

Computer Organization and Networks

(INB.06000UF, INB.07001UF)

Chapter 3 – State Machines

Winter 2022/2023



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Important Note:

Different lecture
time next week!

25.10.2022
18:00 – 20:15
i13

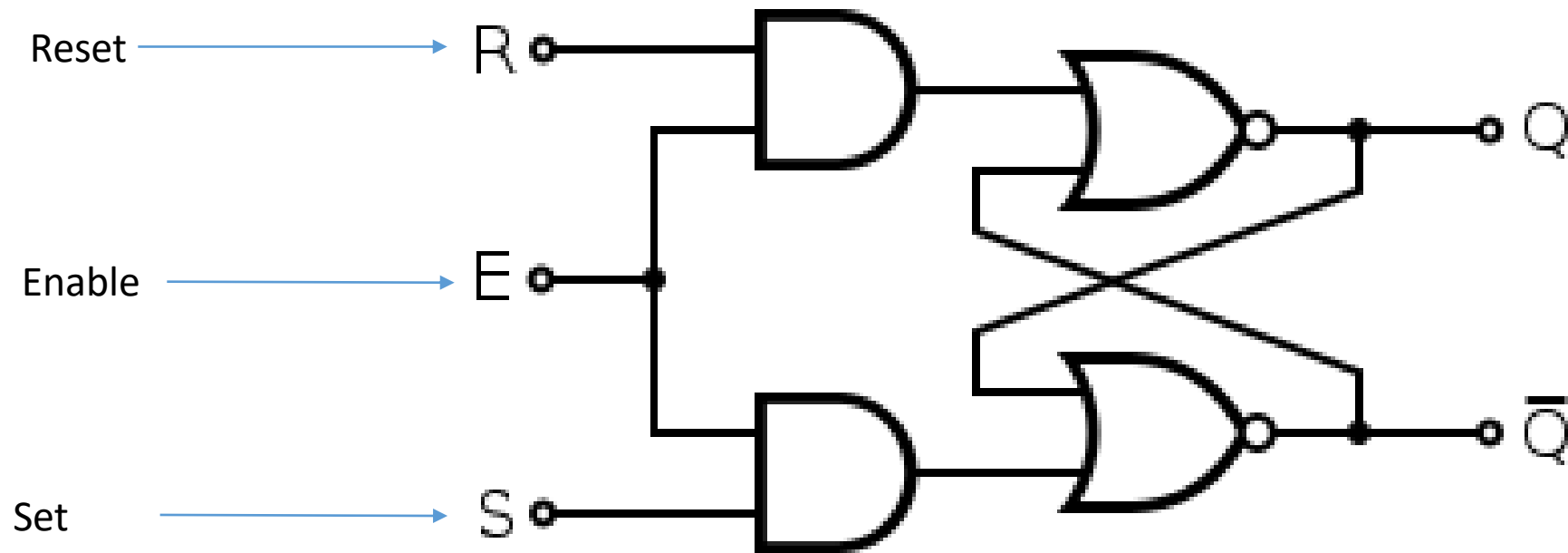
Sequential Circuits

(How to store data)

From Combinational Circuits to Sequential Circuits

- The circuits that we have discussed so far did not contain storage
- A change of an input has directly led to a change at the output
- We now build storage elements from logic gates
 - The basic idea to achieve storage is to create a feedback loop

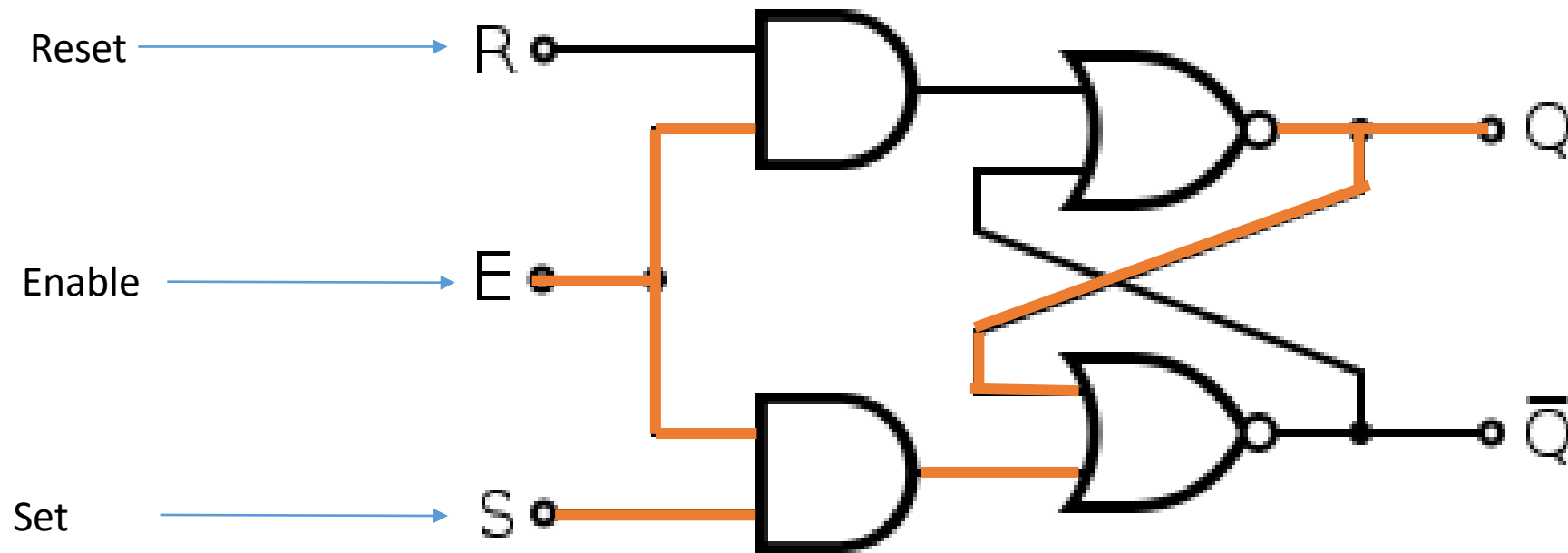
A Simple Set-Reset Latch (NOR Version)



