Graz University of Technology
Institute for Applied Information Processing and Communications

Automata and LTL Model Checking

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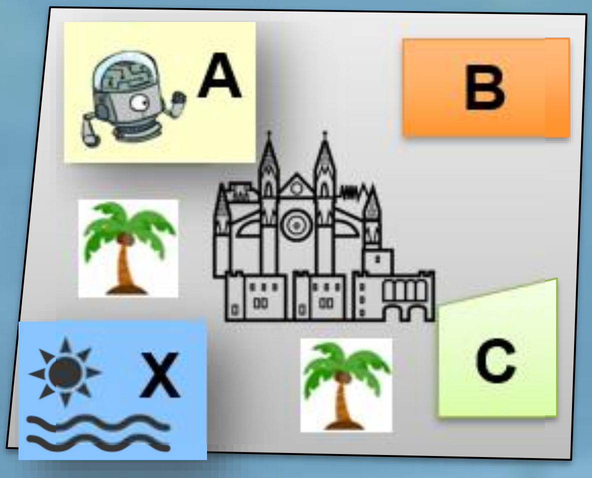
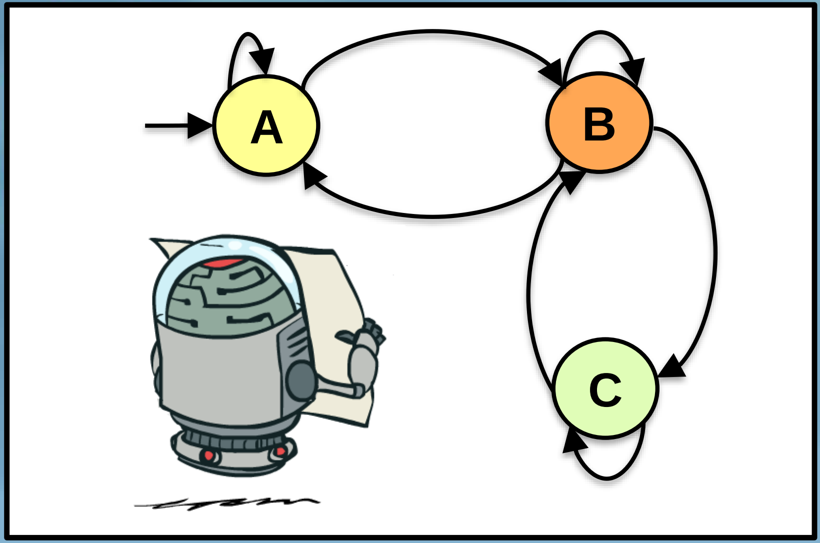




Model Checking SS24

May 12th 2024

Plan for Today

- Part 1 - LTL Model Checking
  - Generalized Büchi Automata
  - Translation of LTL to Büchi Automata

- Part 2 – Reactive Synthesis
  - Safety Games
  - Reachability Games
  - Büchi Games

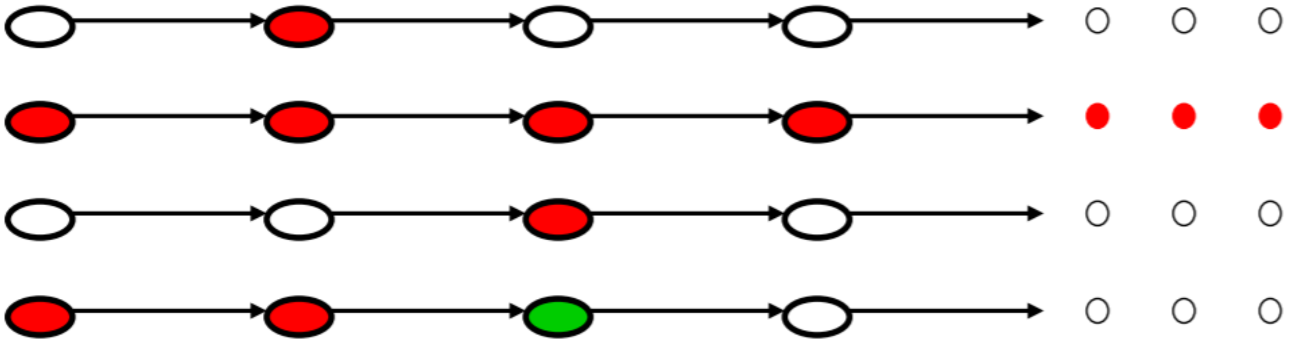
Recall LTL Semantics

Xp

Gp

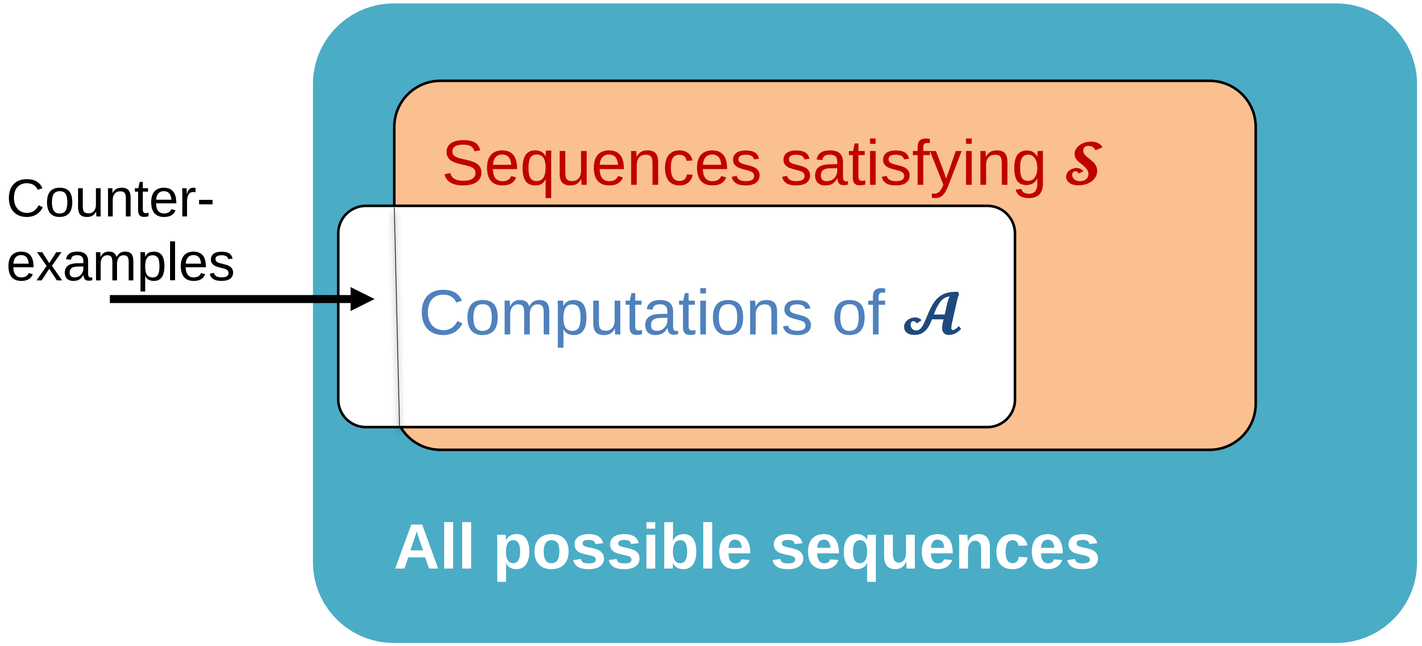
Fp

pUq



Recap – LTL Model Checking

- $\mathcal{A}$ **does not satisfy** $\mathcal{S}$ if $\mathcal{L}(\mathcal{A}) \nsubseteq \mathcal{L}(\mathcal{S})$



Model Checking of LTL

given an LTL property φ and a Kripke structure M
check whether $M \vDash \varphi$

TODAY

1. Construct $\neg\varphi$
2. Construct a Büchi automaton $\mathcal{S}_{\neg\varphi}$
3. Translate M to an automaton $\mathcal{A}$.
4. Construct the automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\mathcal{S}_{\neg\varphi})$
5. If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies φ
6. Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
   - a computation in M that does not satisfy φ

Homework - Intersection of Büchi Automata

- Question
  - How do we define the transition relation for $\mathcal{B}$, if x is over {0,1} only?

With x over {0,1,2} we had:

$\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$ s.t. $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{B}_1) \cap \mathcal{L}(\mathcal{B}_2)$ is defined as follows:

- $Q = Q_1 \times Q_2 \times \{0, 1, 2\}$
- $Q^0 = Q_1^0 \times Q_2^0 \times \{0\}$
- $F = Q_1 \times Q_2 \times \{2\}$

$((q_1,q_2,x), a, (q'_1,q'_2,x')) \in \Delta \Leftrightarrow$

(1) $(q_1,a,q'_1) \in \Delta_1$ and $(q_2,a,q'_2) \in \Delta_2$ and

(2) If $x=0$ and $q'_1 \in F_1$ then $x'=1$
If $x=1$ and $q'_2 \in F_2$ then $x'=2$
If $x=2$ then $x'=0$
Else, $x'=x$

Homework - Intersection of Büchi Automata

- Question
  - How do we define the transition relation for $\mathcal{B}$, if x is over {0,1} only?
- Answer
  - For Δ
    - (2) If $x=0$ and $q_1 \in \mathbf{F}_1$ then $x'=1$
      If $x=1$ and $q_2 \in \mathbf{F}_2$ then $x'=0$
      Else, $x'=x$
  - For $\mathbf{F}$
    - $\mathbf{F} = \mathbf{F}_1 \times \mathbf{Q}_2 \times \{0\}$

Homework - Counterexamples

- Question
  - Show that if a certain LTL property φ can only be falsified by infinite counterexamples, then it also has a counterexample that is lasso shaped.

Homework - Counterexamples

- Question
  - Show that if a certain LTL property φ can only be falsified by infinite counterexamples, then it also has a counterexample that is lasso shaped.
- Answer
  - Is the case for Liveness properties: e.g., Fa
  - We consider Büchi automata with a finite state space
  - Let $n = |Q|$
  - Since the automaton has only n states,
    an infinite long run must contain a loop
  - An infinite counterexample is thus lasso shaped

Outline

- Finite automata on finite words
- Automata on infinite words (Büchi automata)
- Deterministic vs non-deterministic Büchi automata
- Intersection of Büchi automata
- Checking emptiness of Büchi automata
- Automata and Kripke Structures
- Model checking using automata
- Generalized Büchi automata
- Translation of LTL to Büchi automata

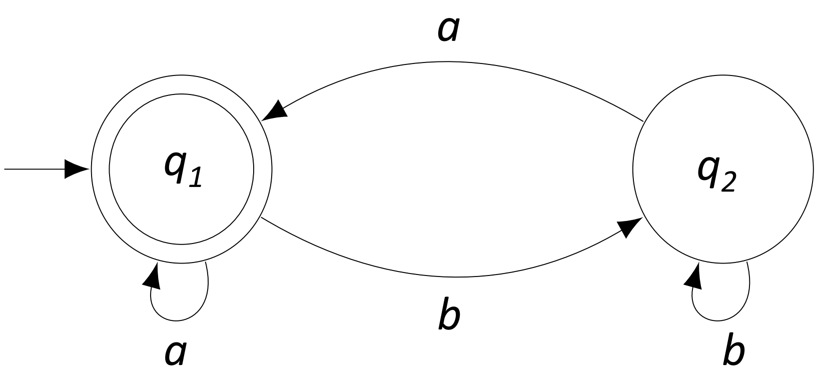
Automata on **Infinite** Words (Büchi)

$$\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$$

- ρ is accepting $\Leftrightarrow$ $\inf(\rho) \cap F \neq \emptyset$

- Language of Büchi automaton: $\mathcal{L}(\mathcal{B})$ = {words with an Infinite number of a's}

