Modeling Systems

Chapter 3

- Exercise handout today

Modeling Systems

Systems and Correctness

- We consider a broad range of systems
  - Hardware (digital circuitry)
  - Software

- We want to check that the system is correct
  - Meets high-level requirements
  - Captured in the form of system properties

Why Model?

Specification

States what you want to prove

System

Abstract away unnecessary details

- How does the OS scheduler work?
- How is the CPU pipeline implemented?
- What are the voltage levels in the CPU?

But careful!

- Carelessly implemented CPUs introduce side channels
- Alpha particles may cause bits to flip
- Your formally verified system will fail when hit with a hammer
- …

What is a Model?

- A model is a description of the behavior of the system
- Behavior is
  - a set of observations
  - as the system evolves its state over time

- We check algorithmically that the model satisfies the properties
- To this end the model…
  - must have sufficient detail to prove the property
  - but should not be too complex

Transition Systems

- A transition system is a formal model
- Formal models enable formal proof

Kripke Structures

Inputs

- Inputs are fully under the control of the environment
- We can use nondeterminism to model inputs

Inputs: Light Switch Example

- Input: “button pressed” or “button released”, controlled by a hand, which is part of the environment

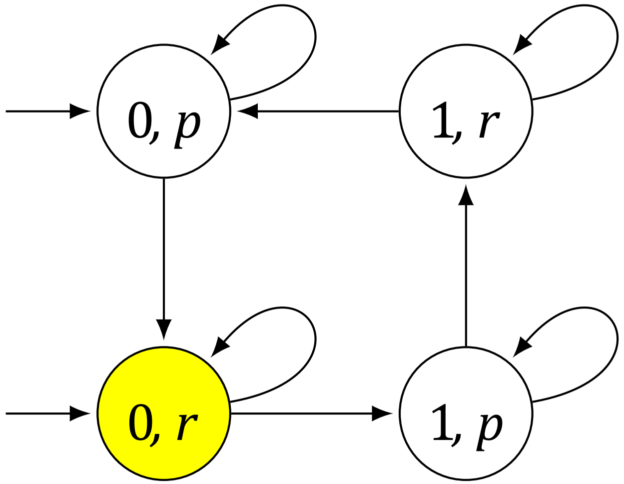
- Output: “light on” or “light off”

- Button is “retractive”, it bounces back
- When the light is off, pushing the button turns the light on
- When the light is on, pushing the button turns the light off

Inputs : Light Switch Example

light switch
“released” = r

light bulb
“off” = 0

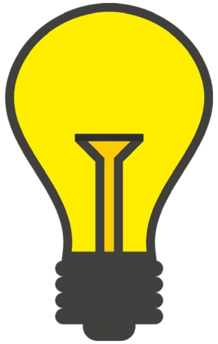


model of the controller

Inputs : Light Switch Example

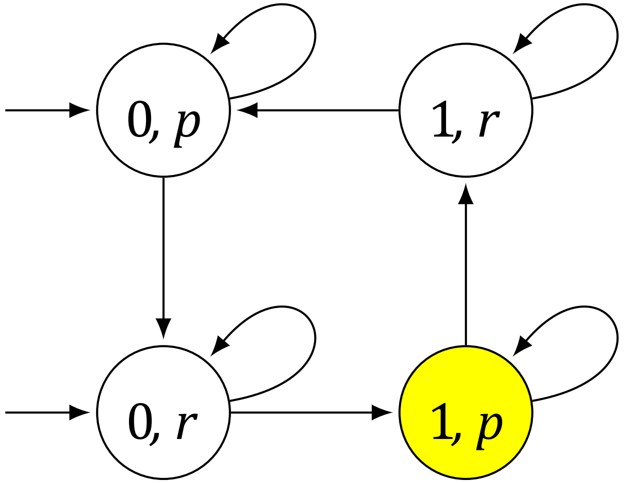
light switch

“pressed” = p



light bulb

“on” = 1



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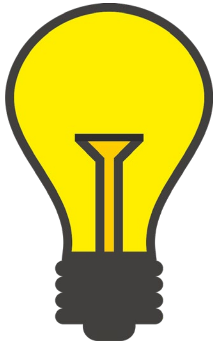

