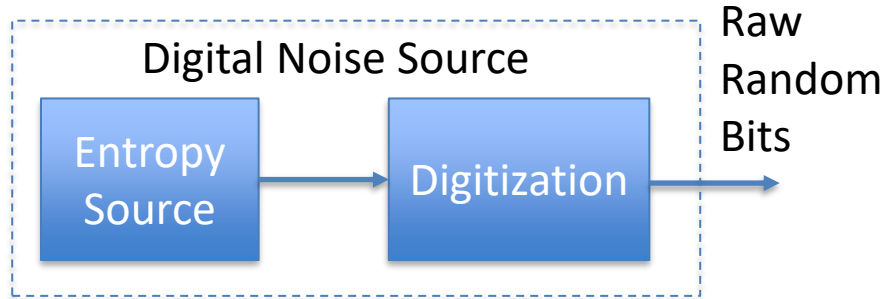


Postprocessing of Raw TRNG Bits

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Raw random numbers produced in this way **are generally not IID**, i.e., independent and identically distributed.

- Bits are biased
- and contain correlation

Could we mitigate or remove statistical defects in raw random data?

Postprocessing (conditioning) of Raw Random Bits

'Postprocessing' is an application of a deterministic algorithm to remove or mitigate statistical defects from TRNG-produced raw random data (which contains defects).

- Increases randomness per bit by performing data compression.
- Some entropy is always lost due to data compression
- It doesn't produce any 'new' randomness

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There are two ways of postprocessing raw random bits:


1. Arithmetic postprocessing → do not rely on cryptographic primitives
2. Cryptographic postprocessing → rely on cryptographic primitives

Arithmetic postprocessing: Parity filter or XOR processing (1)

- Raw random bits are split into blocks of length n_f bits and
- Then the bits within each chunk are XORed

Example:

Raw bit sequence: 1 1 0 1 1 0 0 1 0 0 1 1 1 0 1 0 ...



with $n_f = 2$

XORed bit sequence: 0 1 1 1 0 0 1 1



Arithmetic postprocessing: Parity filter or XOR processing (2)

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Example:

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XORed bit sequence: 0 1 1 1 0 0 1 1

Data compression factor is n_f .

If the raw data has a bias ϵ_{raw}

then the postprocessed data has a bias: $\epsilon = 2^{n_f-1} \epsilon_{raw}^{n_f}$

Arithmetic postprocessing: Von Neuman Processing (1)

This method removes bias completely.

Steps:

1. Partition the input bit string into 2-bit blocks.
2. Discard all '00' and '11' blocks.
3. If a block is '01' then the output bit is 1; If a block is '10' then the output bit is 0.

Example:

Raw bit sequence: 1 1 0 1 1 0 0 1 0 0 1 1 1 0 1 0 ...

✗ ✗ ✗

Output bit sequence: - 1 0 1 - - 0 0

Arithmetic postprocessing: Von Neuman Processing (2)

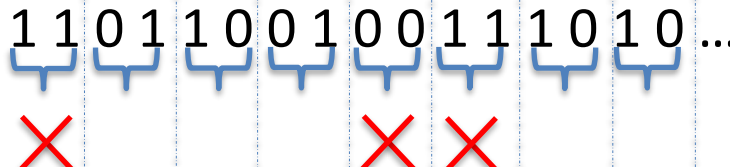
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Output bit sequence: - 1 0 1 - - 0 0

Output is produced at a variable rate.

If input has a throughput T_{in} then the average throughput of output is $T_{in} \cdot p_1 \cdot (1 - p_1)$.

