2^{O(n \log n)}$$

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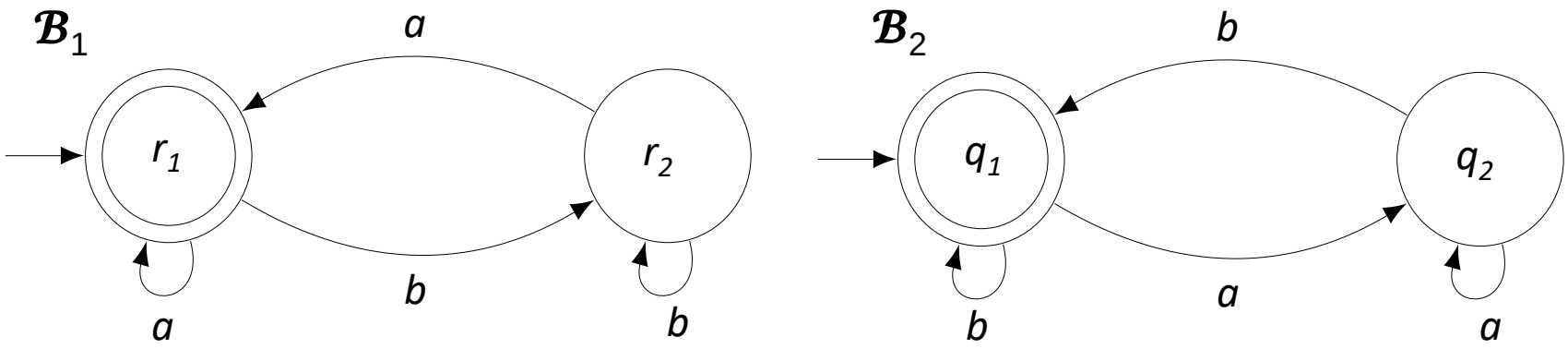
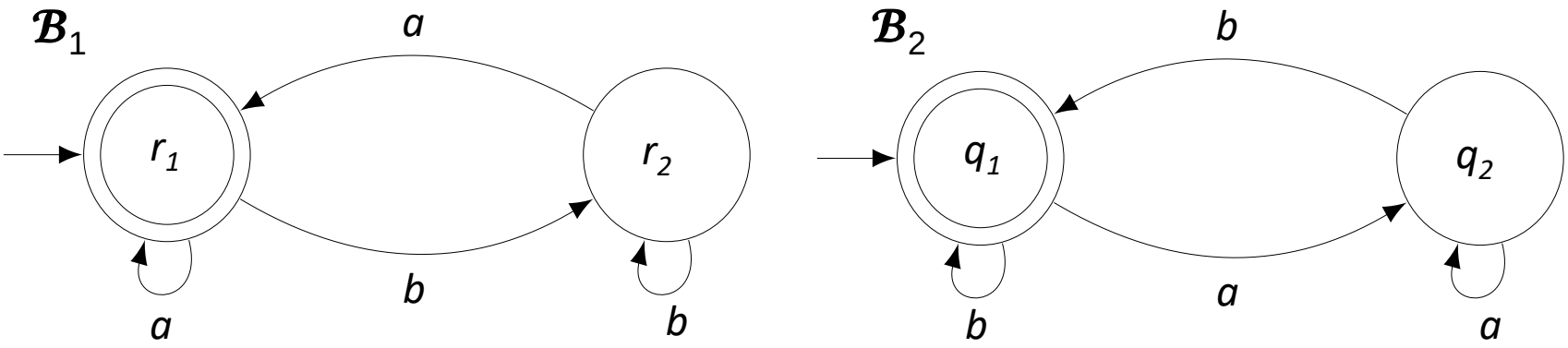
Intersection of Büchi Automata



- $\mathcal{L}(\mathcal{B}_1) \cap \mathcal{L}(\mathcal{B}_2) =$
  {words with an infinite number of a's and infinite number of b's}
  (not empty)

Intersection of Büchi Automata



- $\mathcal{L}(\mathcal{B}_1) \cap \mathcal{L}(\mathcal{B}_2) =$
  {words with an infinite number of a's and infinite number of b's}
- A standard intersection does not work – the automaton will not have any accepting states!
- **Solution: Introduce counter!**