We set the “set” input

Truth Table of NOR

a	b	q
0	0	1
0	1	0
1	0	0
1	1	0

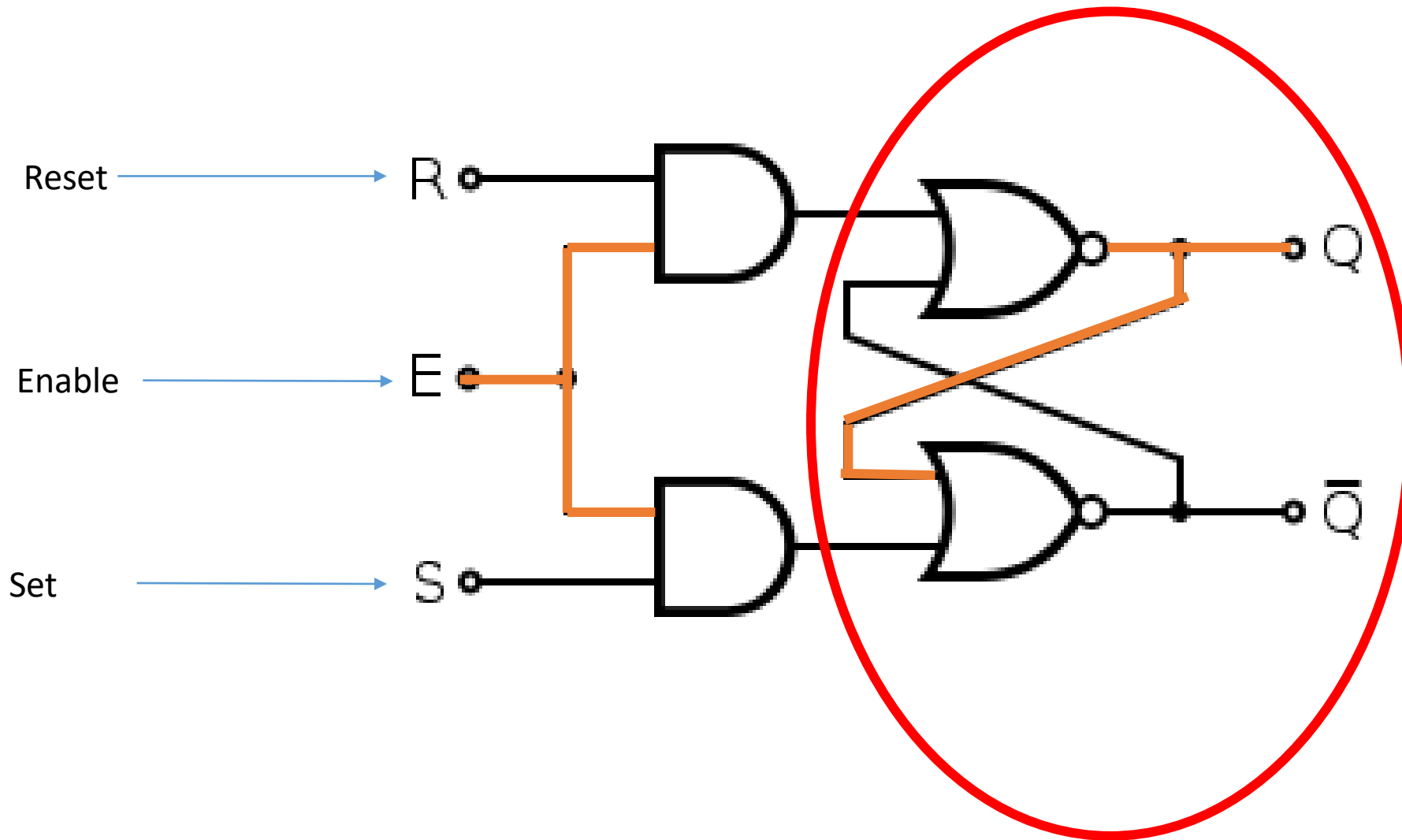


We release the “set” input

Truth Table of NOR

a	b	q
0	0	1
0	1	0
1	0	0
1	1	0

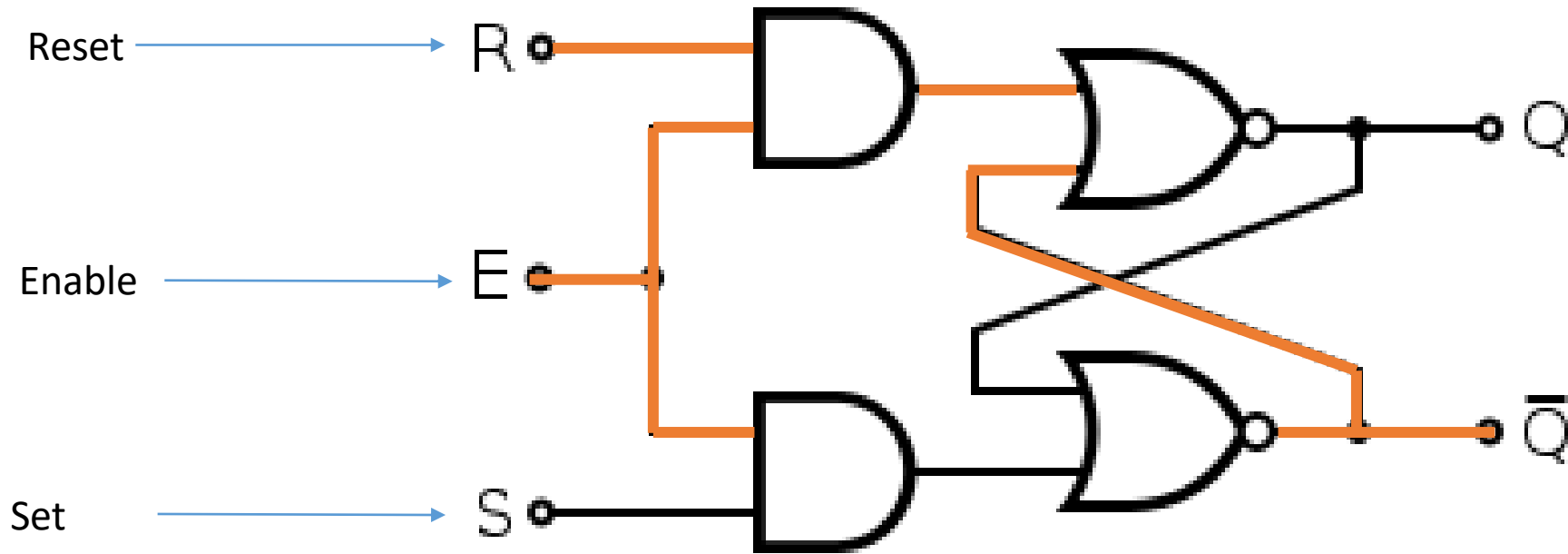
The feedback loop is in a stable state; The output value is kept – even if the “set” input is set to 0