Arithmetic postprocessing: Resilient Function [SMS07]

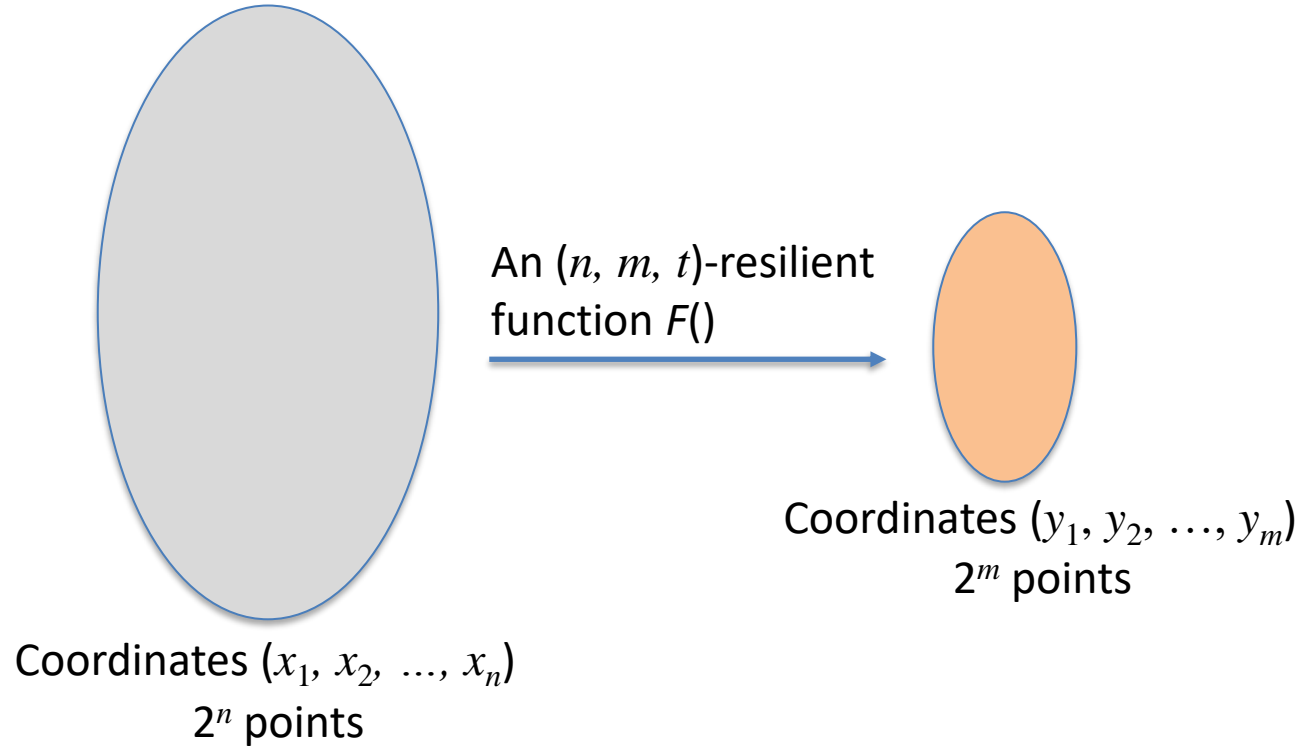
Definition [SMS07]: An (n, m, t) -resilient function is a function

$$F(x_1, x_2, \dots, x_n) = (y_1, y_2, \dots, y_m)$$

from Z_2^n to Z_2^m enjoying the property that for any t coordinates i_1, \dots, i_t , for any constants a_1, \dots, a_t from Z_2 and any element y of the codomain