Intersection of Büchi Automata

- Given $\mathcal{B}_1 = (\Sigma, Q_1, \Delta_1, Q_1^0, F_1)$ and $\mathcal{B}_2 = (\Sigma, Q_2, \Delta_2, Q_2^0, F_2)$
- $\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$ s.t. $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{B}_1) \cap \mathcal{L}(\mathcal{B}_2)$ is defined as follows:
  - $Q = Q_1 \times Q_2 \times \{0, 1, 2\}$
  - $Q^0 = Q_1^0 \times Q_2^0 \times \{0\}$
  - $F = Q_1 \times Q_2 \times \{2\}$

Intersection of Büchi Automata

$((q_1,q_2,x), a, (q'_1,q'_2,x')) \in \Delta \Leftrightarrow$

(1) $(q_1,a,q'_1) \in \Delta_1$ and $(q_2,a,q'_2) \in \Delta_2$ and

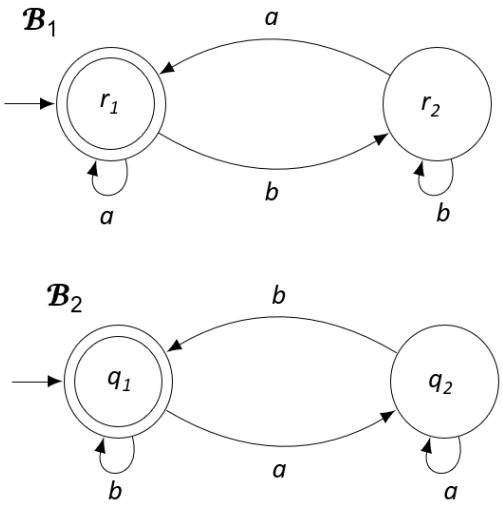
(2) If $x=0$ and $q'_1 \in \mathbf{F}_1$ then $x'=1$
If $x=1$ and $q'_2 \in \mathbf{F}_2$ then $x'=2$
If $x=2$ then $x'=0$
Else, $x'=x$

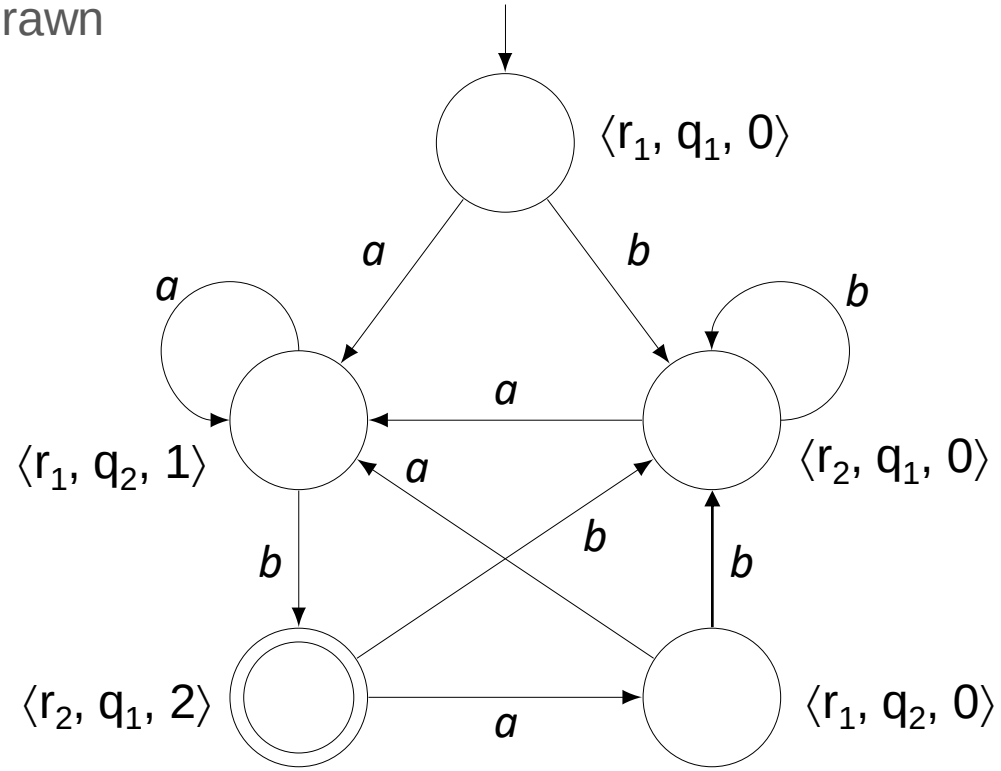
Explanation: $x=0$ is waiting for an accepting state from $\mathbf{F}_1$
$x=1$ is waiting for an accepting state from $\mathbf{F}_2$

If a state with $x=2$ is visited infinitely often, states from $\mathbf{F}_1$ have been visited infinitely often and states from $\mathbf{F}_2$ have been visited infinitely often.