In LTL: *GF(a)*

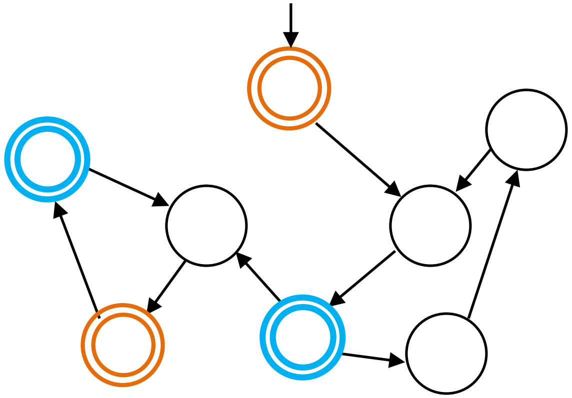


Generalized Büchi automata

- Have several sets of accepting states

- $\mathcal{B}=(\Sigma,Q,\Delta,Q^0,F)$ is a generalized Büchi automaton:
  - $\mathbf{F} = \{F_1, \ldots, F_k\}$, where for every $1 \leq i \leq k, F_i \subseteq Q$

- A run ρ of $\mathcal{B}$ is accepting if for each $F_i \in F$, $\inf(\rho) \cap F_i \neq \emptyset$

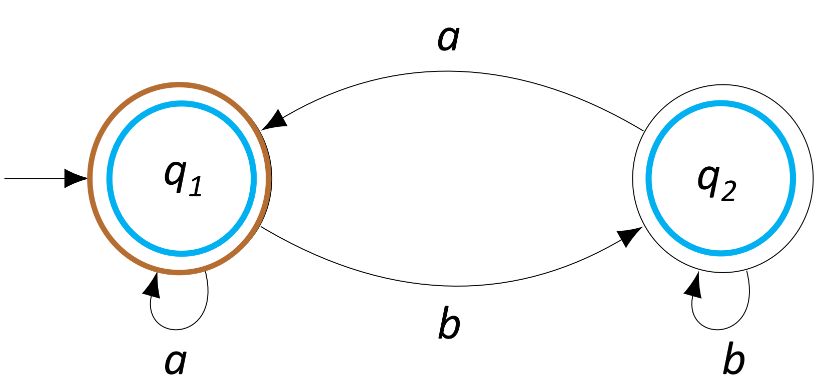


Generalized Büchi automata

- A run ρ of $\mathcal{B}$ is accepting if for each $F_i \in F$, $\inf(\rho) \cap F_i \neq \emptyset$

ToDo

- Is the infinite work b^ω accepted?
- Is the infinite work a^ω accepted?
- Is the infinite work $(ab)^\omega$ accepted?

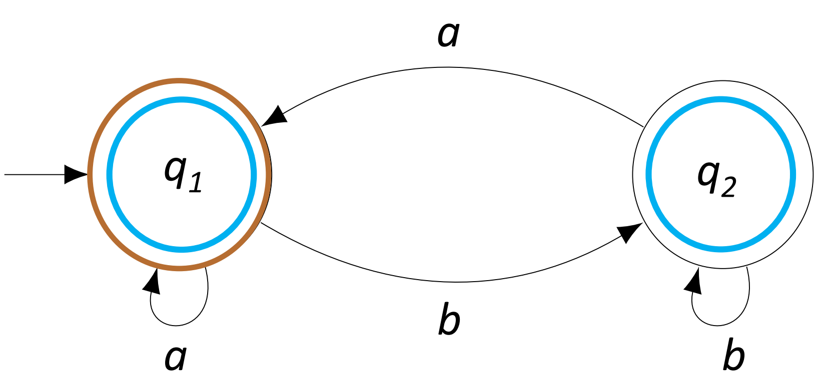


$F_1 = \{q_1, q_2\}$, $F_2 = \{q_1\}$

Generalized Büchi automata

- A run ρ of $\boldsymbol{\mathcal{B}}$ is accepting if for each $F_i \in F$, $\inf(\rho) \cap F_i \neq \emptyset$

- Is the infinite work b^ω accepted? ✗
- Is the infinite work a^ω accepted? ✓
- Is the infinite work $(ab)^\omega$ accepted? ✓



$F_1 = \{q_1, q_2\}$, $F_2 = \{q_1\}$

Translation from Generalized Büchi to Büchi

- Given $\boldsymbol{\mathcal{B}} = (\boldsymbol{\Sigma}, \mathbf{Q}, \boldsymbol{\Delta}, \mathbf{Q}^0, \mathbf{F})$ with $\mathbf{F} = \{F_1, \ldots, F_k\}$

- How does it work to construct a Büchi automaton $\boldsymbol{\mathcal{B}}'$ that accepts the same language?

Translation from Generalized Büchi to Büchi

- Given $\mathcal{B} = (\mathbf{\Sigma},\mathbf{Q},\mathbf{\Delta},\mathbf{Q}^0,\mathbf{F})$ with $\mathbf{F} = \{F_1, \dots, F_k\}$

- How does it work to construct a Büchi automaton $\mathcal{B}$' that accepts the same language?

- Idea:
  - Introduce counter from $0 \dots k$ → $k + 1$ copies of the state space
  - Redirect outgoing edges from accepting states to next copy

  - → Each cycle will contain accepting states from each set $F_1, \dots, F_k$

Translation from Generalized Büchi to Büchi

- $\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$ with $F = \{F_1, \dots, F_k\}$
- $\mathcal{B}' = (\Sigma, Q \times \{0,1,\dots,k\}, \Delta', Q^0 \times 0, Q \times k)$ with:
- The transition relation Δ':
  $((q,x),a,(q',y)) \in \Delta'$ when $(q,a,q') \in \Delta$ and x and y are as follows:
  - If $q' \in F_i$ and $x = i$, then $y = i + 1$ for $i < k$
  - If $x = k$, then $y = 0$.
  - Otherwise, $x = y$.

Size of $\mathcal{B}'$ = (size of $\mathcal{B}$) × (k+1)

Translation from Generalized Büchi to Büchi

- $\mathcal{B} = (\mathbf{\Sigma}, \mathbf{Q}, \mathbf{\Delta}, \mathbf{Q}^0, \mathbf{F})$ with $\mathbf{F} = \{F_1, \dots, F_k\}$
- $\mathcal{B}' = (\mathbf{\Sigma}, \mathbf{Q} \times \{0, 1, \dots, k\}, \mathbf{\Delta}', \mathbf{Q}^0 \times 0, \mathbf{Q} \times k)$ with:
- The transition relation $\mathbf{\Delta}'$:
  $((q, x), a, (q', y)) \in \mathbf{\Delta}'$ when $(q, a, q') \in \mathbf{\Delta}$ and x and y are as follows:
  - If $q' \in F_i$ and $x = i$, then $y = i + 1$ for $i < k$
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Can we safe a copy?

Translation from Generalized Büchi to Büchi

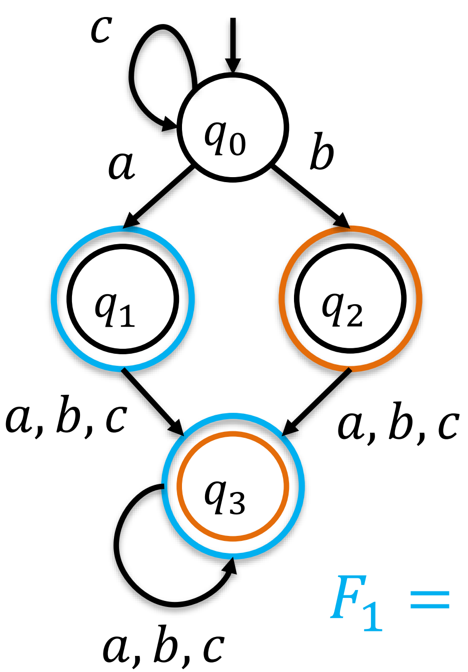
- $\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$ with $F = \{F_1, \ldots, F_k\}$
- $\mathcal{B}' = (\Sigma, Q \times \{1, \ldots, k\}, \Delta', Q^0 \times 1, F_k \times k)$ with:
- The transition relation Δ':
  $((q, x), a, (q', y)) \in \Delta'$ when $(q, a, q') \in \Delta$ and x and y are as follows:
  - If $q' \in F_i$ and $x = i$, then $y = i + 1$ for $i < k$
  - If $x = k$, then $y = 1$.
  - Otherwise, $x = y$.

Size of $\mathcal{B}'$ = (size of $\mathcal{B}$) $\times$ k

Example: Generalized Büchi to Büchi

- What is the language of $\mathcal{B}$?

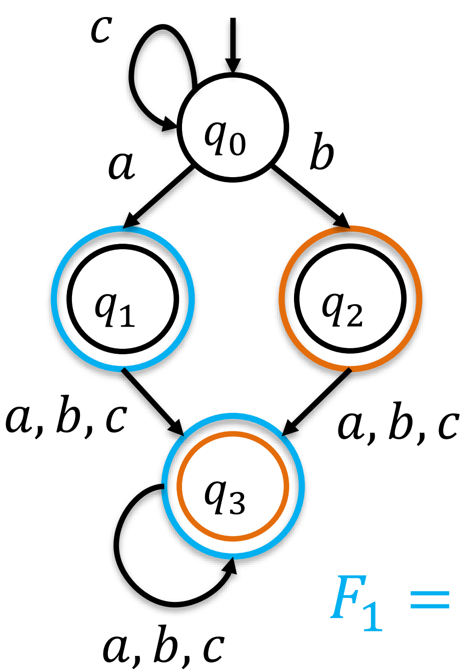
$\mathcal{L}(\mathcal{B}) = c^*(a|b)(a|b|c)^\omega$



$F_1 = \{q_1, q_3\}$, $F_2 = \{q_2, q_3\}$

Example: Generalized Büchi to Büchi

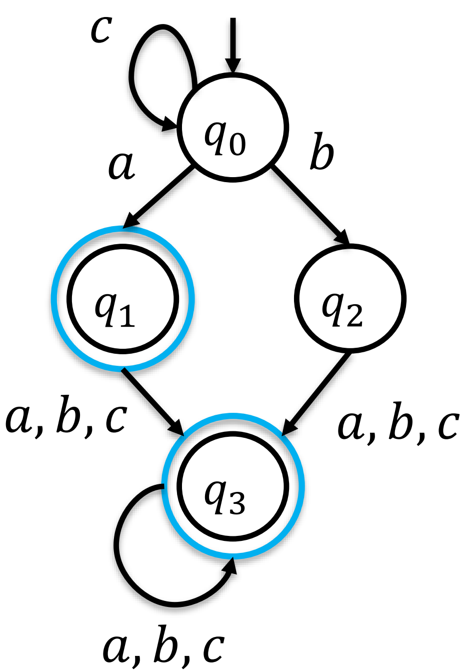
- Translate $\mathcal{B}$ to a Büchi automaton $\mathcal{B}$'

