Logic and Computability

Lecture 5

Introduction to Z3

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- Today: Basics Principles of Z3 and First Problems

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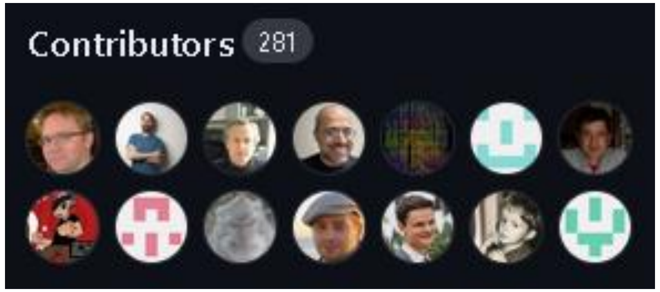

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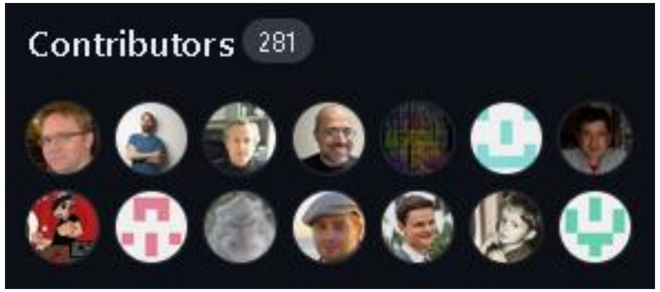

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  - Checking for Satisfiability

    ```
    (check-sat)
    ```

A Simple Example in SMT-LIB2

```
(declare-const a Bool)
(declare-const b Bool)
(assert (not a) )
(assert (or a b) )
(check-sat)
(get-model)
```

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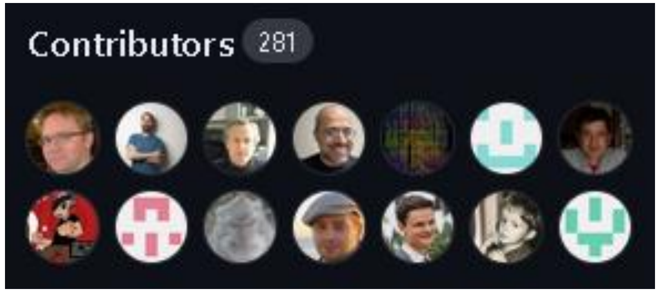

- SMT-LIB2 – A standardized language for Problems in SMT
- API for C++, Python, Julia, etc.

Installing

- We will use the `Python API`:
  - `pip install z3-solver`

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- Optionally, you may install z3 natively:
  - `sudo apt-get install z3` (Via aptitude for Ubuntu, etc.)
  - https://www.nuget.org/packages/Microsoft.Z3/ (Windows)
  - https://jfmc.github.io/z3-play (online)

Python API

- User-friendly interface for `SMT-LIB2`
- Used in the Programming Assignment

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solver.add(Not(a))
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(check-sat)
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→

```
solver.check()
```

Python API

```
from z3 import *

a, b = Bools("a b")

solver = Solver()
solver.add(Not(b))
solver.add(Or(a,b))

print(solver.sexpr())
result = solver.check()
model = solver.model()
print(result)
print(model)
```

Python API

- Constraints

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(assert (not a) )        →    solver.add(Not(a))
(assert (or a b) )       →    solver.add(Or(a,b))
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- Operator overloading:
  - +, -, >>, <<, etc.

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- Reference: https://z3prover.github.io/api/html/namespacez3py.html

A First Example

- We want to show that the following formulas are equal:
  - $p \rightarrow q$
  - $\neg p \vee q$

A First Example

- $p \rightarrow q == \neg p \vee q$?

```
from z3 import *

solver = Solver()
a, b = Bools("a b")
l, r = Bools("l r")

solver.add(l == Implies(a, b))
solver.add(r == Or(Not(a), b))
solver.add(Distinct(r,l) )

result = solver.check()
print(result)
```

Back to SMT-LIB2

- $p \rightarrow q == \neg p \wedge q$?

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- `BitVectors` respect under-/overflow behaviour!
  - In contrast to Z3’s integers

Operations on BitVectors

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  - Use `ULT`, `UGT`, `ULE` for unsigned interpretations

Equivalence Checking for BitVectors

- We want to prove the equivalence of the following
  - `(((y & x)*-2) + (y + x))`
  - x $\oplus$ y

Weird XOR

```
from z3 import *

x = BitVec('x', 32)
y = BitVec('y', 32)

output = BitVec('output ', 32)

s = Solver()
s.add(x^y==output)
s.add(Distinct(((y & x)* -2) + (y + x),output))

print(s.check())
```

Overflow Behaviour

- We want to check whether z3 can find a model for the following
  - `x = BitVector("x", 8)`
  - `(x + 1 < x - 1)`

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- Overflow and Underflow
  - `BVAddNoOverflow, BVAddNoUnderflow`
  - `BVMulNoOverflow, BVMulNoUnderflow`

Variables in a Satisfying Model

- Variables and Expressions are stored in z3-specific classes
- We can use `solver.model().decls()` to iterate through all declared variables
  - Use `.as_long()` to convert a BitVector to a Python Integer

```
model = solver.model()
for var in solver.model.decls():
    print(f"{var}: {model[var]}(:{type(model[var])})")
```

Overflow Behaviour

- We want to check whether the statement TODO
  - `(x + 1 < x - 1)`
- We need to add
  - `BVNoOverflow(x,1,True)`
  - `BVNoUnderflow(x,1,True)`
- Functions that evaluate to `False` when Over-/Underflow would occur in the model

Assignment Sheet

- 4 Exercises + 1 Bonus Exercise

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- Deadline: 05. 06. 2024