We set the “reset” input

Truth Table of NOR

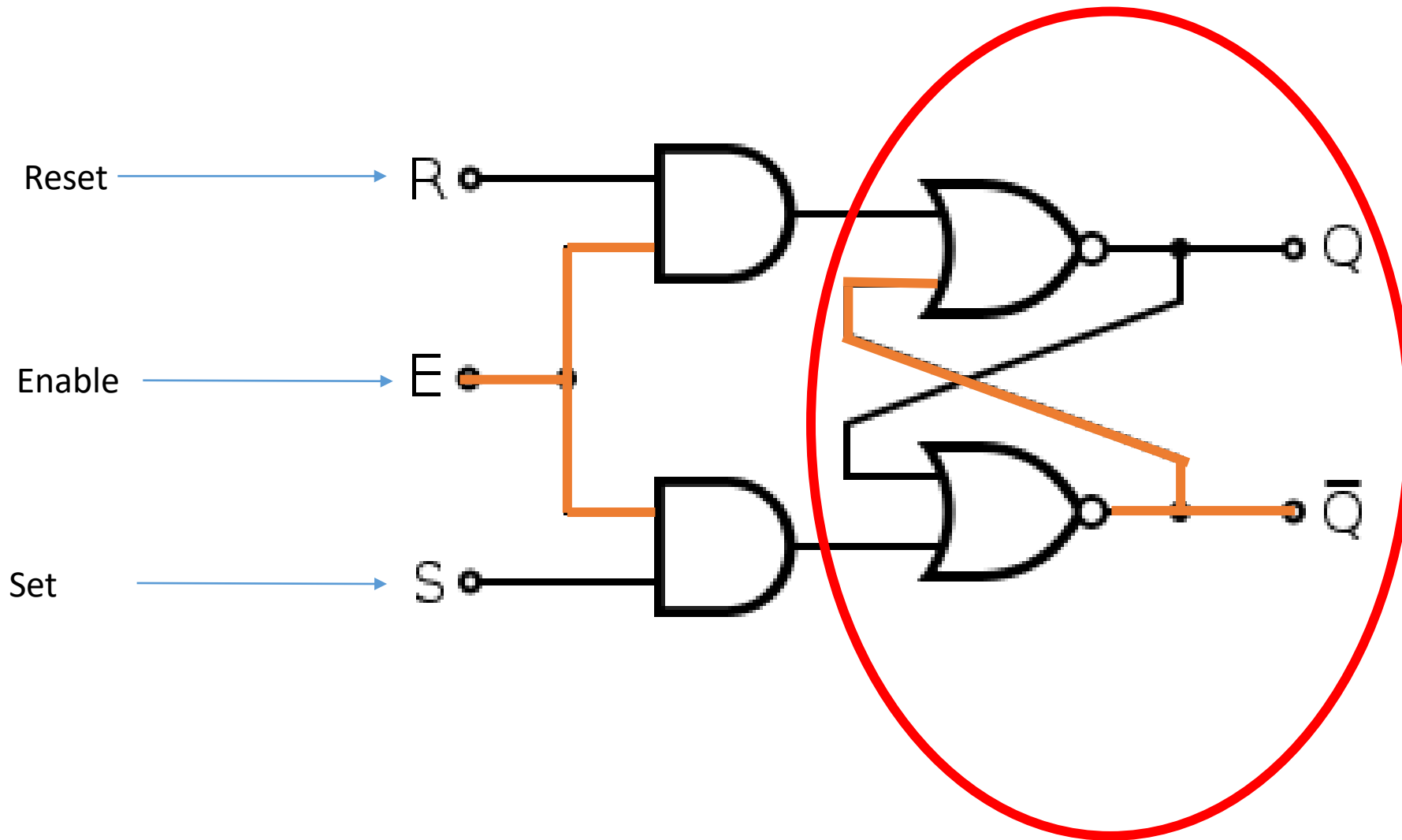
a	b	q
0	0	1
0	1	0
1	0	0
1	1	0



We release the “reset” input