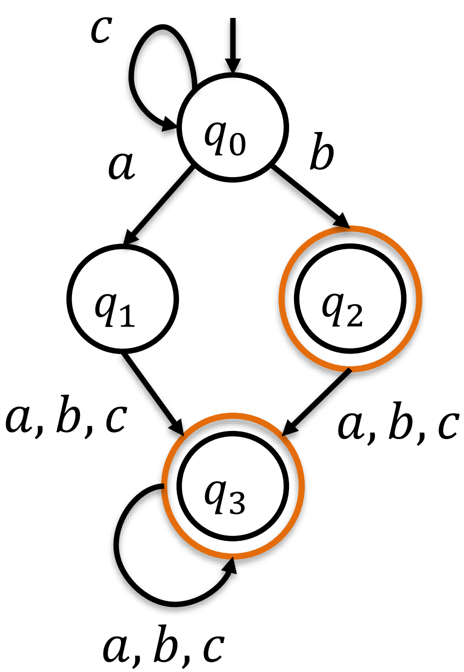


$F_1 = \{q_1, q_3\}$, $F_2 = \{q_2, q_3\}$

Example: Generalized Büchi to Büchi

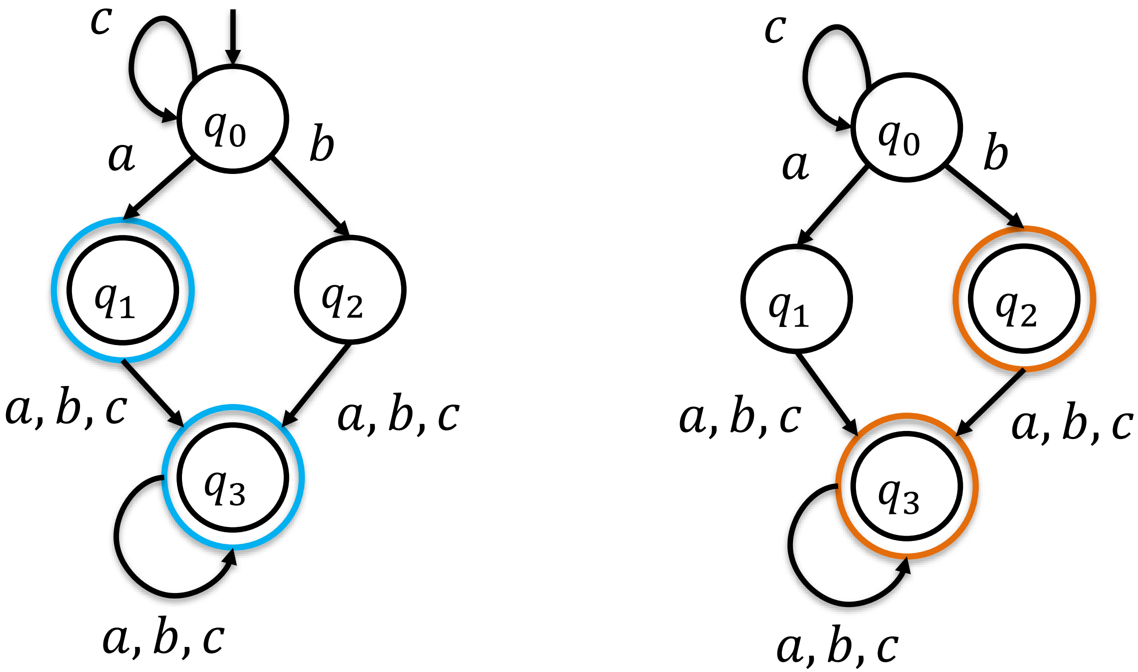
- Translate $\mathcal{B}$ to a Büchi automaton $\mathcal{B}$'
  - Two copies because we have two accepting sets





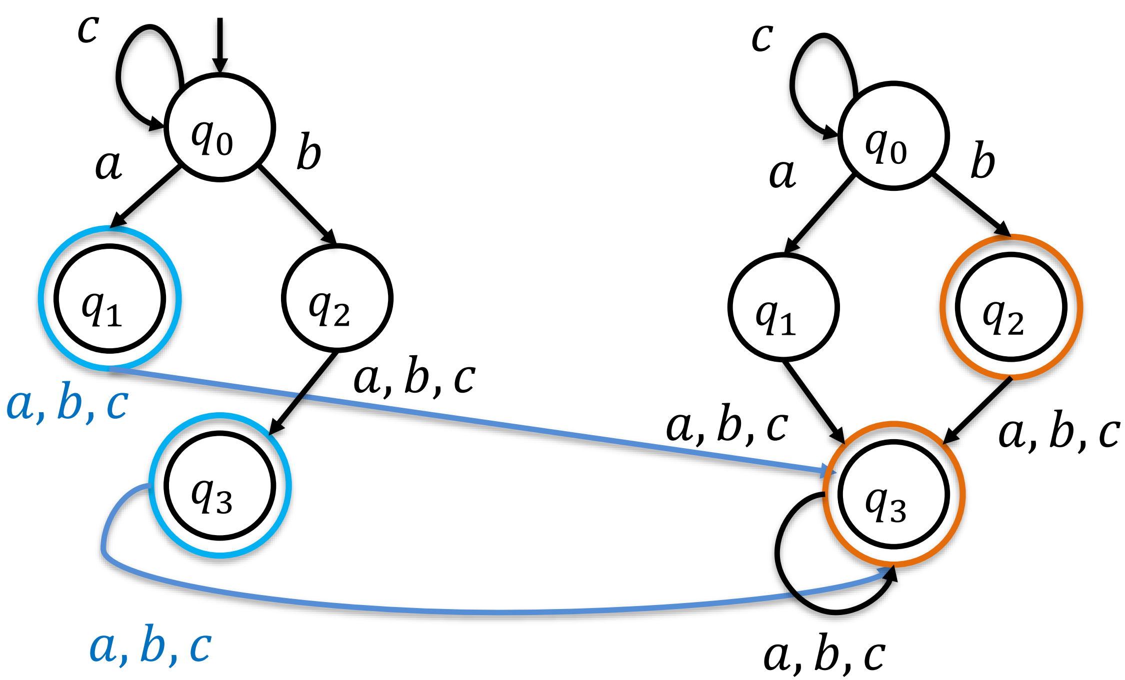
Example: Generalized Büchi to Büchi

- Translate $\mathcal{B}$ to a Büchi automaton $\mathcal{B}$'
  - Choose once copy as initial one



Example: Generalized Büchi to Büchi

- Translate $\mathcal{B}$ to a Büchi automaton $\mathcal{B}$'
  - Redirect outgoing edges from accepting states



Example: Generalized Büchi to Büchi

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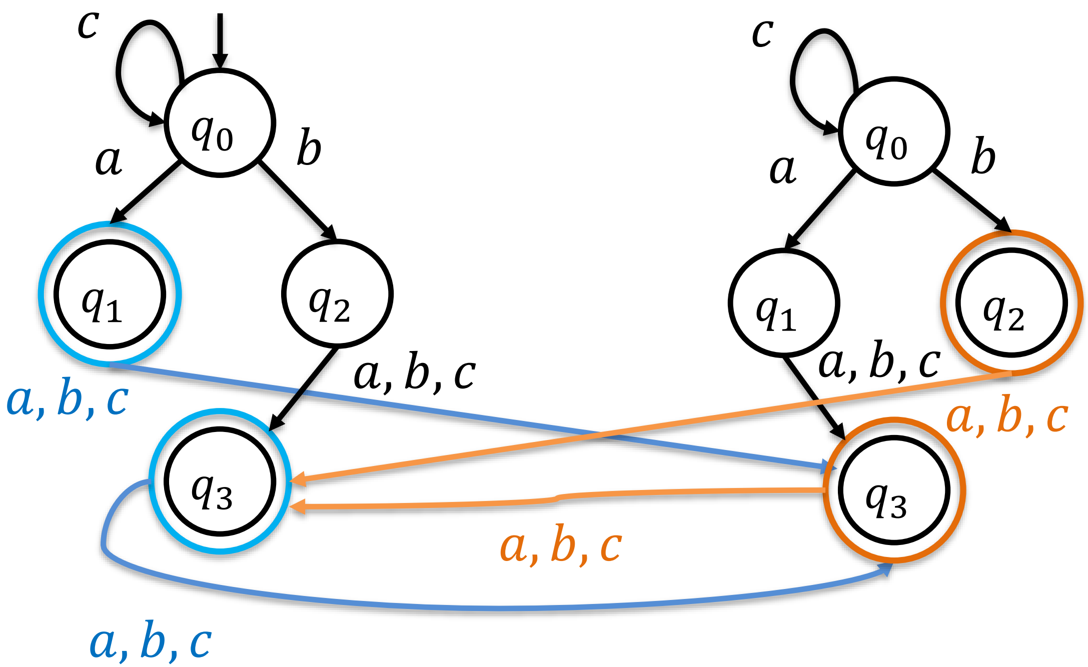
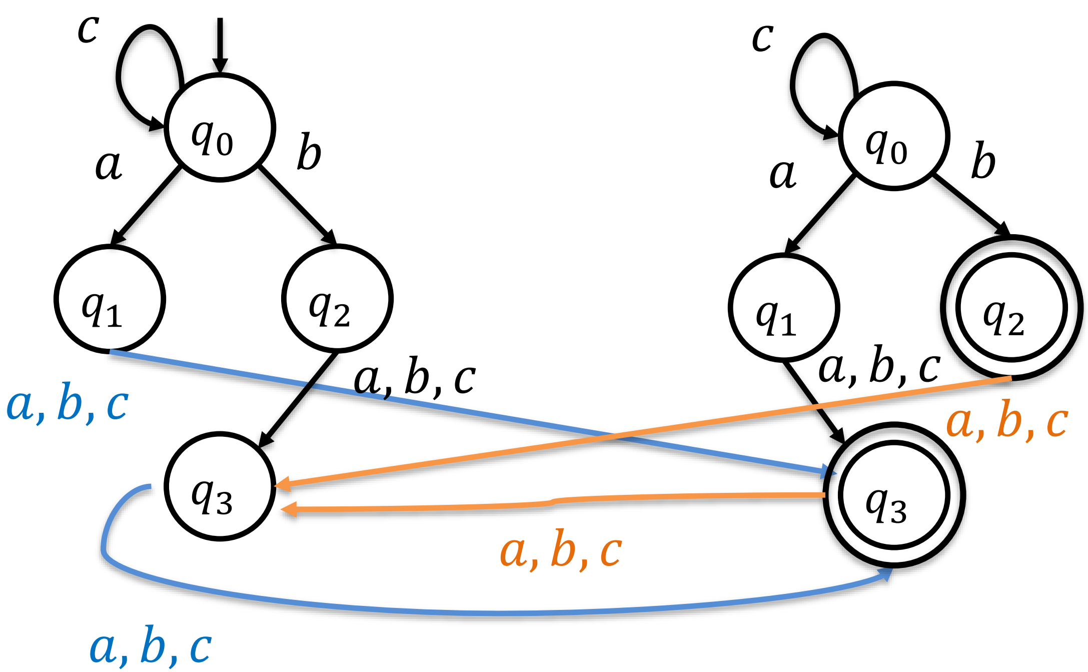