Intersection of Büchi Automata

- The first copy waits for an accepting state of $\mathcal{B}_1$
- The second copy waits for an accepting state of $\mathcal{B}_2$
- All states in the third copy are accepting
- Only the reachable states are drawn





Intersection of Büchi Automata

- Question
  - In every interval we first wait for $\mathbf{F}_1$ and then wait for $\mathbf{F}_2$.
  - We ignore accepting states that don't appear in this order.
  - Might we miss accepting paths in $\mathcal{B}$?
- Answer
  - No. Since on an accepting path there are infinitely many of those, ignoring finite number of them in each interval will still lead us to the conclusion that the run is accepting

Model Checking of LTL

given an LTL property φ and a Kripke structure M
check whether $M \models \varphi$

✓ Construct $\neg\varphi$

2. Construct a Büchi automaton $\mathcal{S}_{\neg\varphi}$
3. Translate M to an automaton $\mathcal{A}$.

✓ Construct the automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\mathcal{S}_{\neg\varphi})$

⇒ If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies φ

6. Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
   - a computation in M that does not satisfy φ

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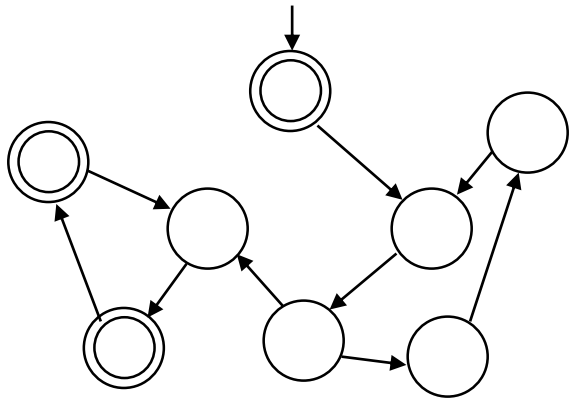
Checking for emptiness of $\mathcal{L}(\mathcal{B})$

- An infinite run $\boldsymbol{\rho}$ is accepting $\Leftrightarrow$ it visits an accepting state an infinite number of times.
  - $\inf(\boldsymbol{\rho}) \cap \mathbf{F} \neq \emptyset$
- How to check for $\boldsymbol{L(A)} = \emptyset$?
- Empty if there is no reachable accepting state on a cycle.