Truth Table of NOR

a	b	q
0	0	1
0	1	0
1	0	0
1	1	0

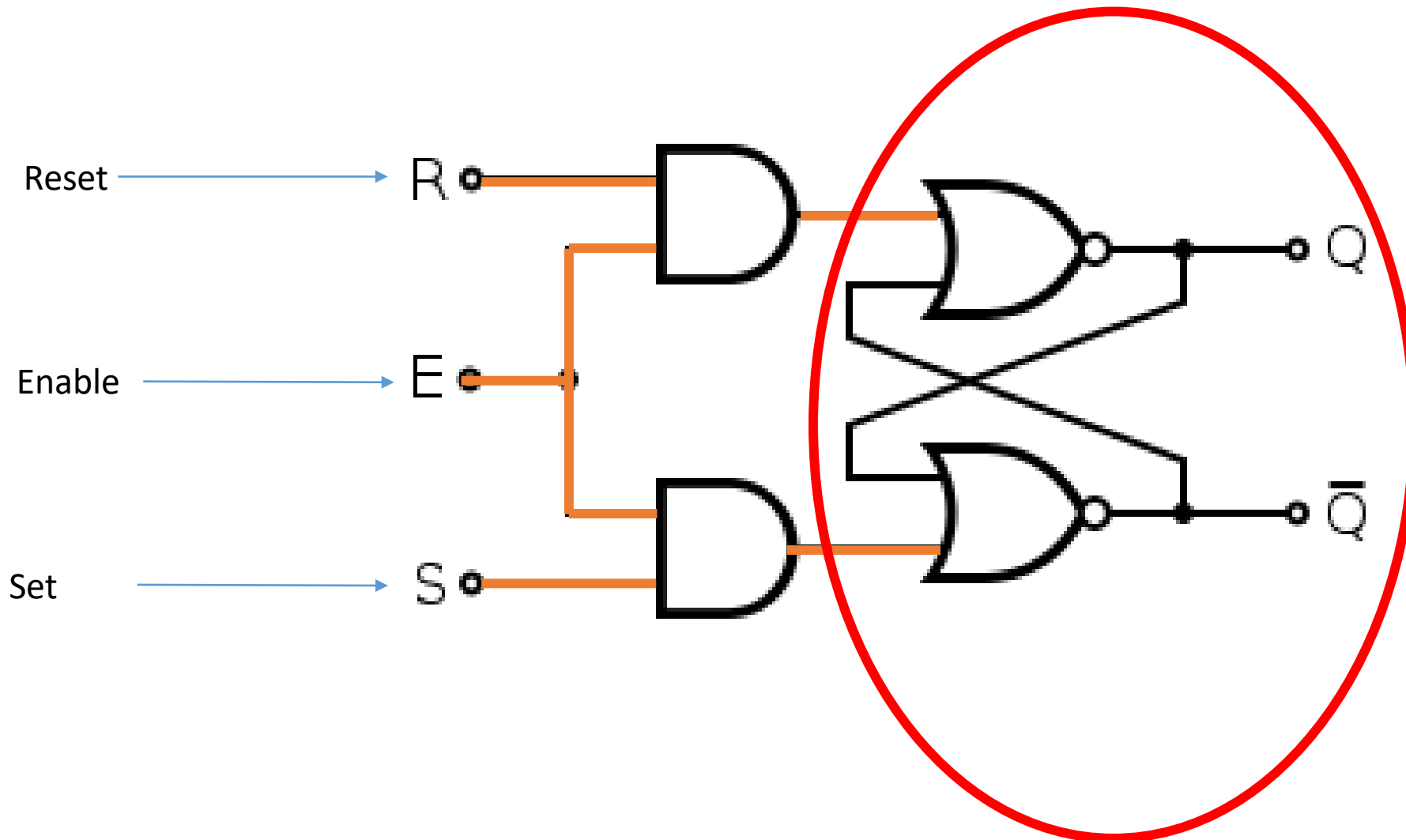


Again, the output value is kept

Illegal Action: We set “Set” and “Reset”