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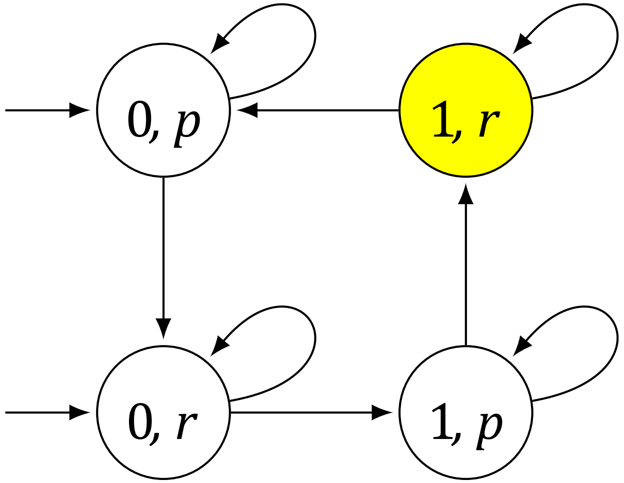

model of the controller

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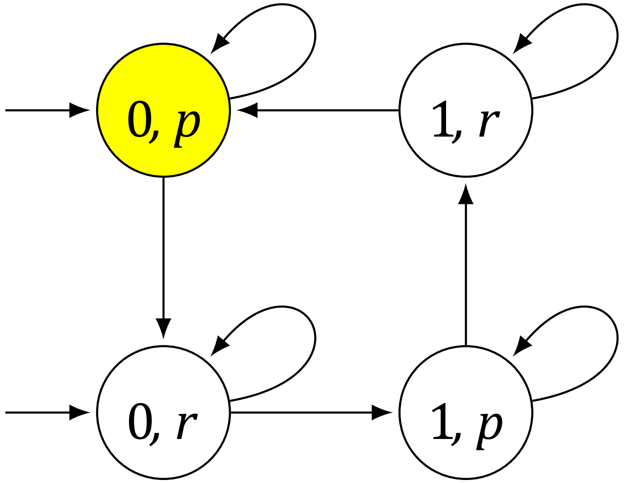

model of the controller

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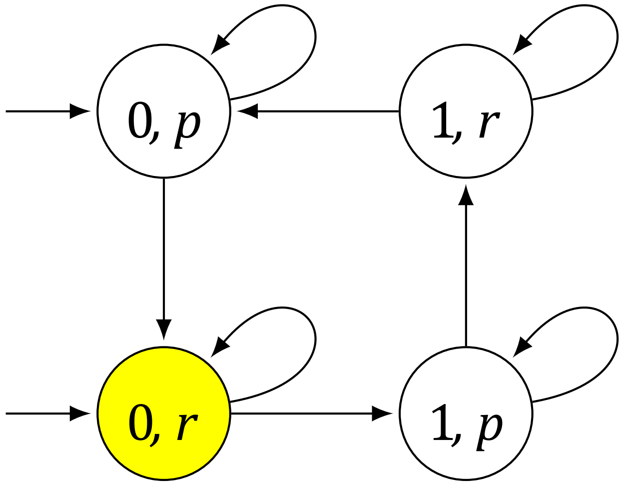

model of the controller

Kripke Structure $M = (S, S_0, R, AP, L)$

- S – (finite) set of states
- $S_0 \subseteq S$ – set of initial states
- $R \subseteq S \times S$ – left-total transition relation
  - For every $s \in S$ there exists $s' \in S$ such that $(s, s') \in R$
  - Left-total implies that every path is infinite
- AP – finite set of atomic propositions
- $L : S \rightarrow 2^{AP}$ – labeling function that associates every state with the atomic propositions true in that state

First-Order Logic and Symbolic Representations

Symbolic Representation

$V = \{v_1, \dots, v_n\}$ system variables

D_v domain of v

$s: V \to \bigcup_{v \in V} D\_v$ valuation, state

Example

Symbolic Representation

$V = \{v_1, \ldots, v_n\}$ system variables

D_v domain of v

$s: V \rightarrow \bigcup_{v \in V} D\_v$ valuation, state

Example

$V = \{v_1, v_2, v_3\}, D_{v_i} = \boldsymbol{N}$

State space: $\boldsymbol{N}^3$

examples of state: $\{(v_1, 2), (v_2, 3), (v_3, 8)\}$ (short: (2,3,8))

Characteristic Functions

In general, a formula is a set of states.

Characteristic Functions

In general, a formula is a set of states.