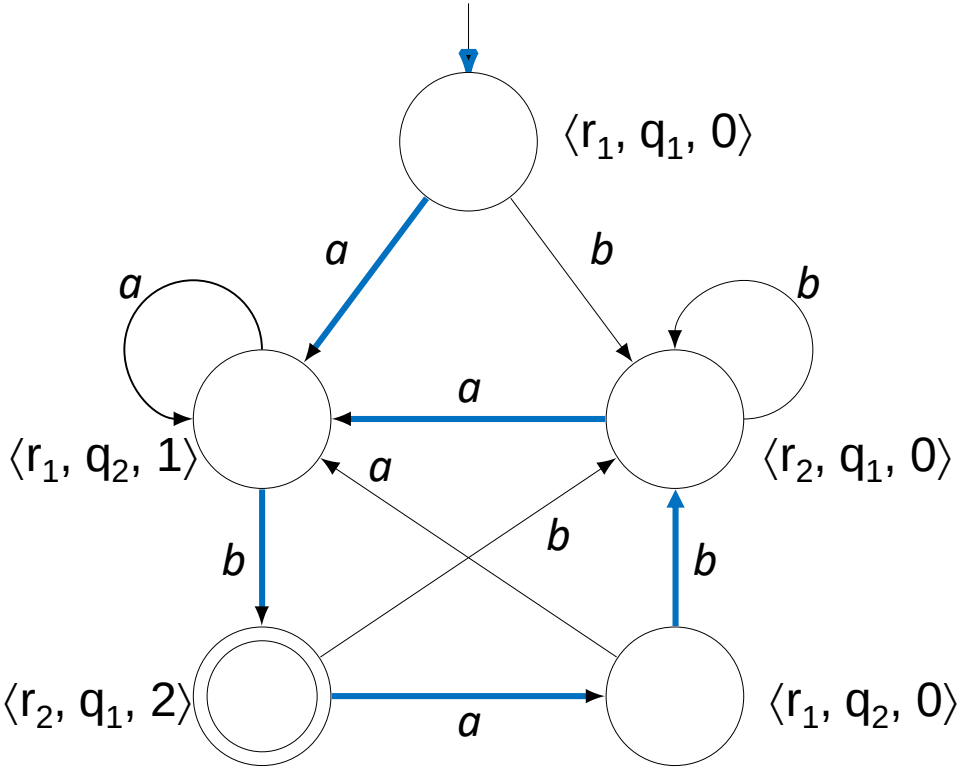


Non-emptiness ⇔ Existence of reachable accepting cycles

- $\mathcal{L}(\mathcal{B})$ *is nonempty* ⇔
- *The graph induced by* $\mathcal{B}$ *contains a path from an initial state of* $\mathcal{B}$ *to a state* $t \in F$ *and a path from* t *back to itself.*

Example

- $\langle r_2,q_1,2\rangle$ is accepting and reachable from $\langle r_1,q_1,0\rangle$ and reachable from itself



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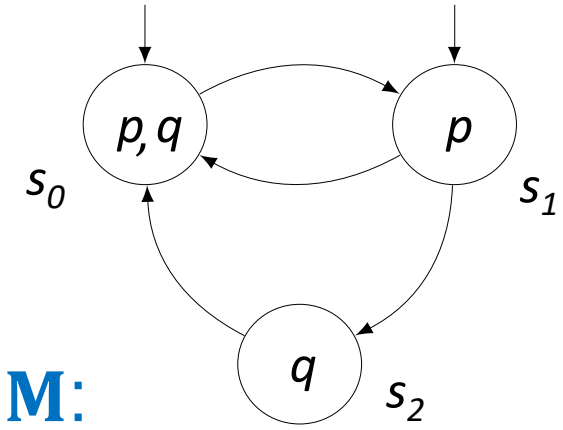
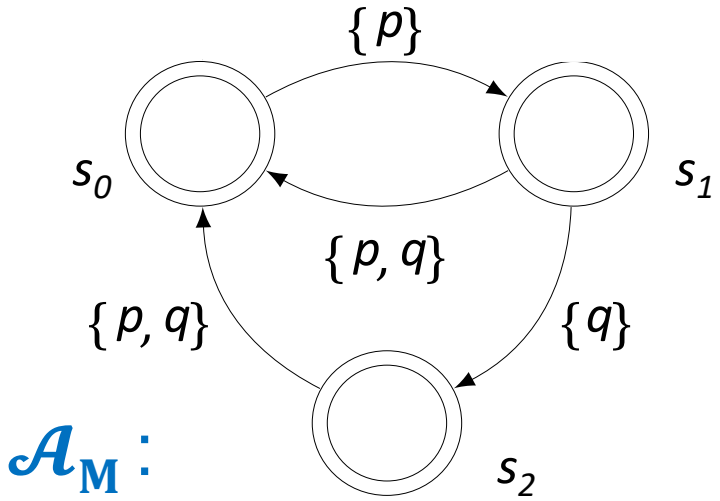
Kripke Structure $\mathbf{M}$ to Büchi Automaton $\mathcal{A}_{\mathbf{M}}$

- Move labels to incoming transitions
  - Push labels backwards
- All states are accepting

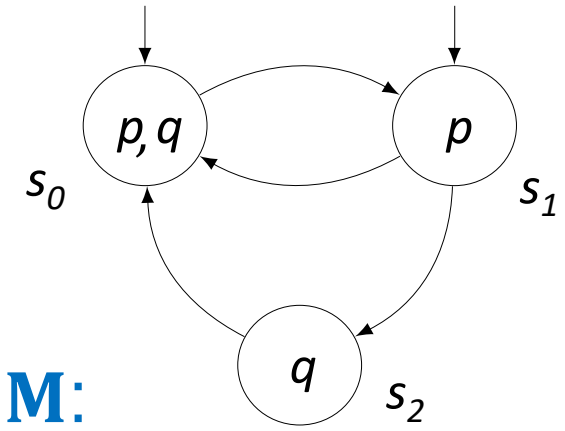
What about initial states?