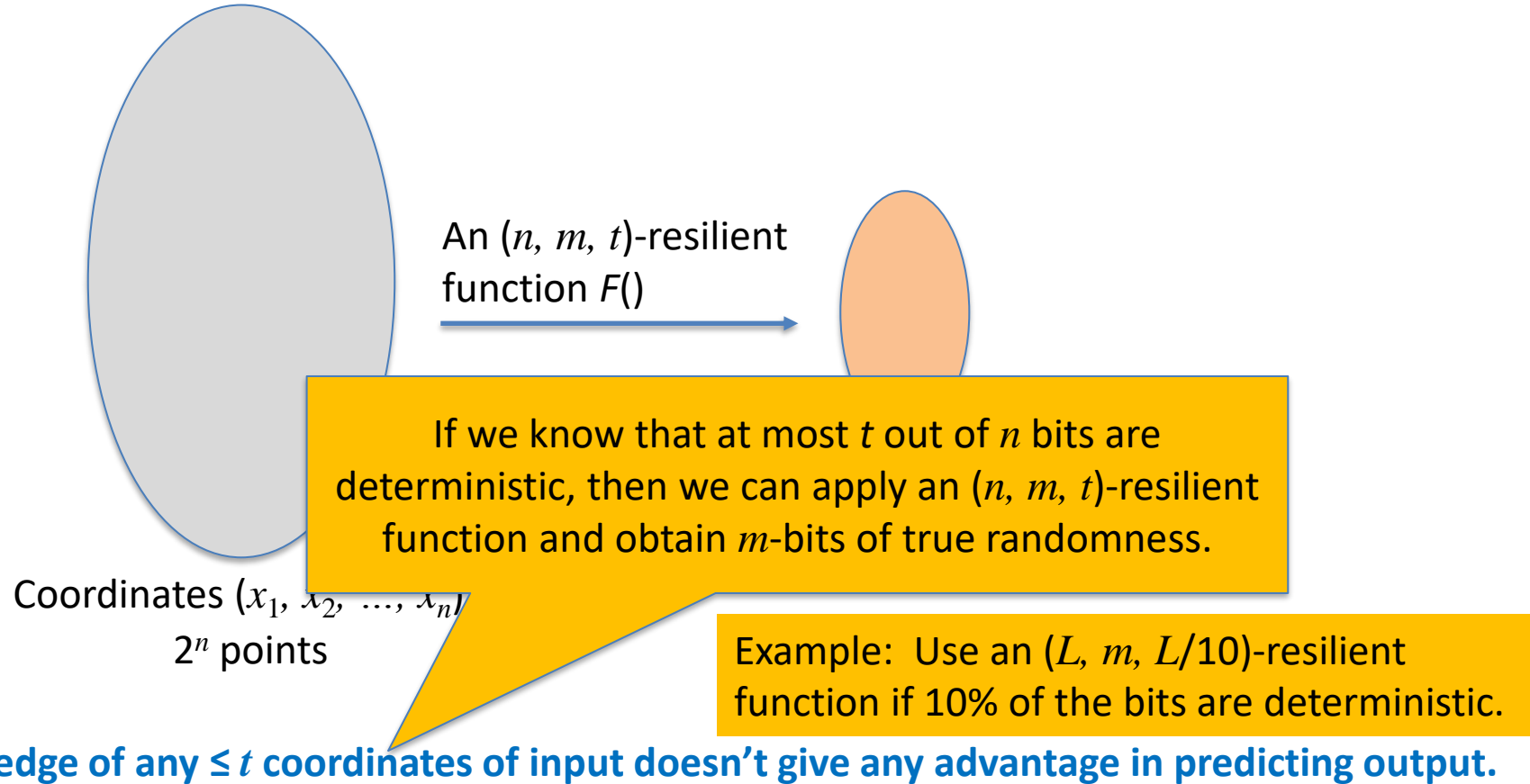
$$Pr(F(x) = y \mid x_{i_1} = a_1, \dots, x_{i_t} = a_t) = 1/2^m.$$

Arithmetic postprocessing: Resilient Function [SMS07]



Knowledge of any $\leq t$ coordinates of input doesn't give any advantage in predicting output.

Arithmetic postprocessing: Resilient Function [SMS07]



Arithmetic postprocessing: Example of a Resilient Function

[SMS07] used a linear error correcting code $C = [n, m, d]$ to implement a $[n, m, d-1]$ resilient function.

$$f(x) = x \cdot \left(G \right)^T$$



This code can correct up to $(d - 1)$ “errors”

Arithmetic postprocessing: Example of a Resilient Function

[SMS07] used a linear error correcting code $C = [n, m, d]$ to implement a $[n, m, d-1]$ resilient function.

$$f(x) = x \cdot \begin{pmatrix} G \end{pmatrix}^T$$

[SPV06] used a cyclic code for compact implementation on hardware platforms.

$$G = \begin{pmatrix} g_0 & 0 & \dots & 0 \\ g_1 & g_0 & & 0 \\ \vdots & \vdots & \ddots & \\ g_{n-m-1} & g_{n-m-2} & \dots & g_0 \\ 0 & g_{n-m-1} & \dots & g_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & g_{n-m-1} \end{pmatrix}^T$$

Summary: Postprocessing (conditioning) of Raw Random Bits

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Cryptographic postprocessing

A cryptographic postprocessing uses a cryptographic primitive to process the raw random bits and then produce uniformly distributed random bits.