$v_1 = 2 \wedge v_2 = 3 \wedge v_3 = 8$	$(2,3,8)$
$v_1 = 2 \wedge v_2 = 3$	$\{ (2,3,n_3) \mid n_3 \in \boldsymbol{N} \}$
$v_2 = 3 \wedge v_3 = v_1 + v_2$	$\{ (n_1, 3, n_1 + 3) \mid n_1 \in \boldsymbol{N} \}$
$true$	$\boldsymbol{N}^3$

Sets and Formulas

Formula	Set
$\mathcal{A}, \mathcal{B}$	A, B
	$A \cup B$
	$A \cap B$
	S = $D_{v_1} \times \cdots \times D_{v_n}$
	$S \setminus A$

Example

$v_1 = 2 \wedge v_2 = 3$	$\{ (2,3,n_3) \mid n_3 \in \boldsymbol{N} \}$
$v_2 = 3 \wedge v_3 = v_1 + v_2$	$\{ (n_1, 3, n_1 + 3) \mid n_1 \in \boldsymbol{N} \}$

.

Sets and Formulas

Formula	Set
$\mathcal{A}, \mathcal{B}$	A, B
$\mathcal{A} \vee \mathcal{B}$	$A \cup B$
$\mathcal{A} \wedge \mathcal{B}$	$A \cap B$
$true$	S = $D_{v_1} \times \cdots \times D_{v_n}$
$\neg\mathcal{A} \wedge \mathcal{B}$	S $\setminus A$

Example

$v_1 = 2 \wedge v_2 = 3$	$\{ (2,3,n_3) \mid n_3 \in \boldsymbol{N} \}$
$v_2 = 3 \wedge v_3 = v_1 + v_2$	$\{ (n_1, 3, n_1 + 3) \mid n_1 \in \boldsymbol{N} \}$
$v_1 = 2 \wedge v_2 = 3 \wedge v_2 = 3 \wedge v_3 = v_1 + v_2$	$(2,3,5)$
$v_1 = 2 \wedge v_2 = 3 \vee v_2 = 3 \wedge v_3 = v_1 + v_2$	$\{ (2,3,n_3) \mid n_3 \in \boldsymbol{N} \} \cup \{ (n_1, 3, n_1 + 3) \mid n_1 \in \boldsymbol{N} \}$

Transition Systems

Example

System with variables x, y that range over $\{0,1\}$.

Initially, $(x, y) = (1,1)$ and then

$x \ := (x + y)\ mod\ 2.$

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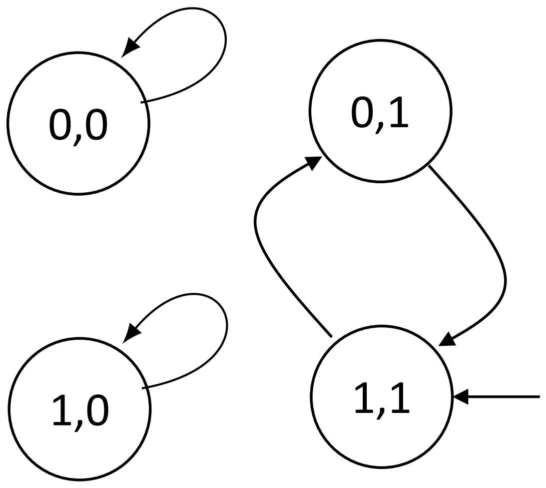


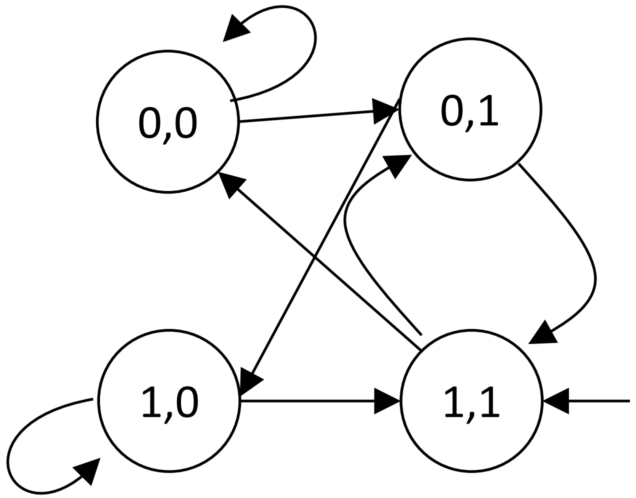
Kripke structure

Initial states: $\mathcal{S}_0(x, y) = x = 1 \wedge y = 1$

Transitions: $\mathcal{R}\ (x, y, x', y') = (x' = (x + y)\ mod\ 2) \wedge (y' = y)$

Modeling Digital Circuits

- Inputs are fully under the control of the environment
- We can use nondeterminism to model inputs



Kripke structure

Example

System with variables x, y that range over $\{0,1\}$.