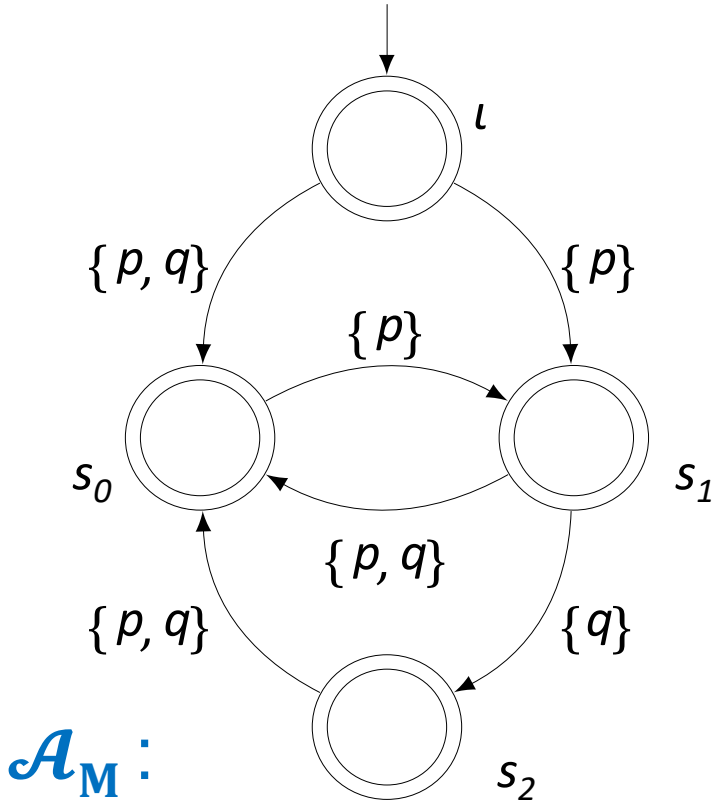


Kripke Structure $\mathbf{M}$ to Büchi Automaton $\mathcal{A}_{\mathbf{M}}$

- Move labels to incoming transitions
- All states are accepting



$\mathbf{M}$:

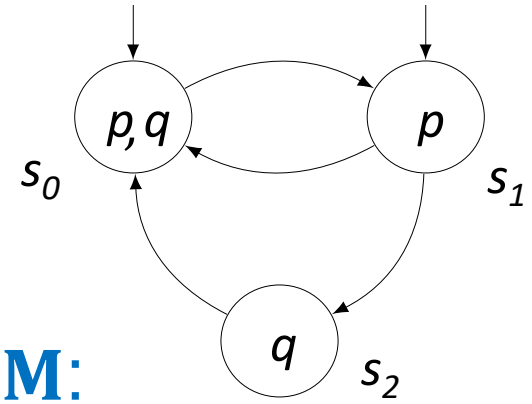


$\mathcal{A}_{\mathbf{M}}$:

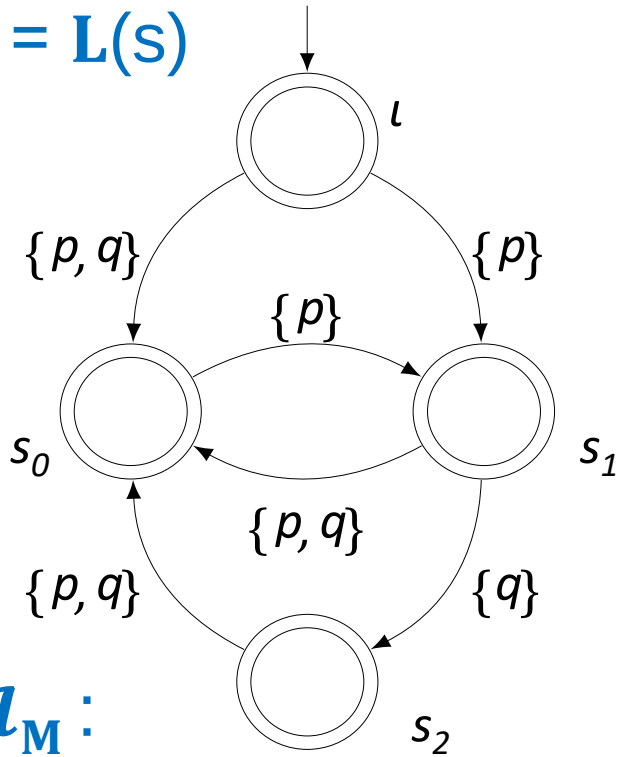
Automata and Kripke Structures

$\mathbf{M} = (\mathbf{S}, \mathbf{S}_0, \mathbf{R}, \mathrm{AP}, \mathbf{L}) \Rightarrow \mathcal{A}_\mathbf{M} = (\Sigma, \mathbf{S} \cup \{\iota\}, \Delta, \{\iota\}, \mathbf{S} \cup \{\iota\})$,
where $\Sigma = \mathrm{P}(\mathrm{AP})$.

- $(s,\alpha,s') \in \Delta \Leftrightarrow (s,s') \in R$ and $\alpha = \mathbf{L}(s')$
- $(\iota,\alpha,s) \in \Delta \Leftrightarrow s \in \mathbf{S}_0$ and $\alpha = \mathbf{L}(s)$



$\mathbf{M}$:



$\mathcal{A}_\mathbf{M}$:

Model Checking of LTL

given an LTL property φ and a Kripke structure M
check whether M $\models \varphi$