Truth Table of NOR

a	b	q
0	0	1
0	1	0
1	0	0
1	1	0

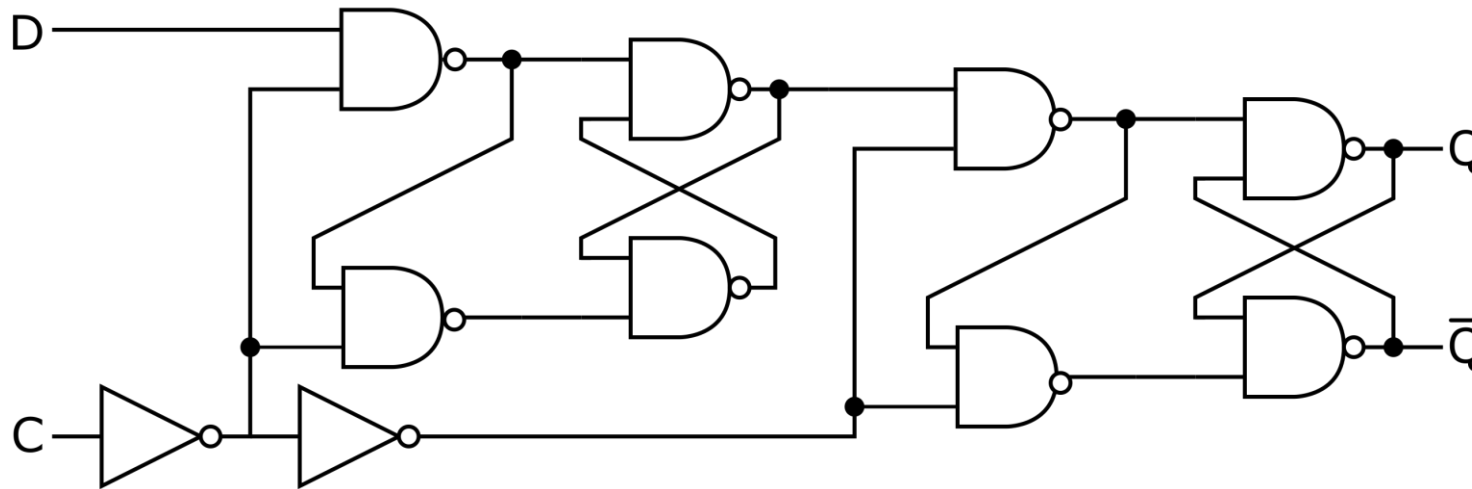


Invalid output state

Combining Computation and Storage

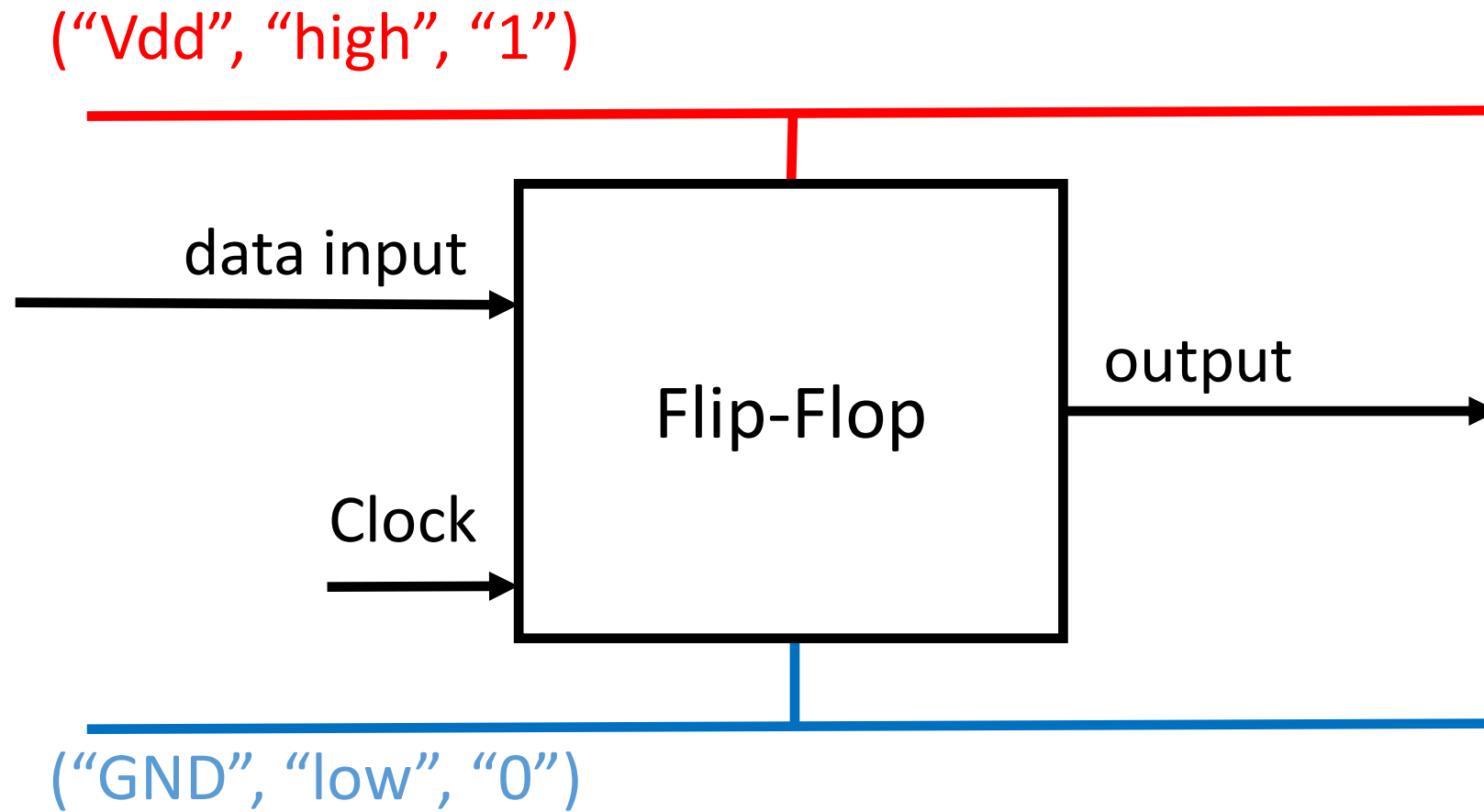
- There are many ways to combine gates for computation and for storage
 - It has turned out that only few scale to large circuit designs
- Nearly all digital circuits are built as **synchronous circuits** with a global clock signal
 - These circuits don't use latches as storage, but Flip-Flops that are connected to a clock signal
 - This course focuses on synchronous circuits only
- There is also a design methodology for **asynchronous circuits** (self-timed circuits), but they are a nice topic