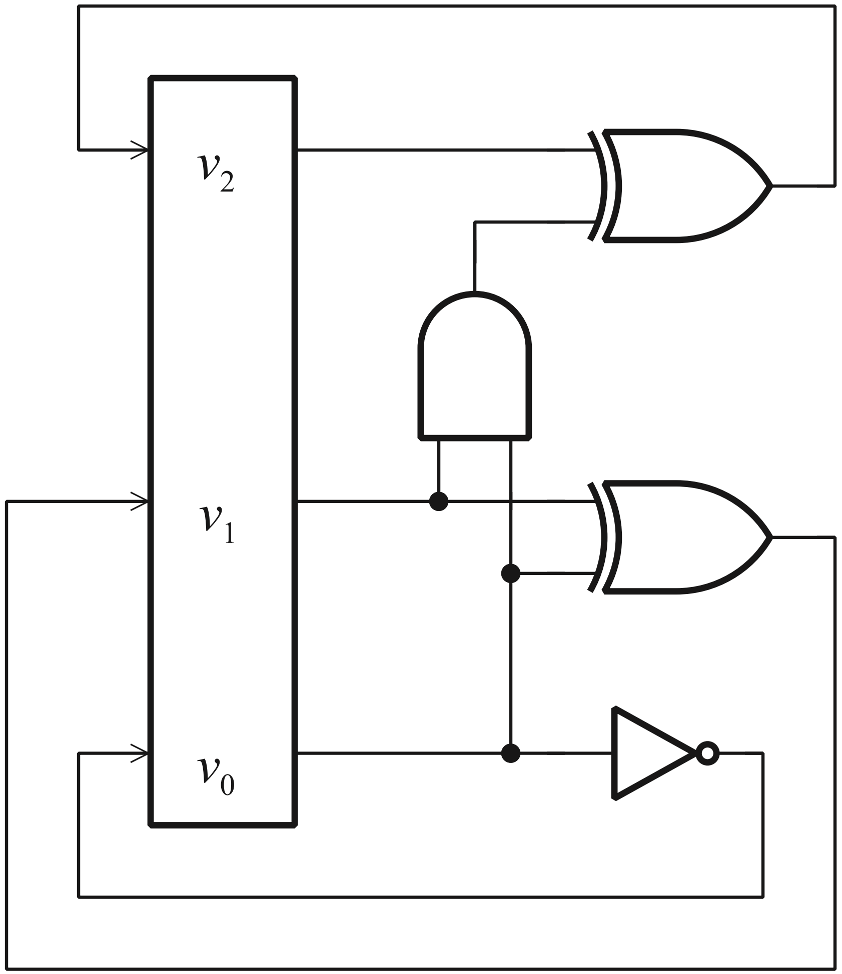
Initially, $(x, y) = (1,1)$ and then

- $x \coloneqq (x + y)\ mod\ 2.$
- y is an input

Initial states: $\mathcal{S}_0(x, y) = x = 1 \wedge y = 1$

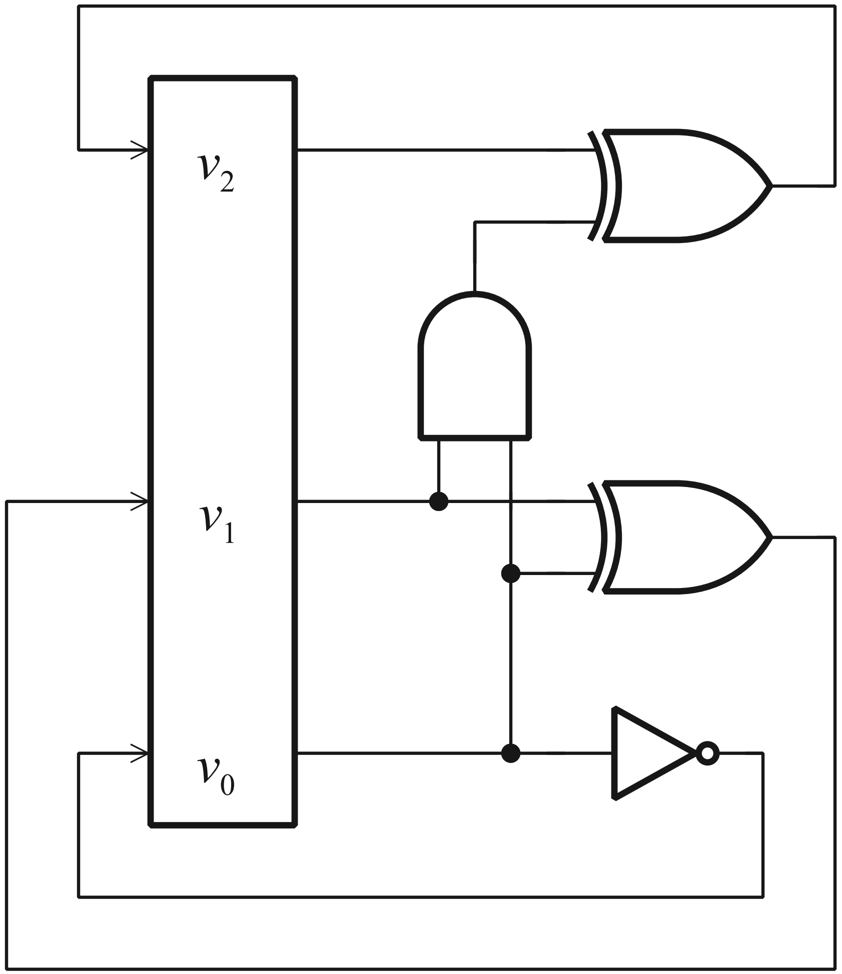
Transitions: $\mathcal{R}\ (x, y, x', y') = (x' = (x + y)\ mod\ 2)$