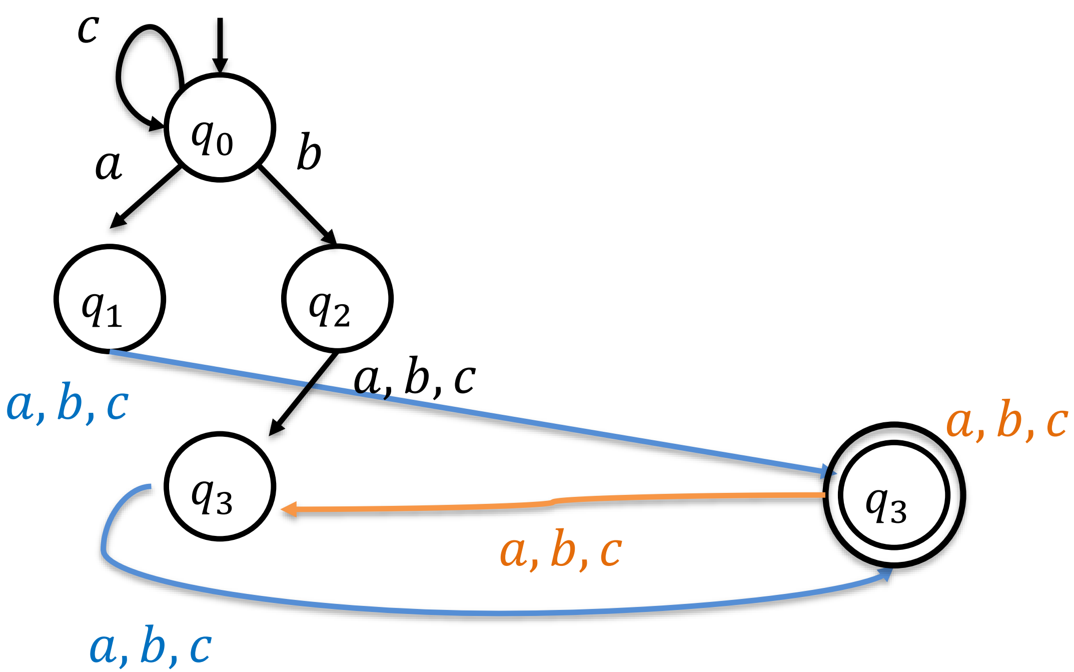
Example: Generalized Büchi to Büchi

- Translate $\mathcal{B}$ to a Büchi automaton $\mathcal{B}$'
  - Only one copy is accepting



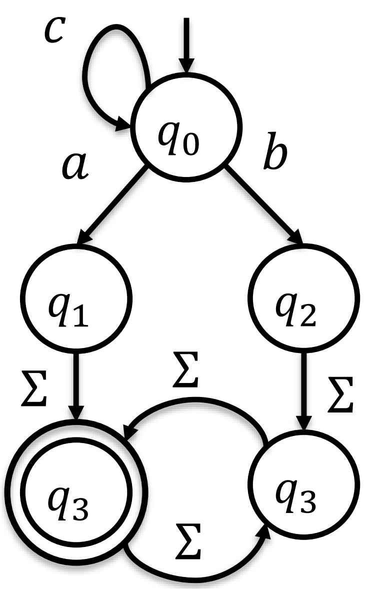
Example: Generalized Büchi to Büchi

- Translate $\mathcal{B}$ to a Büchi automaton $\mathcal{B}$'
  - Remove unreachable states



Example: Generalized Büchi to Büchi

- Translate $\mathcal{B}$ to a Büchi automaton $\mathcal{B}$'
  - And here is our beautiful Büchi automaton.
  - What is the language of $\mathcal{B}$'?



Outline

- Finite automata on finite words
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Translation of LTL to Büchi automata

- We discuss simple algorithm from Vardi and Wolper (book p. 98)
- Size of automaton always exponential in the size of the specification

M.Y. Vardi, and P. Wolper.
An automata-theoretic approach to automatic program verification.
In Logic in Computer Science (LICS), pages 332-344, 1986

- More efficient algorithm by Gerth, Peled, Vardi and Wolper (book page 101)

R. Gerth, D. Peled, M.Y. Vardi, and P. Wolper.
Simple on-the-fly automatic verification of linear temporal logic.
In Protocol Specification Testing and Verification, pages 3-18, 1995

Translation of LTL to Büchi automata

Input: LTL specification φ
Output: Büchi automaton $\mathcal{A}_\varphi$

- $\mathcal{A}_\varphi$ accepts exactly all the traces that satisfy φ

Steps of the Algorithm:

1. Rewriting of φ
2. Translate φ into generalized Büchi Automaton
3. Translate generalized Büchi to Büchi automaton

Translation of LTL to Büchi automata

Input: LTL specification φ
Output: Büchi automaton $\mathcal{A}_\varphi$

- $\mathcal{A}_\varphi$ accepts exactly all the traces that satisfy φ

Steps of the Algorithm:

1. Rewriting of φ
   - Algorithm only handles $\neg, \wedge, \vee, X, U$
   - Use rewriting rules
     - $F\varphi = true\ U\varphi$
     - $G\varphi = \neg F \neg \varphi$

From LTL formula φ to GBA $\mathcal{A}_\varphi$

- Step 1: Based on φ, we define the **state space** of $\mathcal{A}_\varphi$

- Each state of the automata is **labelled** with **a set of sub-formulas** that should be satisfied **on paths starting at that state.**
  - Thus, we only consider consistent subsets

Form State Space of $\mathcal{A}_\varphi$

- Build the closure cl(φ) of φ
  - … subformulas of φ and their negation

Form State Space of $\mathcal{A}_\varphi$

- Build the closure cl(φ) of φ
  - … subformulas of φ and their negation
- Formally:
  - $\varphi \in cl(\varphi)$.
  - **If $\boldsymbol{\varphi_1 \in cl(\varphi), then \neg\varphi_1 \in cl(\varphi)}$.**
  - **If $\boldsymbol{\neg\varphi_1 \in cl(\varphi), then\ \varphi_1 \in cl(\varphi)}$.**
  - If $\varphi_1 \vee \varphi_2 \in cl(\varphi), then\ \varphi_1 \in cl(\varphi)\ and\ \varphi_2 \in cl(\varphi)$.
  - If $X\ \varphi_1 \in cl(\varphi), then\ \varphi_1 \in cl(\varphi)$.
  - If $\varphi_1\ U\ \varphi_2 \in cl(\varphi), then\ \varphi_1 \in cl(\varphi)\ and\ \varphi_2 \in cl(\varphi)$.

Form State Space of $\mathcal{A}_\varphi$

- Build the closure cl(φ) of φ
  - … subformulas of φ and their negation
- Compute the **good** sets in cl(φ)
  - $S \subseteq cl(\varphi)$ is ***good** in cl(φ)* if S is a maximal set of formulas in *cl(φ)* that is **consistent**
    - For all $\varphi_1 \in cl(\varphi)$: $\varphi_1 \in S \Leftrightarrow \neg \varphi_1 \notin S$,
    - For all $\varphi_1 \vee \varphi_2 \in cl(\varphi)$: $\varphi_1 \vee \varphi_2 \in S \Leftrightarrow$ at least one of φ_1, φ_2 is in S.

Form State Space of $\mathcal{A}_\varphi$

- Build the closure cl(φ) of φ
  - … subformulas of φ and their negation
- Compute the **good** sets in cl(φ)
  - *S* ⊆ *cl*(φ) is ***good*** *in cl(φ)* if *S* is a maximal set of formulas in *cl(φ)* that is **consistent**
- The set of all **good** sets of cl(φ) defines the state space of $\mathcal{A}_\varphi$

Form State Space of $\mathcal{A}_\varphi$

- **Closure** *cl*(φ): subformulas of φ and their negation
- **Good Sets**: $S \subseteq cl(\varphi)$ if S is a maximal set of formulas in *cl(φ)* that is **consistent**

- Given: $\varphi \coloneqq \neg h \; \mathrm{U} \; c$.   What is the state space of $\mathcal{A}_\varphi$?
  - $cl(\varphi) \coloneqq \{h, \neg h, c, \neg c, \neg h \; \mathrm{U} \; c, \neg(\neg h \; \mathrm{U} \; c)\}$
  - $Q = \{\{h, c, \varphi\}, \{\neg h, c, \varphi\}, \{h, c, \varphi\}, \{\neg h, \neg c, \varphi\},$
    $\{h, c, \neg\varphi\}, \{\neg h, c, \neg\varphi\}, \{h, c, \neg\varphi\}, \{\neg h, \neg c, \neg\varphi\}\}$

Transition Relation of GBA $\mathcal{A}_\varphi$

$\mathcal{A}_\varphi = (\mathcal{P}(AP), \mathbf{Q}, \mathbf{\Delta}, \mathbf{Q}^0, \mathbf{F})$

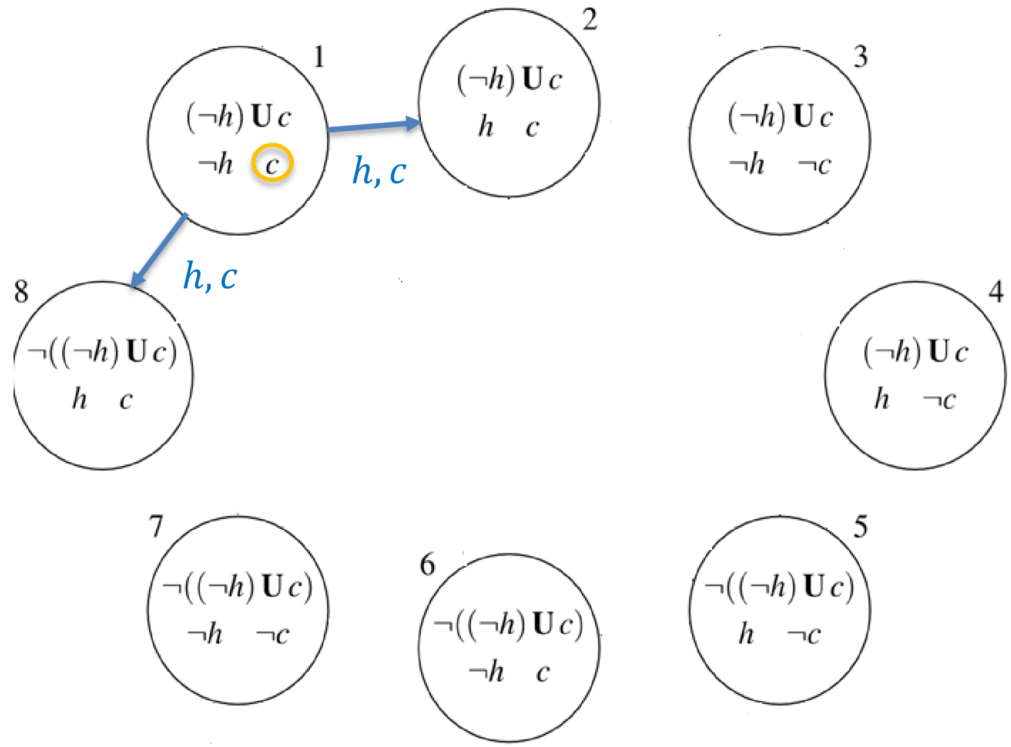
- $\mathbf{Q} \subseteq \mathcal{P}$ ($cl(\varphi)$) is the set of all the good sets in $cl(\varphi)$.
- For $q, q' \in Q$ and $\sigma \subseteq AP$, $(q, \sigma, q') \in \Delta$ if:
  - $\sigma = q' \cap AP$
  - For all $\boldsymbol{X}\varphi_1 \in cl(\varphi)$:
    - $\boldsymbol{X}\varphi_1 \in q \Leftrightarrow \varphi_1 \in q'$
  - For all $\varphi_1 U\, \varphi_2 \in cl(\varphi)$:
    - φ_1 U $\varphi_2 \in$ q $\Leftrightarrow$ either $\varphi_2 \in$ q or both $\varphi_1 \in$ q **and** φ_1 U $\varphi_2 \in$ q'

Each state of $\mathcal{A}_\varphi$ is **labelled** with **a set of properties** that should be satisfied **on all paths starting at that state**

Transition Relation of GBA $\mathcal{A}_\varphi$

$\varphi = (\neg h \; U \; c)$

Draw the transitions of $\mathcal{A}_\varphi$



For $q, q' \in Q$ and $\sigma \subseteq$ AP, $(q, \sigma, q') \in \Delta$ if:

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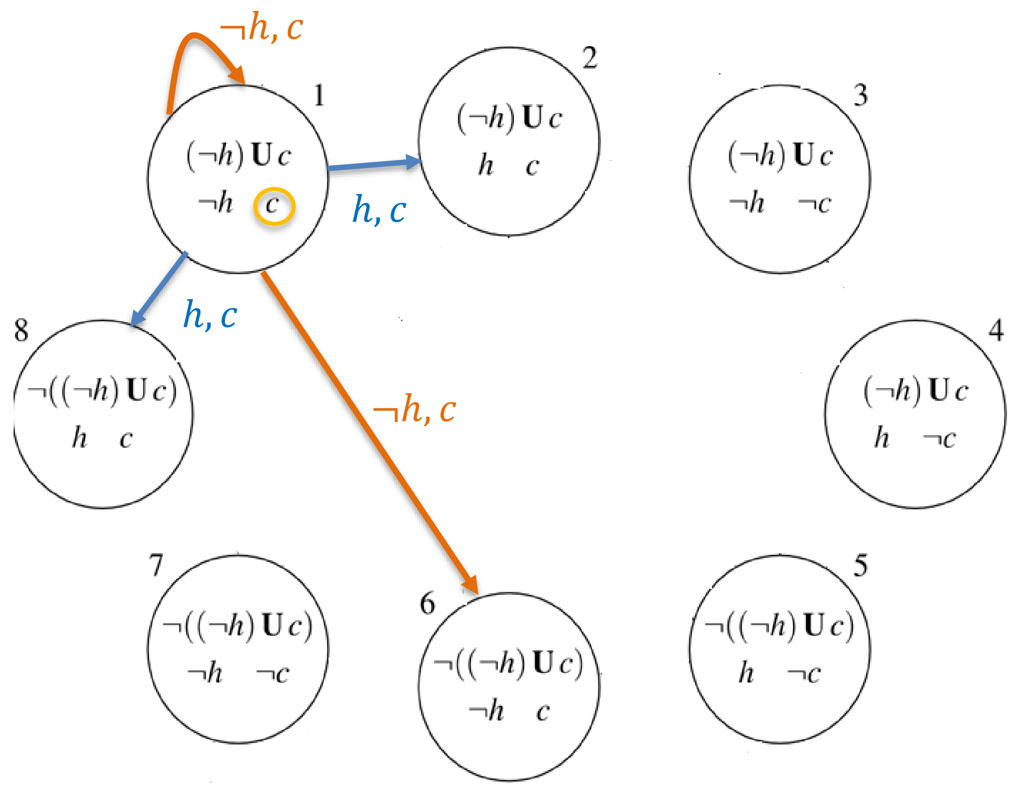
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \,\mathrm{U}\, c)$

Draw the transitions of $\mathcal{A}_\varphi$



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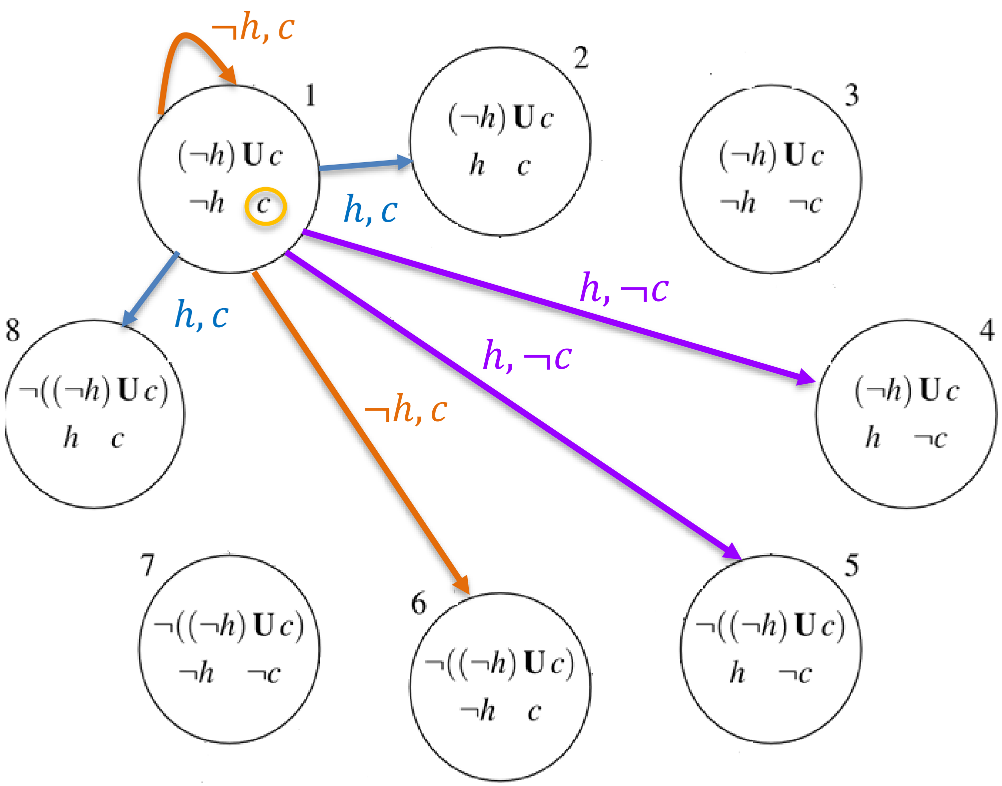
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \text{ U } c)$

Draw the transitions of $\mathcal{A}_\varphi$



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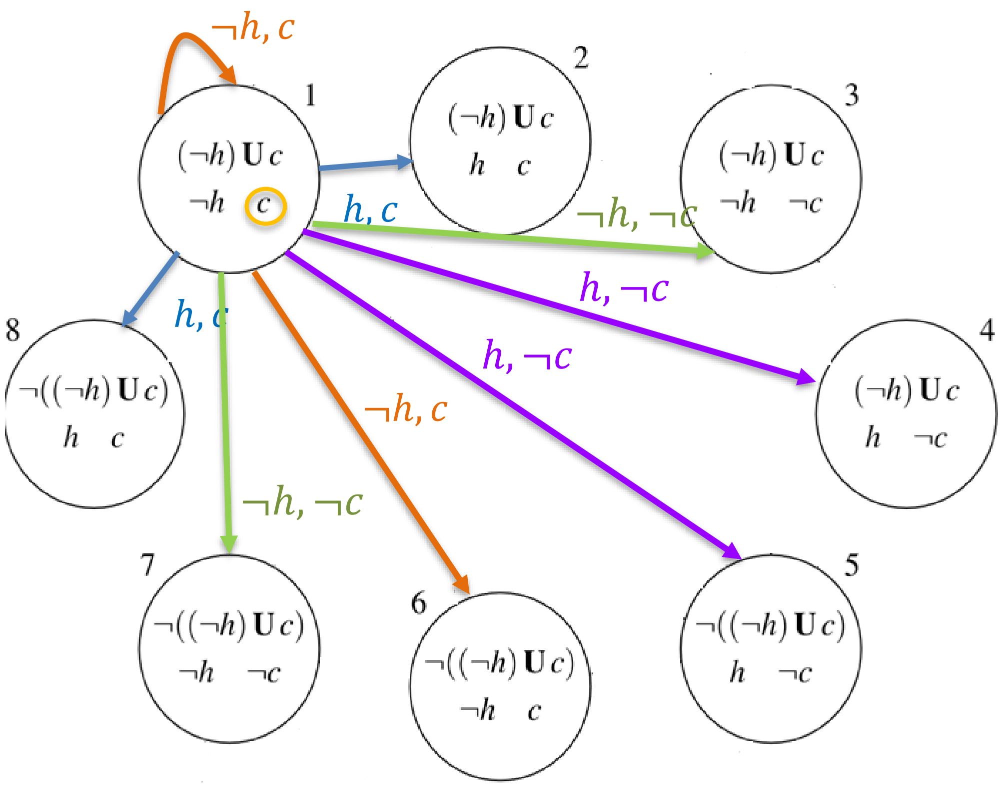
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \, U \, c)$

Draw the transitions of $\mathcal{A}_\varphi$



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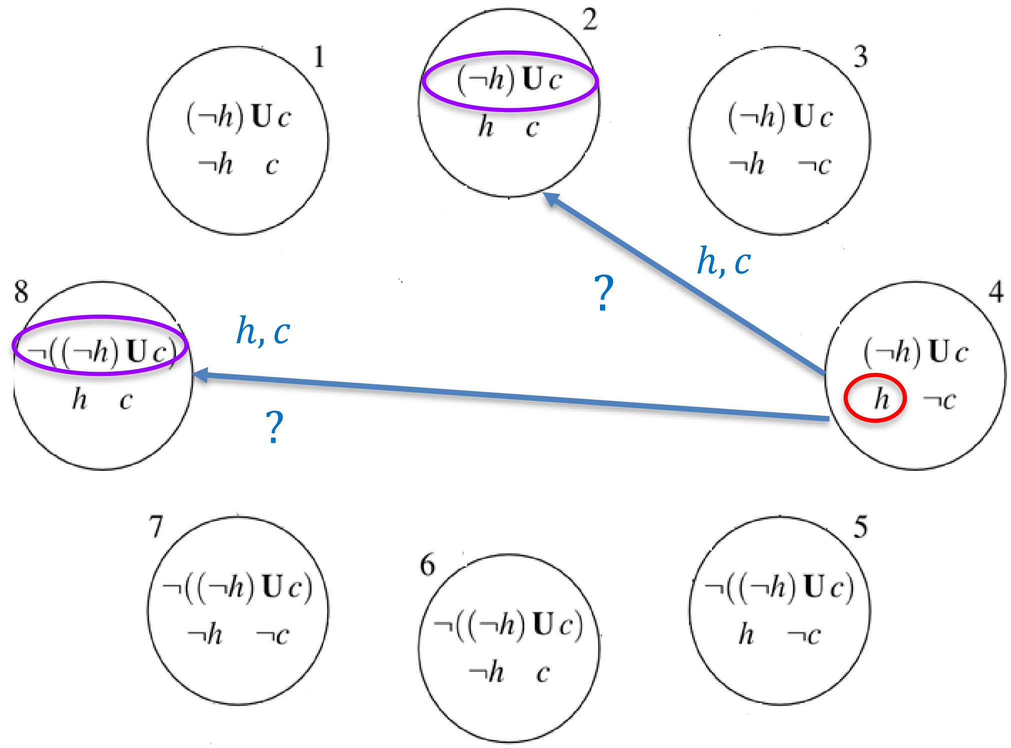
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \; U \; c)$

Draw the transitions of $\mathcal{A}_\varphi$



For $q, q' \in Q$ and $\sigma \subseteq AP$, $(q, \sigma, q') \in \Delta$ if:

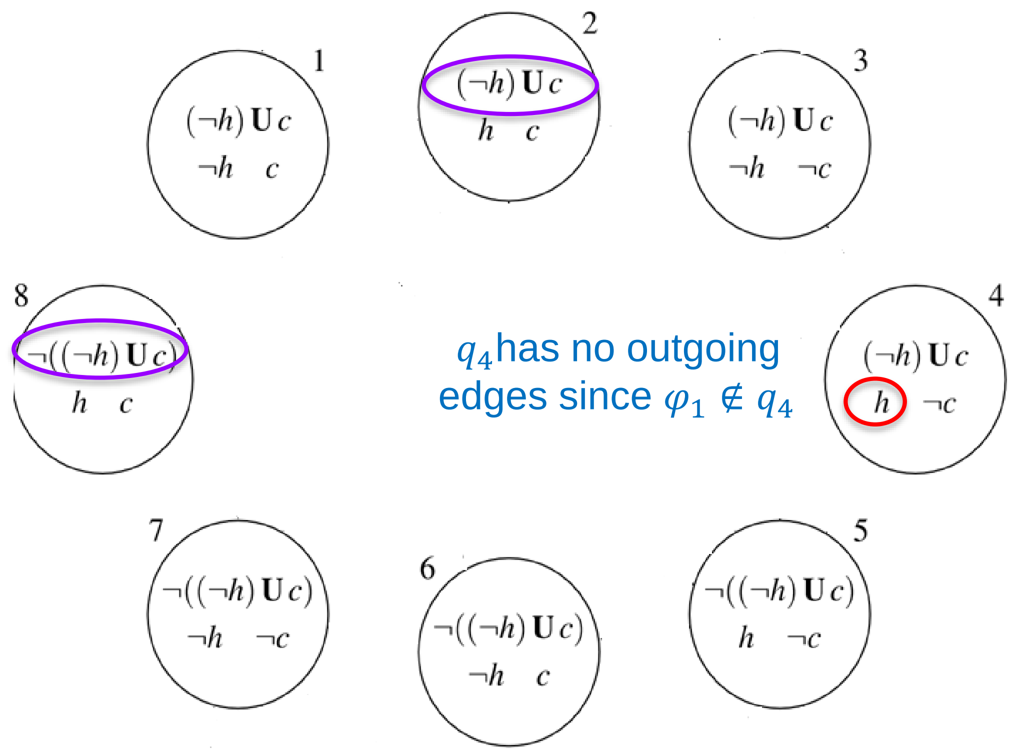
- $\sigma = q' \cap AP$
- For all $X\varphi_1 \in cl(\varphi)$:
  - $X\varphi_1 \in q \Leftrightarrow \varphi_1 \in q'$
- For all $\varphi_1 U \varphi_2 \in cl(\varphi)$:
  - $\varphi_1 \, U \, \varphi_2 \in q \Leftrightarrow$ either $\varphi_2 \in q$ or both $\varphi_1 \in q$ and $\varphi_1 \, U \, \varphi_2 \in q'$

Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \text{ U } c)$

Draw the transitions of $\mathcal{A}_\varphi$



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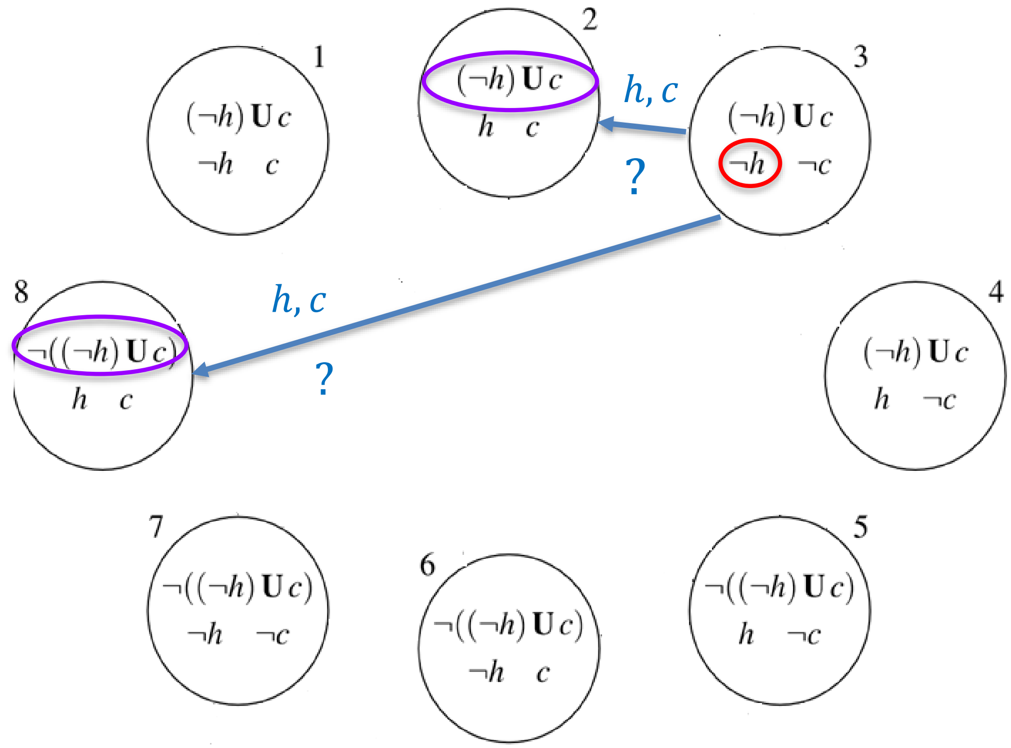
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \text{ U } c)$

Draw the transitions of $\mathcal{A}_\varphi$



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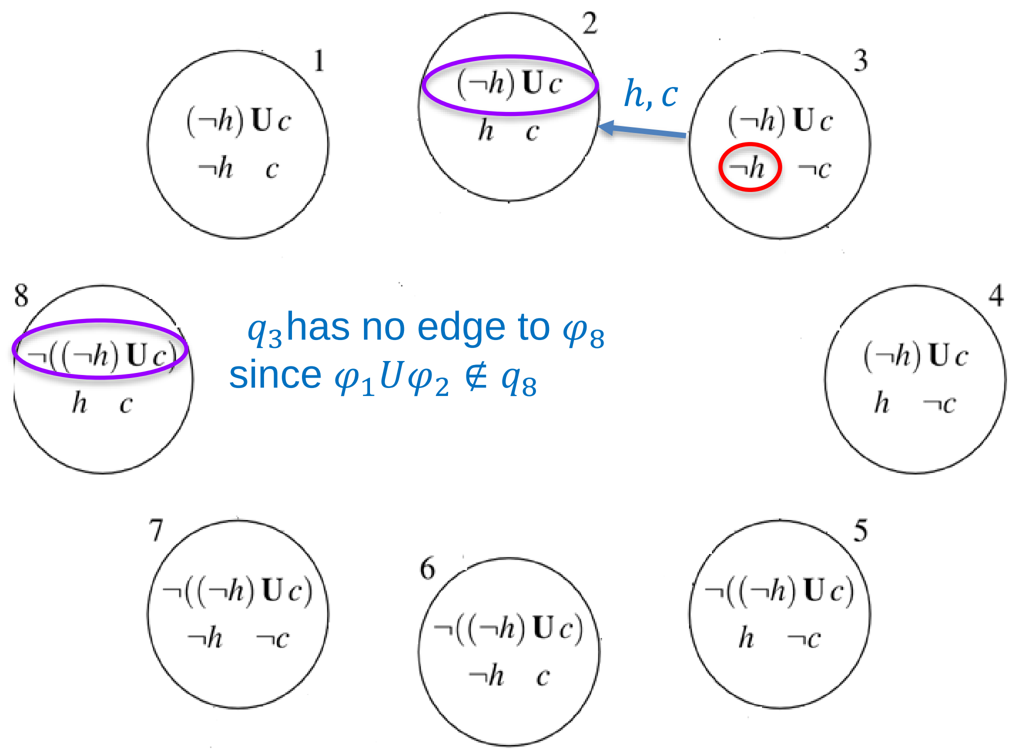
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \, U \, c)$

Draw the transitions of $\mathcal{A}_\varphi$



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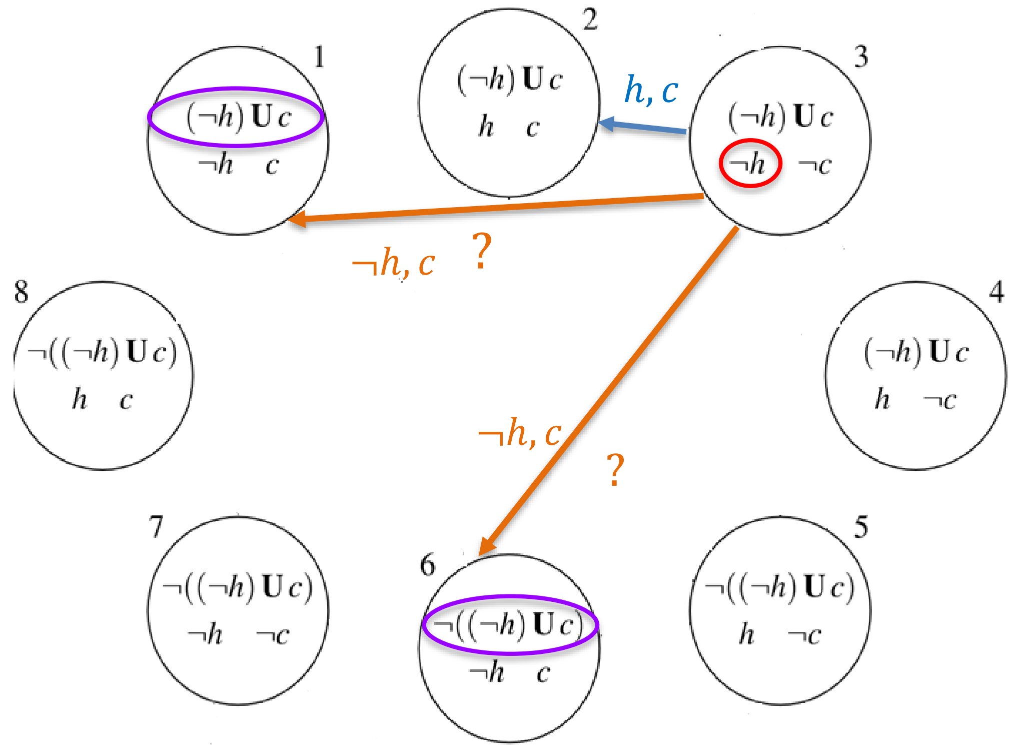
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \ U \ c)$

Draw the transitions of $\mathcal{A}_\varphi$



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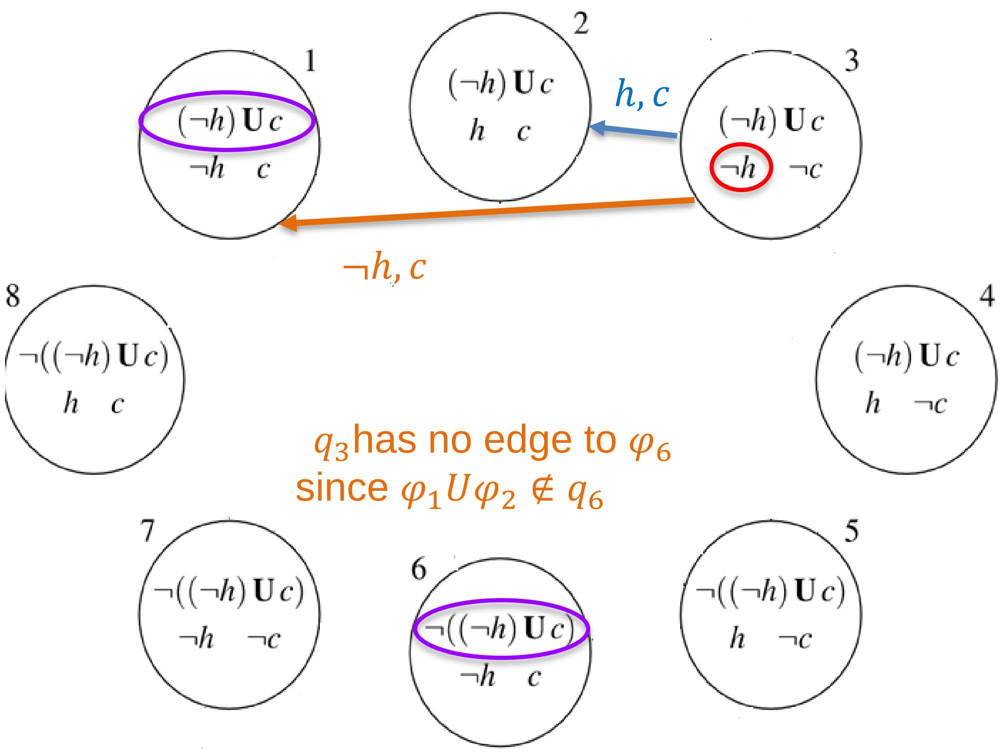
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \ U \ c)$

