Logic and Computability SS24, Assignment 3

Due: 24. 04. 2024, 23:59

1 Combinational Equivalence Checking

We list the *Tseitin-rewriting rules* to be applied for the following examples.

$$\chi \leftrightarrow (\varphi \vee \psi) \quad \Leftrightarrow (\neg\varphi \vee \chi) \wedge (\neg\psi \vee \chi) \wedge (\neg\chi \vee \varphi \vee \psi)$$
$$\chi \leftrightarrow (\varphi \wedge \psi) \quad \Leftrightarrow (\neg\chi \vee \varphi) \wedge (\neg\chi \vee \psi) \wedge (\neg\varphi \vee \neg\psi \vee \chi)$$
$$\chi \leftrightarrow \neg\varphi \quad \Leftrightarrow (\neg\chi \vee \neg\varphi) \wedge (\chi \vee \varphi)$$

1. [3 points] Apply the Tseitin transformation to $\varphi = \neg(\neg b \wedge \neg c) \vee (\neg c \wedge a)$. For each variable you introduce, clearly indicate which subformula it represents.

2. [3 points] Apply the Tseitin transformation to $\varphi = \neg(p \rightarrow q) \wedge (r \wedge p)$. For each variable you introduce, clearly indicate which subformula it represents. Derive the Tseitin transformation rule for $\rightarrow$ or transform the input such that you can use the rules above.

2 Predicate Logic

3. [2 points] Translate the following sentence into predicate logic. Be as precise as possible. Give the meaning of any function and predicate symbols you use.

   Every even integer greater than 2 is equal to the sum of two prime numbers.

4. [2 points] Translate the following sentence into predicate logic. Be as precise as possible. Give the meaning of any function and predicate symbols you use.

   Every person who has the same parents as John Doe and is different from John Doe himself is a sibling of John Doe.

5. [2 points] Given is the following formula in predicate logic

$$\varphi = \exists x \forall y \Big(\big(P(x,y) \to Q(x,y) \big) \vee \big(P(y,x) \to R(x,y) \big) \Big)$$

   and the model $\mathcal{M}$:

   - $\mathcal{A} = \{a, b\}$
   - $P^M = \{(a,b), (b,b), (b,a)\}$
   - $Q^M = \{(a,a)\}$
   - $R^M = \{(b,b)\}$

   Does the model $\mathcal{M}$ satisfy the formula φ? Evaluate $\mathcal{M}$ using a syntax tree.

6. [3 points] For the formula below, state one model that satisfies the formula, and one model that does not satisfy the formula. Explain your answer by drawing a syntax tree and evaluate your models with the help of this syntax tree.

$$\forall x \exists y (P(f(x), y) \wedge \neg P(x, f(y)))$$