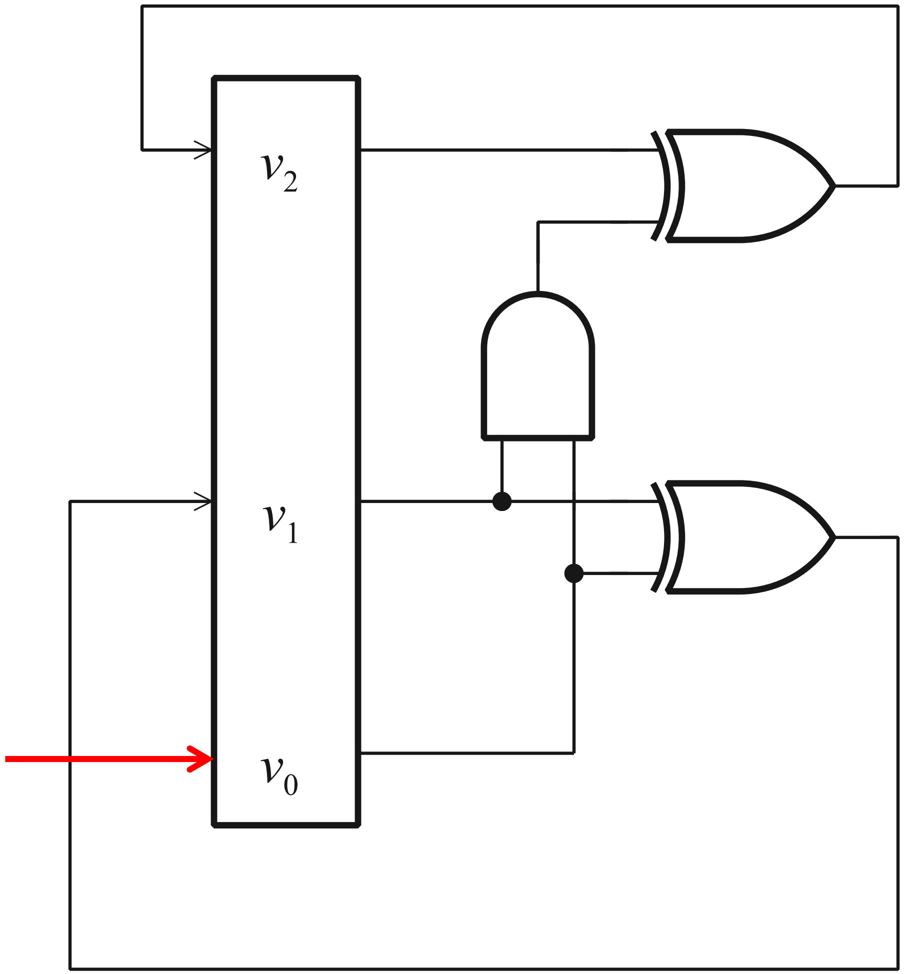
3-bit Counter



$$\mathcal{R}_0(V,V') = (v_0' \leftrightarrow \neg v_0)$$
$$\mathcal{R}_1(V,V') = (v_1' \leftrightarrow v_0 \oplus v_1)$$
$$\mathcal{R}_2(V,V') = \left(v_2' \leftrightarrow v_2 \oplus (v_0 \wedge v_1)\right)$$
$$\mathcal{R}(V,V') = \mathcal{R}_0 \wedge \mathcal{R}_1 \wedge \mathcal{R}_2$$