Draw the transitions of $\mathcal{A}_\varphi$



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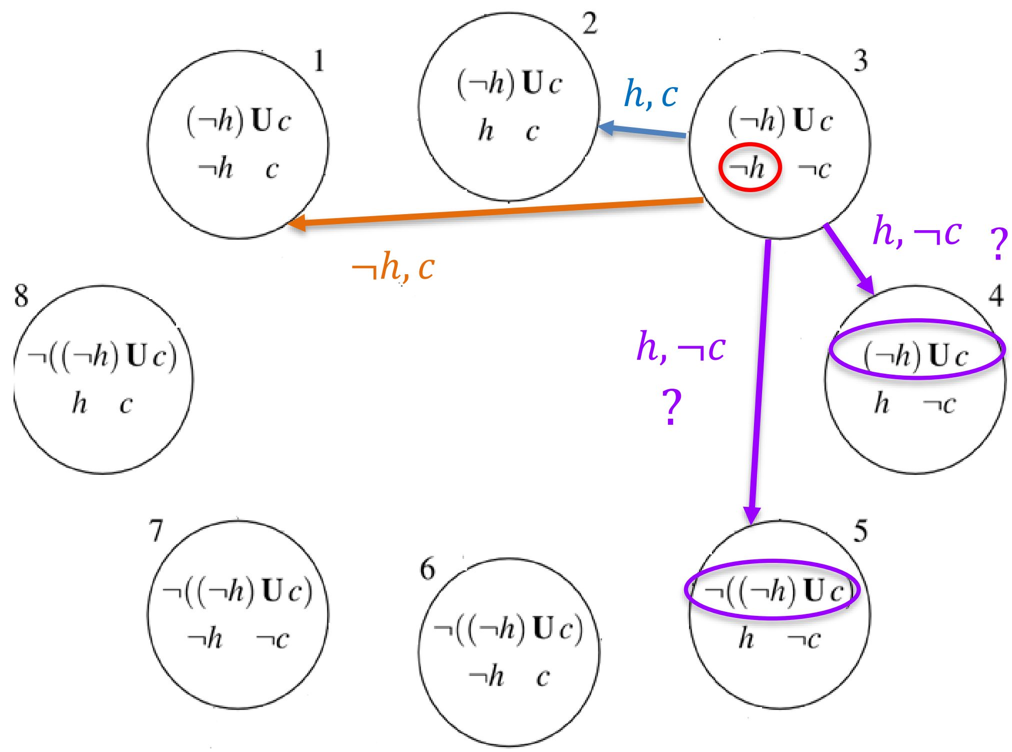
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \ U \ c)$

Draw the transitions of $\mathcal{A}_\varphi$



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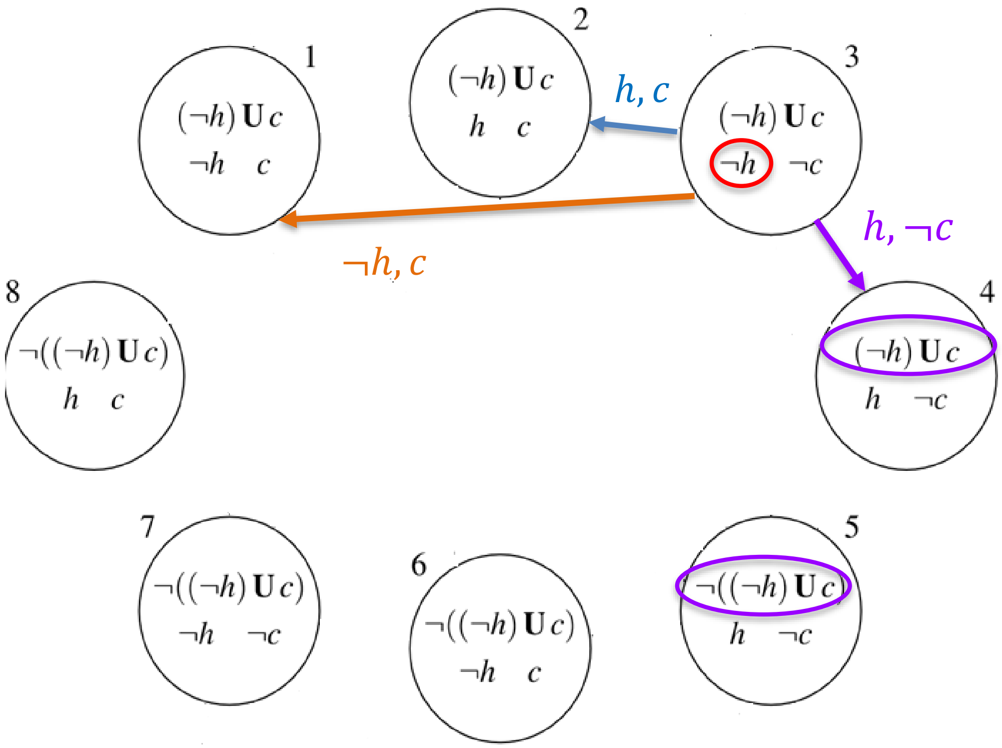
Transition Relation of GBA $\mathcal{A}_\varphi$

ToDo

$\varphi = (\neg h \; U \; c)$

Draw the transitions of $\mathcal{A}_\varphi$



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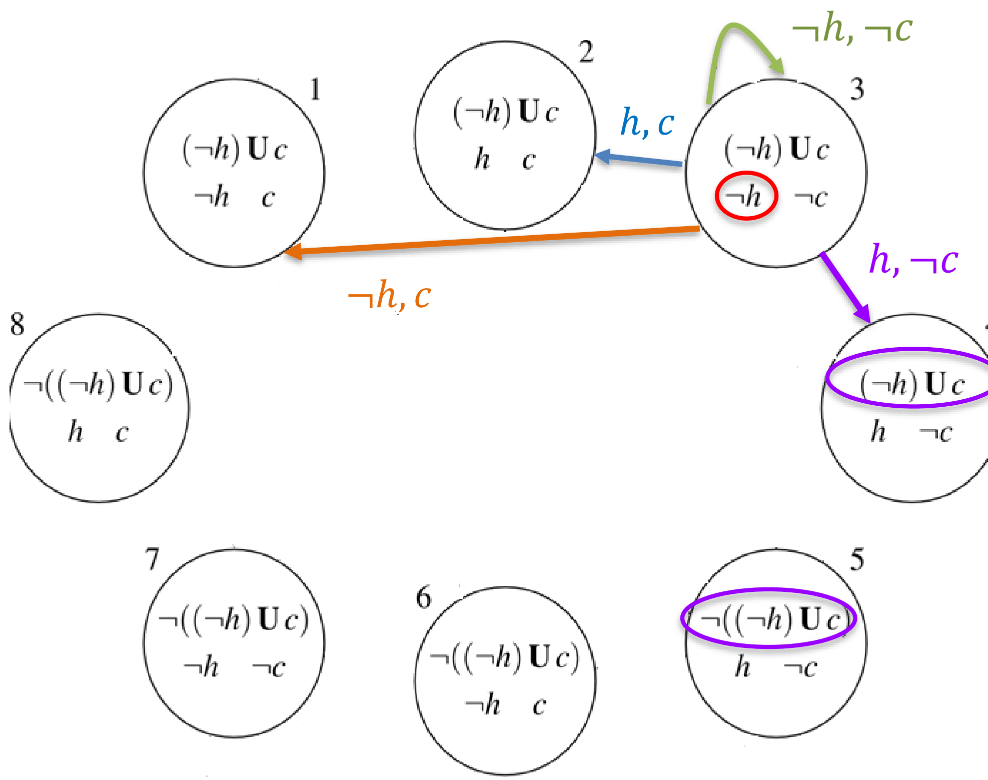
Transition Relation of GBA $\mathcal{A}_\varphi$

$\varphi = (\neg h \text{ U } c)$

Draw the transitions of $\mathcal{A}_\varphi$



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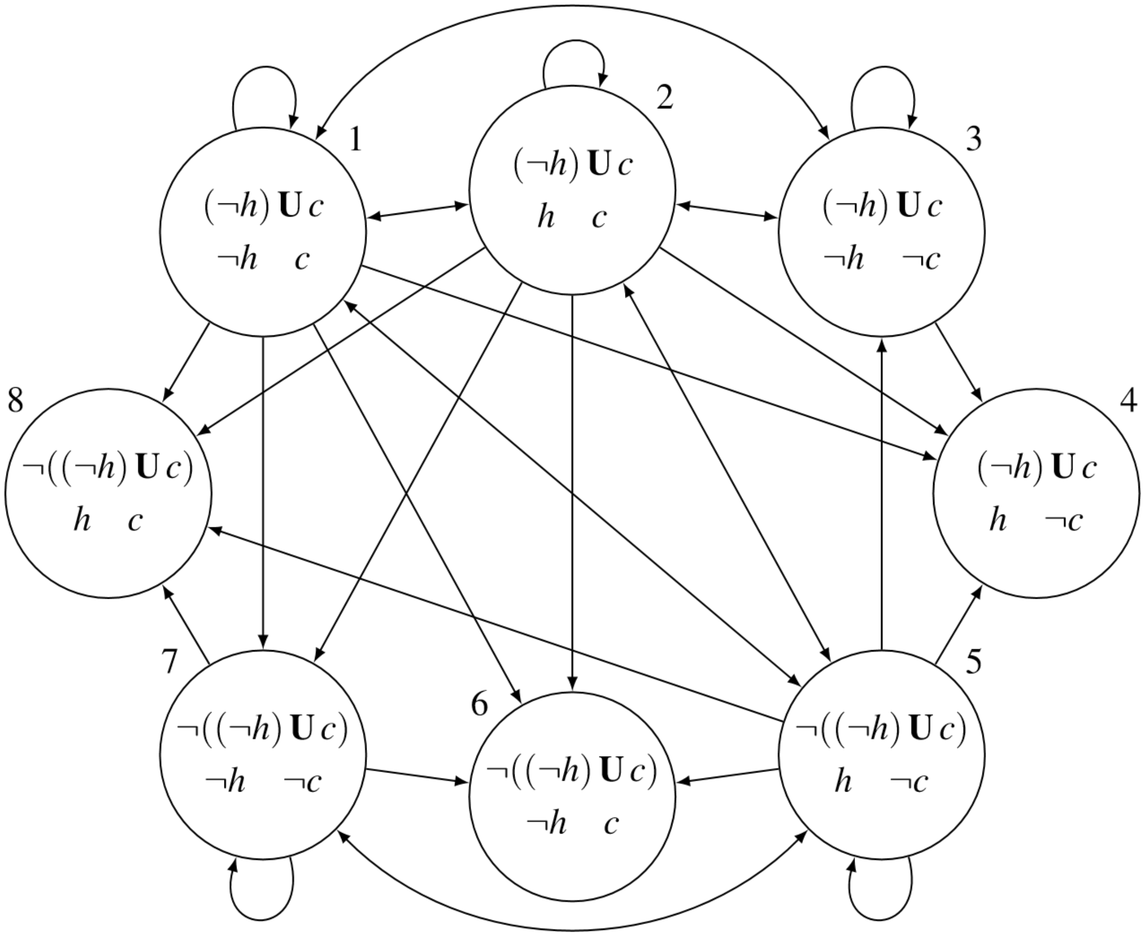
Transition Relation of GBA $\mathcal{A}_\varphi$