Flip-Flop based on CMOS Gates



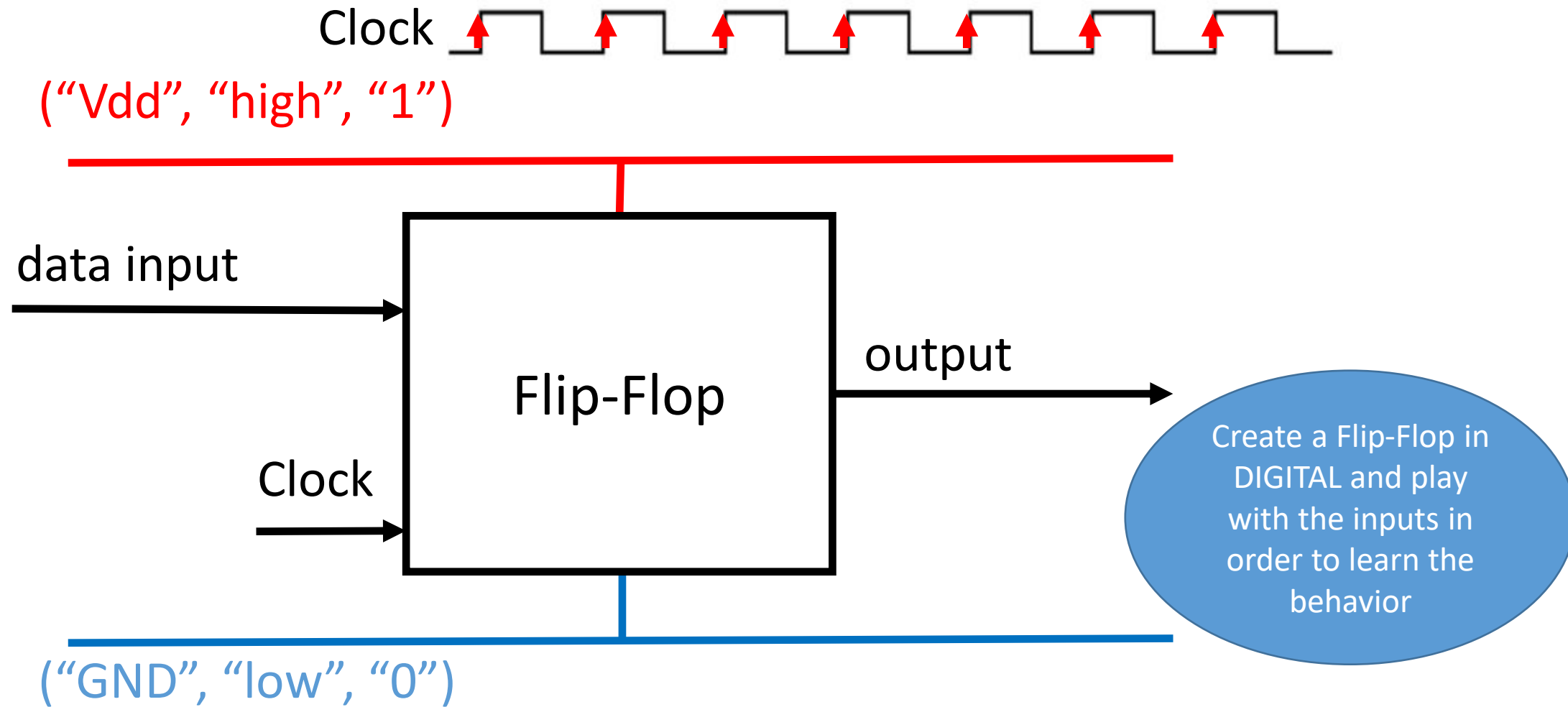
Note: A flip-flop simply consists of two latches