- ✓ Construct $\neg\varphi$
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- ✓ Translate M to an automaton $\mathcal{A}$.
- ✓ Construct the automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\mathcal{S}_{\neg\varphi})$
- ✓ If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies φ
- ✓ Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
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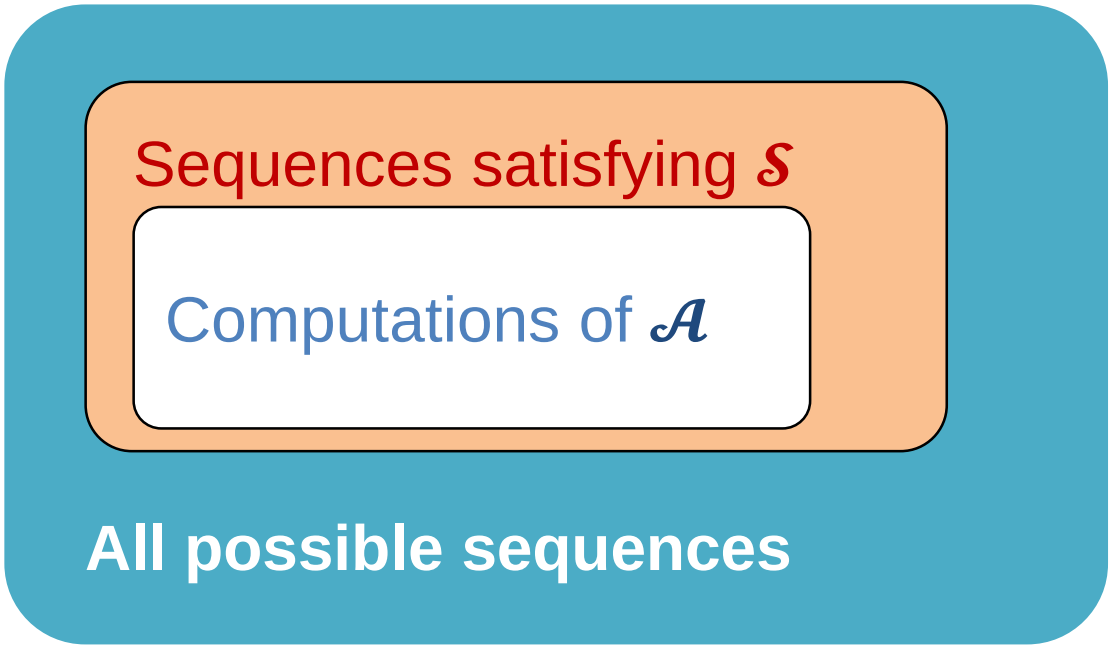
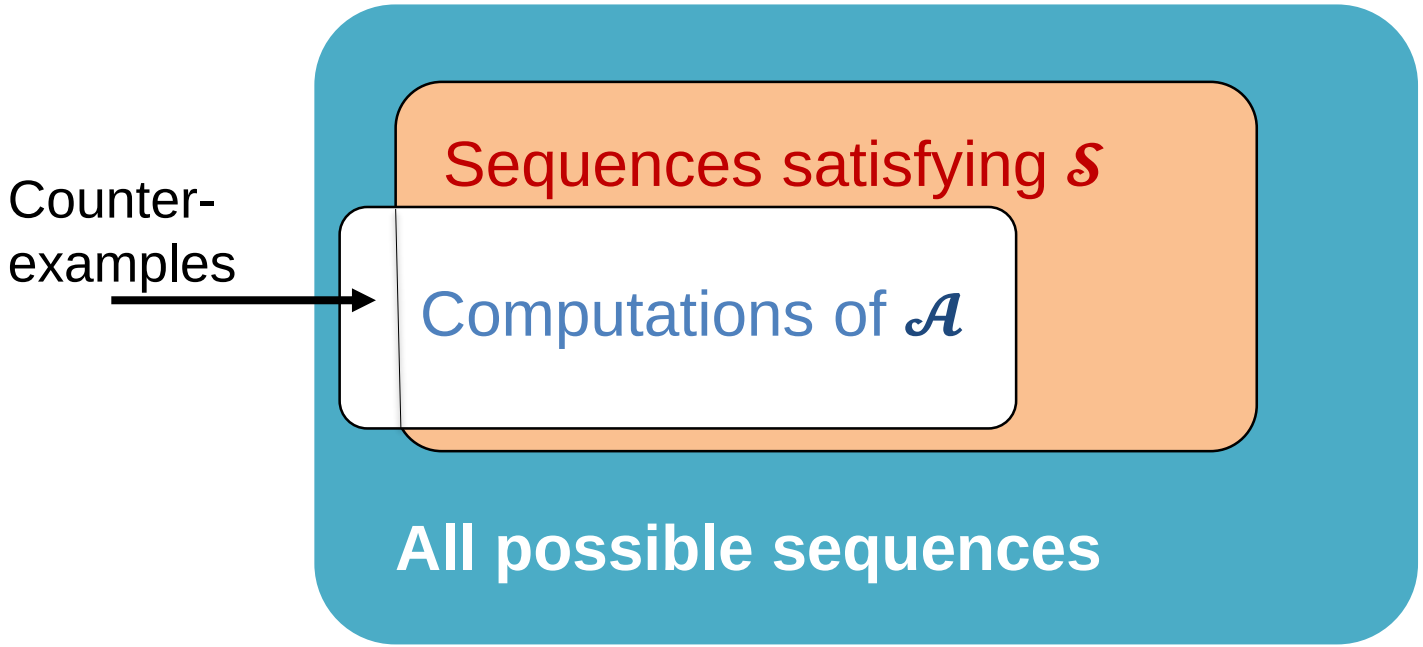
Model Checking when system $\mathcal{A}$ and spec $\mathcal{S}$ are given as Büchi automata

- **$\mathcal{A}$ satisfies $\mathcal{S}$** if $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{S})$
  - Is any behavior of $\mathcal{A}$ allowed by $\mathcal{S}$?



Model Checking when System $\mathcal{A}$ and Spec $\mathcal{S}$ are given as Büchi automata

- **$\mathcal{A}$ does not satisfy $\mathcal{S}$** if $\mathcal{L}(\mathcal{A}) \not\subseteq \mathcal{L}(\mathcal{S})$

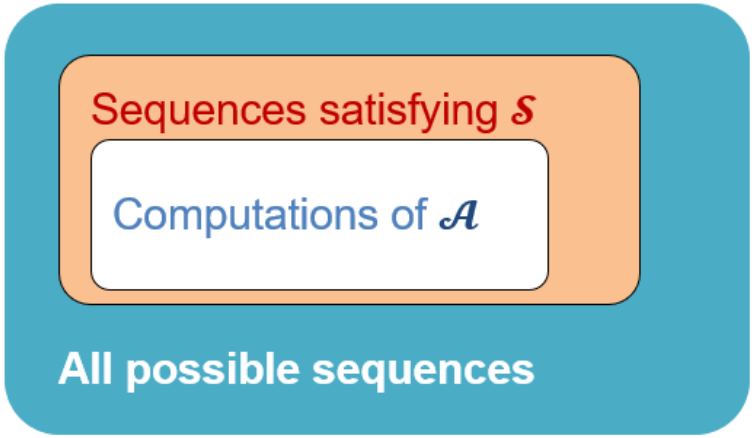


Model Checking when system $\mathcal{A}$ and spec $\mathcal{S}$ are given as Büchi automata