3-bit Counter





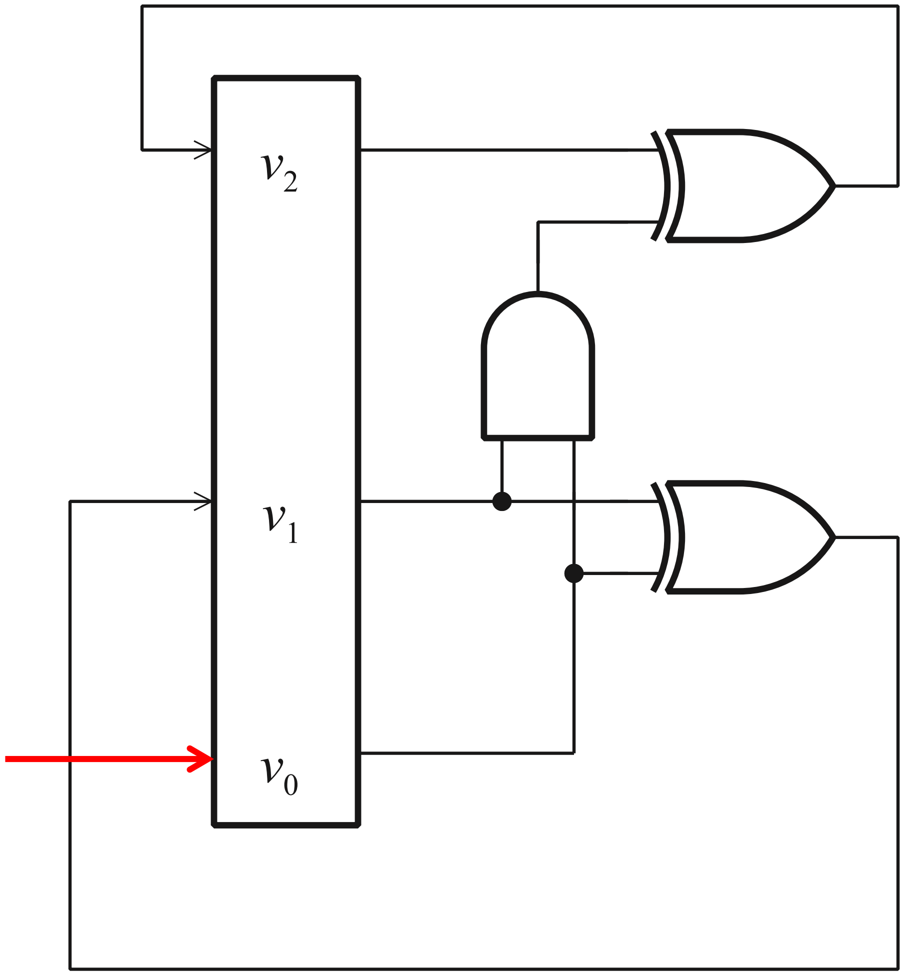
Inputs

Inputs can be anything - model as nondeterministic

$\mathcal{R}_0(V, V') = .$

$\mathcal{R}_1(V, V') = (v_1' \leftrightarrow v_0 \oplus v_1)$ $\mathcal{R}_2(V, V') = \left(v_2' \leftrightarrow v_2 \oplus (v_0 \wedge v_1)\right)$

Inputs



Inputs can be anything - model as nondeterministic

$\mathcal{R}_0(V, V') = true$ no constraints on v_1

$\mathcal{R}_1(V, V') = (v_1' \leftrightarrow v_0 \oplus v_1)$

$\mathcal{R}_2(V, V') = \big(v_2' \leftrightarrow v_2 \oplus (v_0 \wedge v_1)\big)$

What does the Kripke structure look like?

Symbolic Representations

Hope: Sets (transition relation, all reachable states) will have small formulas

We know

+ size of transition relation $\cong$ size of circuit, software
- To represent a subset of $\{1, \ldots, 2^k\}$ we need 2^k bits in general

We will try to find algorithms that tend to produce small formulas

Asynchronous Systems

skipped

Software

Modeling Software

Programs

Consist of

- consecution (`;`)
- `if`
- `while`
- `x:=e`
- `skip`
- labels `L:`

Assume every line has a label.