Storage



Storage

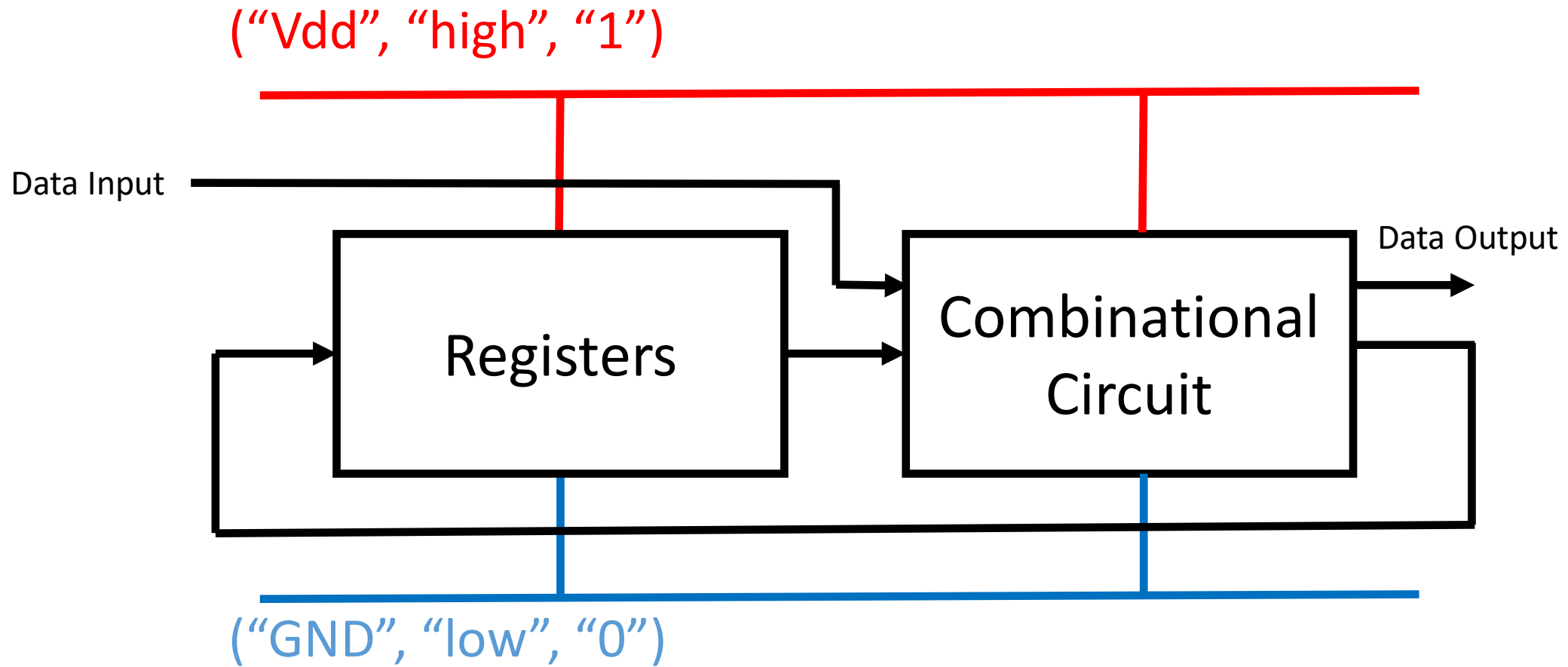
- The flip-flop sets output = input when the clock switches from low to high;
- In all other cases, the input is ignored; the last “sampled” value is kept at the output