NIST recommended **keyed** algorithms for cryptographic postprocessing:

1. HMAC with any standardized hash function
2. CMAC with AES block cipher
3. CBC-MAC with AES block cipher

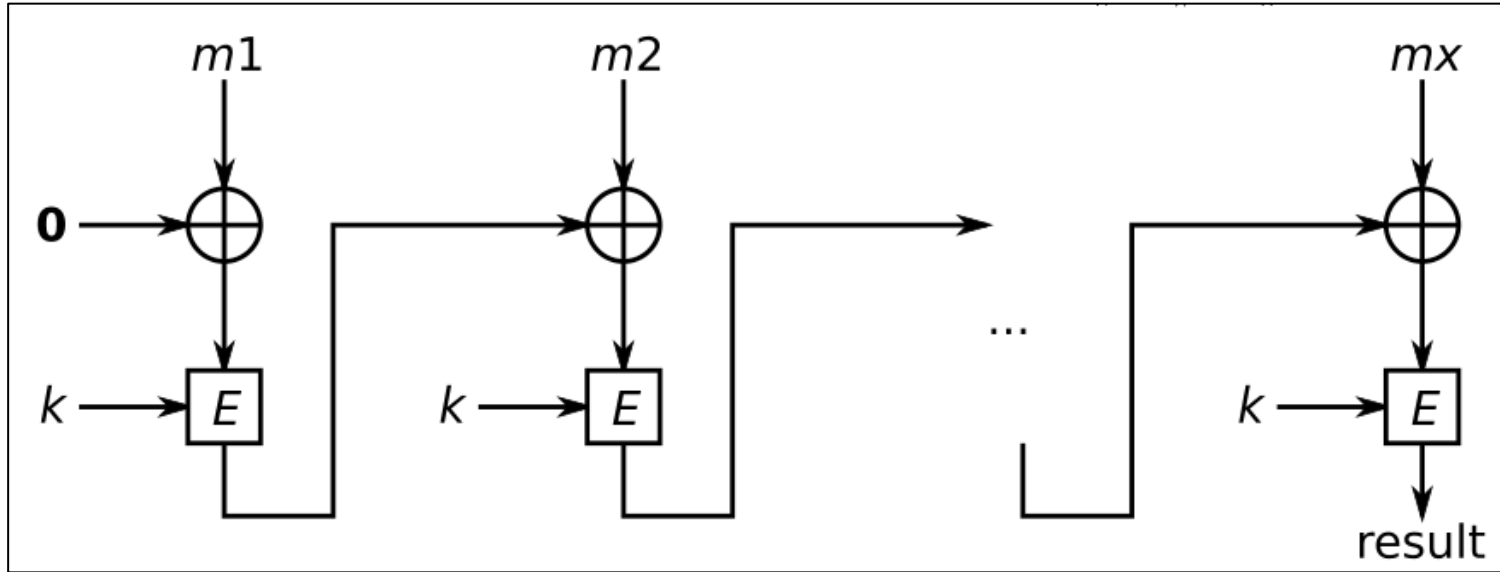
NIST recommended **un-keyed** algorithms for cryptographic postprocessing:

1. Any standardized hash function
2. Hash_df with any standardized hash function
3. Block_Cipher_df with AES block cipher

(Note: df stands for derivative function)

Cryptographic postprocessing: Example using CBC-MAC

Partition raw random bits into 128-bit blocks and use each block as a message-block.



E is AES-128.

The number of blocks ≥ 2 .

Cryptographic postprocessing

Detailed technical information available on the NIST special publication SP 800-90A

NIST Special Publication 800-90A
Revision 1

**Recommendation for Random
Number Generation Using
Deterministic Random Bit Generators**

Elaine Barker
John Kelsey

References

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