- Check whether $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{S})$
- Equivalent:

$$\mathcal{L}(\mathcal{A}) \nsubseteq \mathcal{L}(\mathcal{S}) \equiv \boxed{\mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\overline{\mathcal{S}}) \neq \emptyset}$$

$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{S})$



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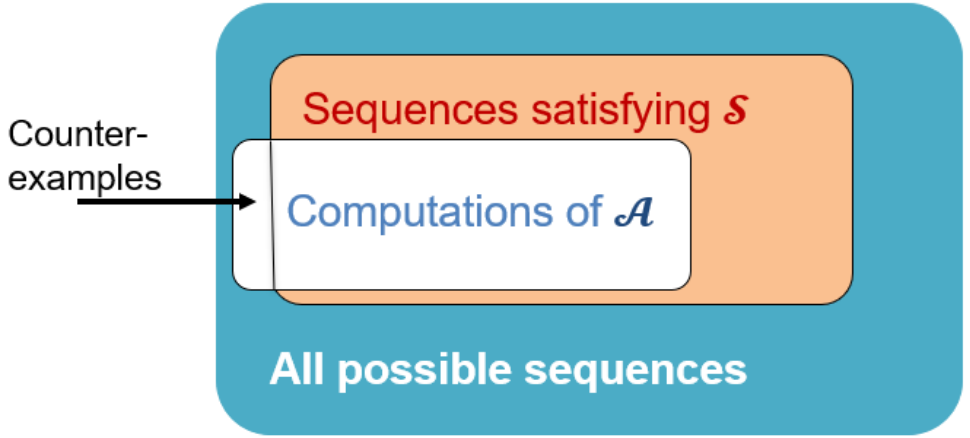


Model Checking – Suggested Algorithm

when system $\mathcal{A}$ and spec $\mathcal{S}$ are given as Büchi Automata

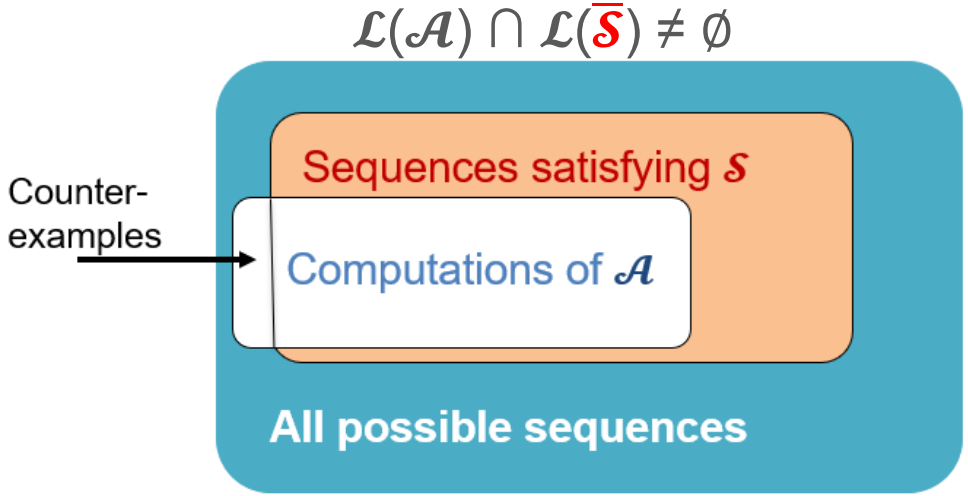
1. Complement $\mathcal{S}$. The resulting Büchi automaton is $\overline{\mathcal{S}}$ — very hard!

✓ Construct the automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\overline{\mathcal{S}})$

✓ If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies $\mathcal{S}$

✓ Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample

- a computation in $\mathcal{A}$ that does not satisfy $\mathcal{S}$



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5. If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies φ
6. Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
   - a computation in M that does not satisfy φ


THANK
YOU!