Example

```
P::
l: cobegin P0 || P1 coend;

P0::
l0: while true do
        NC0: wait(turn = 0);
        CR0: turn := 1
end while

P1::
l1: while true do
        NC1: wait(turn = 1);
        CR1: turn := 0
end while
```

Translation

Define $same(Y) = \bigwedge_{y \in Y} y = Y'$

Define $\mathcal{C}(l, s, l')$

- l: label of statement
- s: statement
- l': label of next statement

$\mathcal{C}(l, v := e, l') =$

$\mathcal{C}(l, skip, l') =$

$\mathcal{C}(l, (P; l' : P'), l'') =$

Translation

Define $same(Y) = \bigwedge_{y \in Y} y = Y'$

label of statement

statement

Define $\mathcal{C}(l, s, l')$ label of next statement

$\mathcal{C}(l, v := e, l') = pc = l \wedge pc' = l' \wedge v' = e \wedge same(V \setminus \{v\})$,

$\mathcal{C}(l, skip, l') = pc = l \wedge pc' = l' \wedge same(V)$,

$\mathcal{C}(l, (P; l' : P'), l'') = \mathcal{C}(l, P, l') \vee \mathcal{C}(l', P', l'')$,

Translation

$\mathcal{C}(l, \textbf{if}\ \mathrm{b}\ \textbf{then}\ \mathrm{l}_1\!: \mathrm{P1}\ \textbf{else}\ \mathrm{l}_2\!: \mathrm{P2}\ \textbf{end}\ \mathrm{if}, l') =$

$\mathcal{C}(l, \textbf{while}\ \mathrm{b}\ \textbf{do}\ \mathrm{l}_1\!: \mathrm{P1}\ \textbf{end}\ \textbf{while}, l') =$

Translation

$\mathcal{C}(l, \textbf{if}\ \mathrm{b}\ \textbf{then}\ \mathrm{l}_1\!: \mathrm{P1}\ \textbf{else}\ \mathrm{l}_2\!: \mathrm{P2}\ \textbf{end}\ \mathrm{if}, l') =$

$[pc = l \wedge b \wedge pc' = l_1 \wedge same(V)] \vee$

$[pc = l' \wedge \neg b \wedge pc' = l_2 \wedge same(V)] \vee$

$\mathcal{C}(l_1, P1, l') \vee$

$\mathcal{C}(l_2, P2, l')$

$\mathcal{C}(l, \textbf{while}\ \mathrm{b}\ \textbf{do}\ \mathrm{l}_1\!: \mathrm{P1}; \mathrm{l}_2\!:\ \textbf{end while}, l') =$

$[pc = l \wedge b \wedge pc' = l_1 \wedge same(V)] \vee$

$[pc = l \wedge \neg b \wedge pc' = l' \wedge same(V)] \vee$

$[pc = l_2 \ \wedge pc' = l \wedge same(V)] \vee$

$\mathcal{C}(l_1, P1, l_2)$

Concurrency

```
P:: cobegin
        l1: P1 l1’ ||
        l2: P2 l2’
coend
```

Three program counters:

1. pc for the program that invokes cobegin
2. pc_1 for Thread 1
3. pc_2 for Thread 2

$pc = susp$ means that the program is not running.

$$\mathcal{C}(l, \mathbf{P}, l') = (pc = l \wedge pc' = susp \wedge pc_1' = l_1 \wedge pc_2' = l_2 \wedge same(V)) \vee$$
$$\left(pc = susp \wedge pc_1 = l'_1 \wedge pc_2 = l'_2 \wedge pc' = l' \wedge pc'_1 = susp \wedge pc'_2 = susp \wedge same(V)\right) \vee$$
$$\bigvee_{i=1}^{n} (C(l_i, P_i, l_i') \wedge same(V \setminus V_i) \wedge same(PC \setminus \{pc_i\})$$

Example

P0::

l0: **while** true **do**

NC0: **wait**(turn = 0);