$\mathcal{A}_\varphi = (\mathcal{P}(\mathrm{AP}), \mathbf{Q}, \mathbf{\Delta}, \mathbf{Q}^0, \mathbf{F})$

- For $q, q' \in Q$ and $\sigma \subseteq \mathrm{AP}$, $(q, \sigma, q') \in \Delta$ if:
  - For all $\varphi_1 U \varphi_2 \in cl(\varphi)$:
    - φ_1 U φ_2 ∈ q ⇔ either φ_2 ∈ q or both
      φ_1 ∈ q **and** φ_1 U φ_2 ∈ q'
  - **For all** $\neg(\boldsymbol{\varphi_1 U \varphi_2}) \in \boldsymbol{cl(\varphi)}$**:**
    - **¬(φ_1 U φ_2) ∈ q ⇔ either ¬ φ_2 ∈ q and either**
      ¬ φ_1 ∈ q or ¬ (φ_1 U φ_2) ∈ q'

Transition Relation of GBA $\mathcal{A}_\varphi$



Initial States & Sets of Accepting States of $\mathcal{A}_\varphi$

$\mathcal{A}_\varphi = (\mathcal{P}(\text{AP}), \mathbf{Q}, \boldsymbol{\Delta}, \mathbf{Q}^0, \mathbf{F})$

- $\mathbf{Q} \subseteq \mathcal{P}(cl(\varphi))$ is the set of all the good sets in $cl(\varphi)$.
- Δ : Slide before

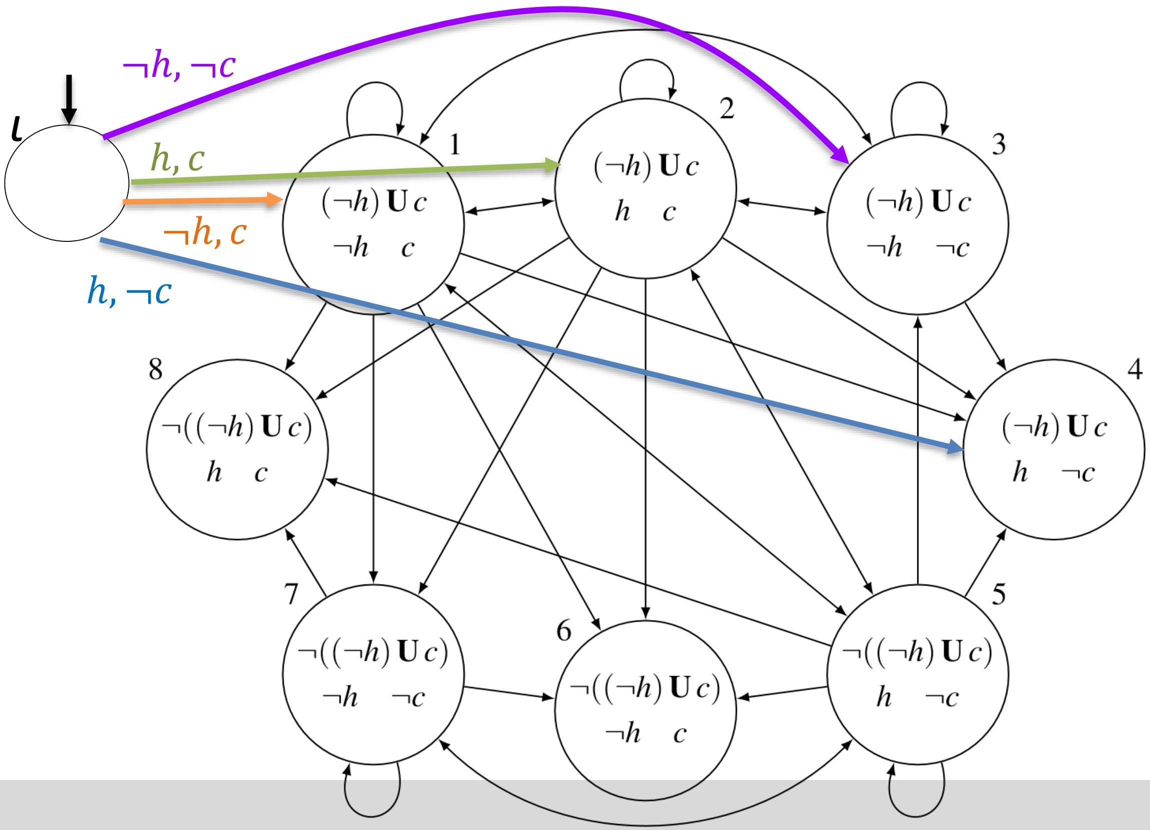
ToDo

- What is the set of initial state?
- What are the sets of accepting states?

Initial States of $\mathcal{A}_\varphi$

$\mathcal{A}_\varphi = (\mathcal{P}(AP), \mathbf{Q}, \boldsymbol{\Delta}, \mathbf{Q}^0, \mathbf{F})$

- $\mathbf{Q} \subseteq \mathcal{P}(cl(\varphi)) \cup \{\iota\}$ is the set of all the good sets in $cl(\varphi) \cup \{\iota\}$.
- Δ : Slide before + $(\iota, \sigma, q) \in \boldsymbol{\Delta} \Leftrightarrow \varphi \in \mathbf{q}$ and $\sigma = q \cap AP$



Set of Accepting States of $\mathcal{A}_\varphi$

$\mathcal{A}_\varphi = (\mathcal{P}(AP), \mathbf{Q}, \mathbf{\Delta}, \mathbf{Q}^0, \mathbf{F})$

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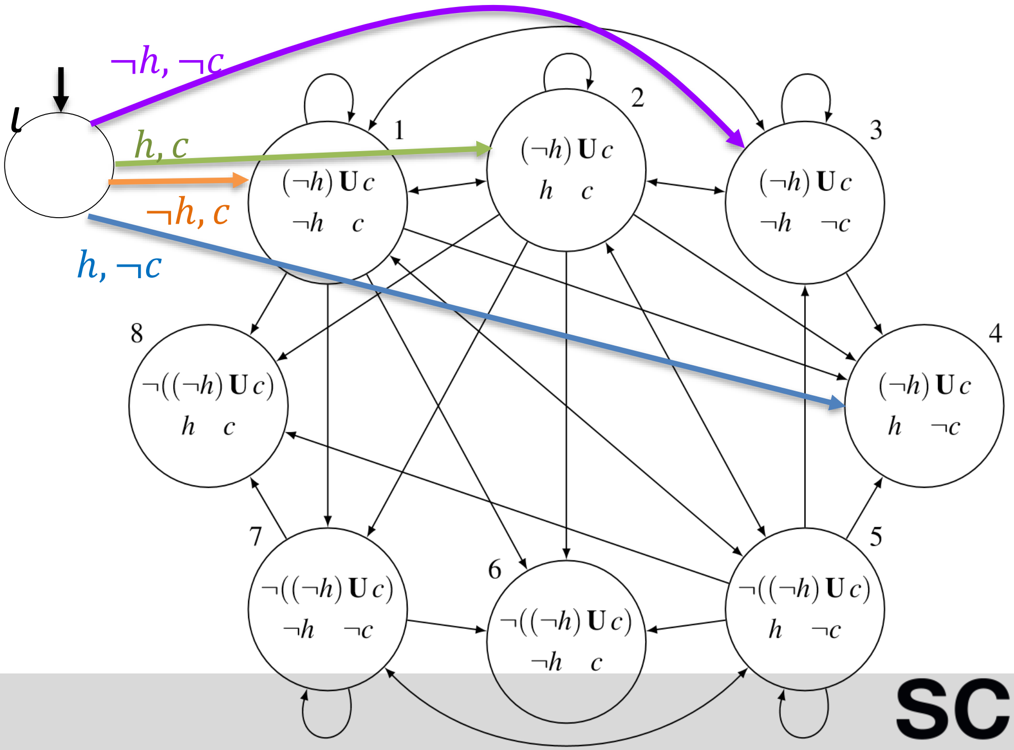
- Δ : Slide before + $(\iota, \sigma, q) \in \mathbf{\Delta} \Leftrightarrow \varphi \in q$ and $\sigma = q \cap AP$
- **For every φ_1 U $\varphi_2 \in cl(\varphi)$, $\mathbf{F}$ includes the set**
  - $F_{\varphi_1 \mathbf{U} \varphi_2} = \{q \in \mathbf{Q} \mid \varphi_2 \in q$ or $\neg(\varphi_1 \mathbf{U} \varphi_2) \in q\}$.

What are the sets of accepting states?



Set of Accepting States of $\mathcal{A}_\varphi$

$\mathcal{A}_\varphi = (\mathcal{P}(AP), \mathbf{Q}, \mathbf{\Delta}, \mathbf{Q}^0, \mathbf{F})$

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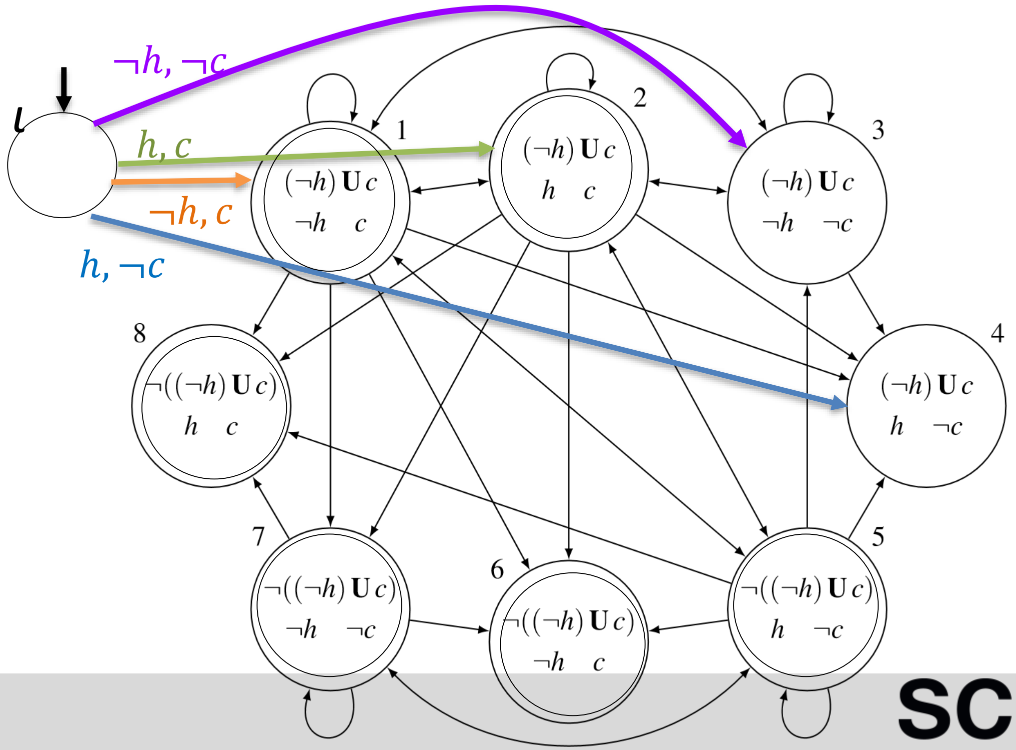
F = {{1, 2, 5, 6, 7, 8}}




THANK
YOU!