CR0: turn := 1

end while

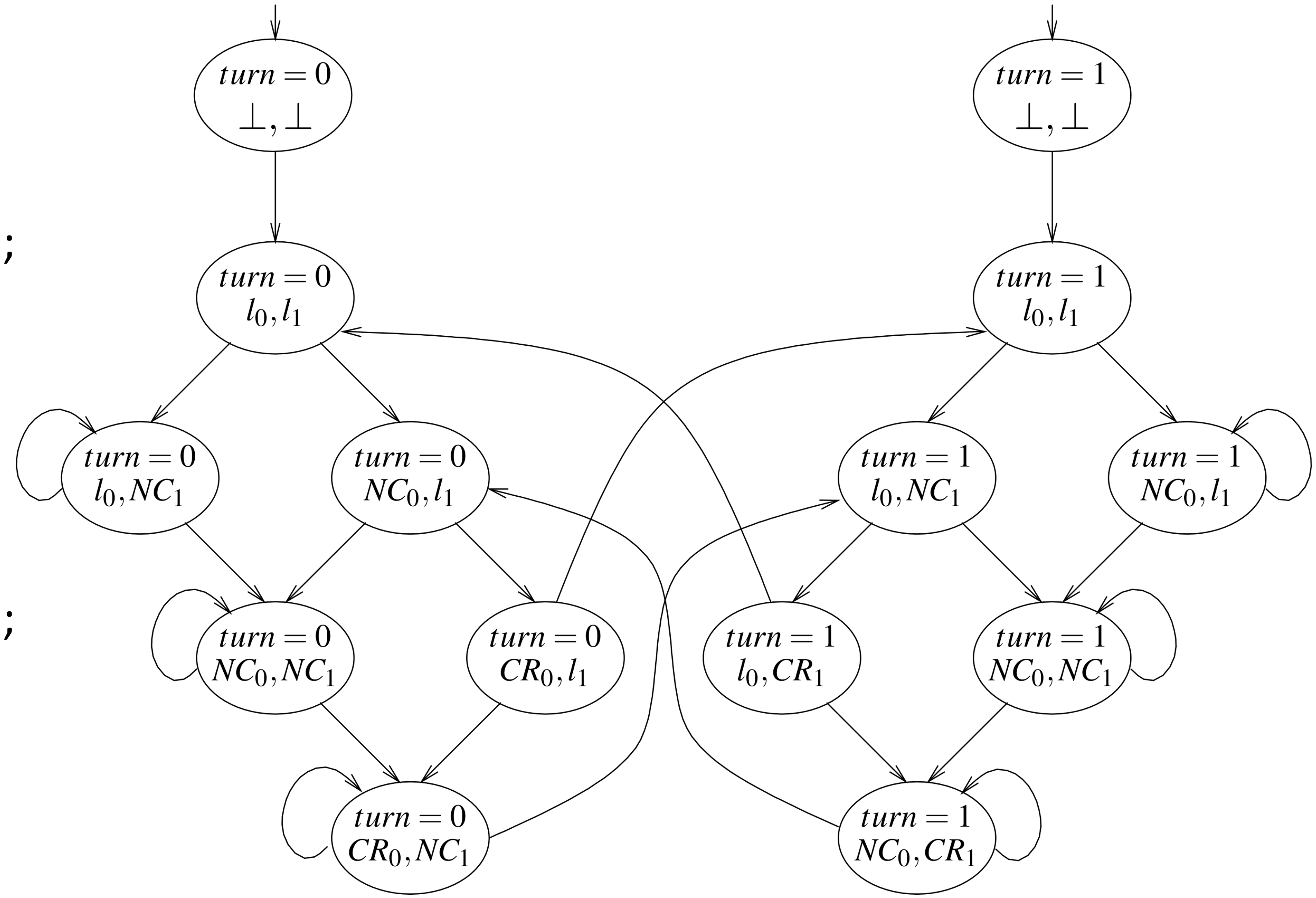
P1::

l1: **while** true **do**

NC1: **wait**(turn = 1);

CR1: turn := 0

end while



Termination

- Programs can end
- Kripke structures are not allowed to have dead ends, reminder:
  - $R \subseteq S \times S$ – left-total transition relation
    - For every $s \in S$ there exists $s' \in S$ such that $(s, s') \in R$
    - Left-total implies that every path is infinite
- We solve this contradiction by assuming that programs end in a self loop that does nothing

Fairness

Fairness

- skipped

Model Checking (SS 2023) Homework 1

Deadline: March 23, 2023, 4:00 pm
Send your solution to `modelchecking@iaik.tugraz.at`

You are given the following program P.

```
l1: while  x > y do
l2:     y := y + 2
l3:     if y%4 == 0 then
l4:         x := x + 1
l5:     else
l6:         x : = x + 2
l7:     end if
l8: end while
l9:
```

The initial value of x can be 0 or 2. The initial value of y is 0.

Task 1. [**1 point**] Write the formula S_0 that represents the set of initial states.

Task 2. [**4 points**] Write the formula C that represents the transition relation of P.

Task 3. [**5 points**] Draw the Kripke structure $M = (S, S_0, R, AP, L)$ that represents P.

Group size 1 or 2.

Email
Subject: homework 1
Includes Name and immatriculation number of both group members and a PDF with your solution