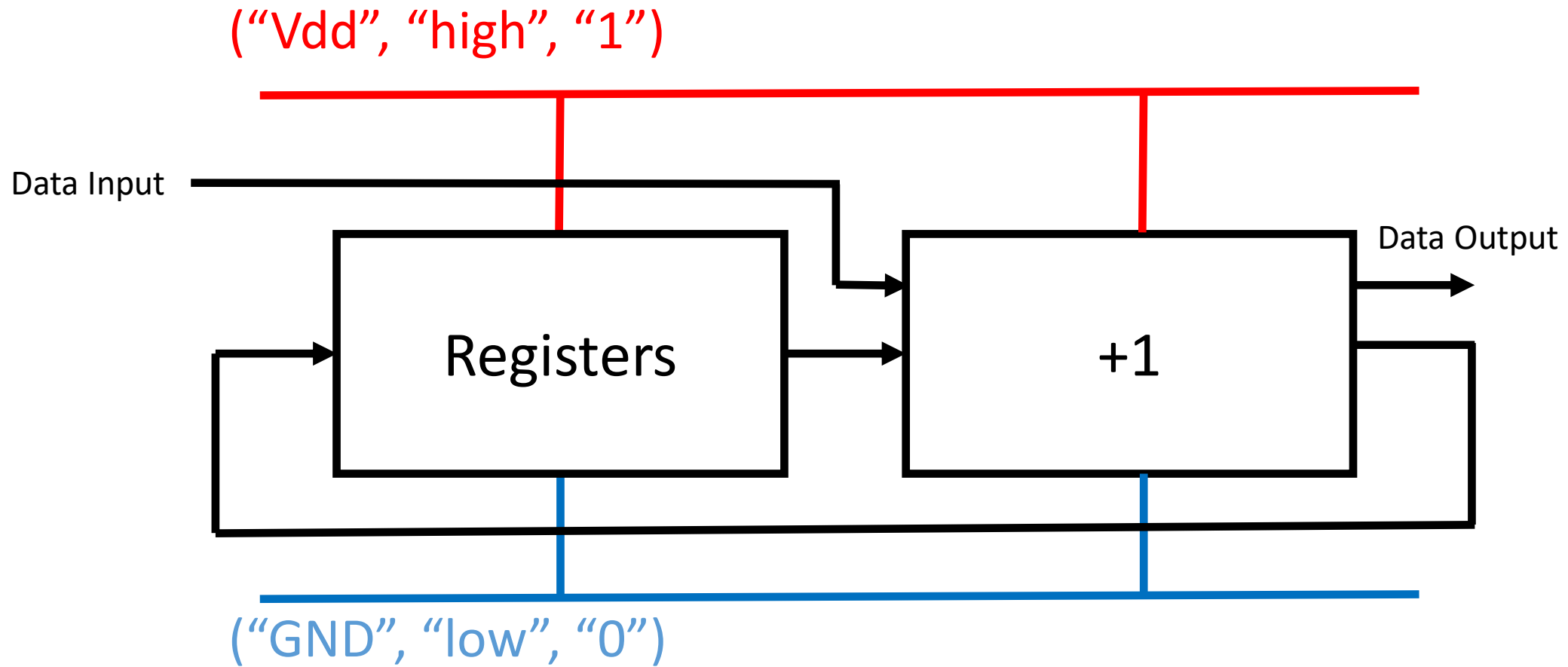
Naming Conventions

- Flip-Flop: A 1-bit storage sampling data on the rising clock edge
- Register: An n-bit storage sampling data on the rising clock edge

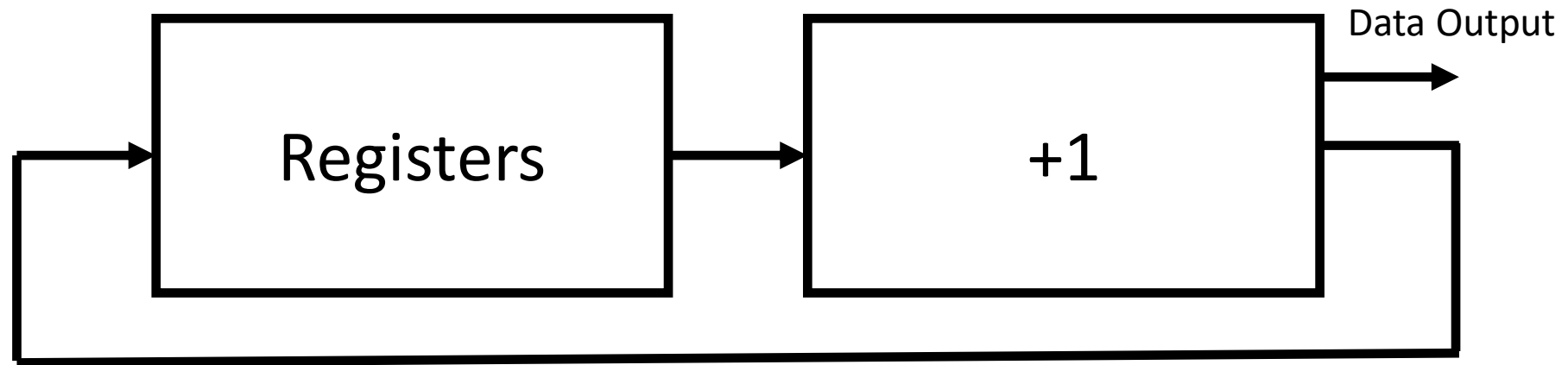
Combining



Example Counter



Example Counter



Clock



Register

0 1 2 3 4 5 6 7

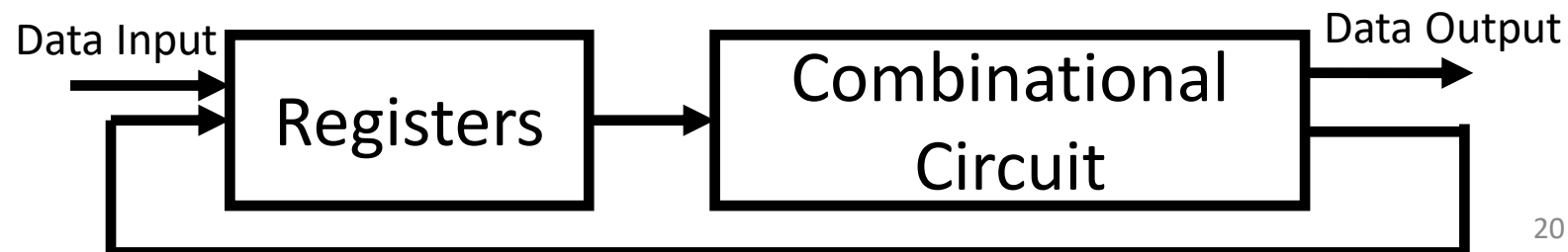
Let's Build This in SystemVerilog

- See example con03.01_addsub

<https://extgit.iaik.tugraz.at/con/examples-2022.git>

The Clock Frequency

- Can we increase the clock frequency arbitrarily?
- The clock frequency is limited by the time the combinational circuit needs to compute its outputs.
- The critical path is the path with the longest propagation delay in the combinational circuit. It defines the maximum clock rate

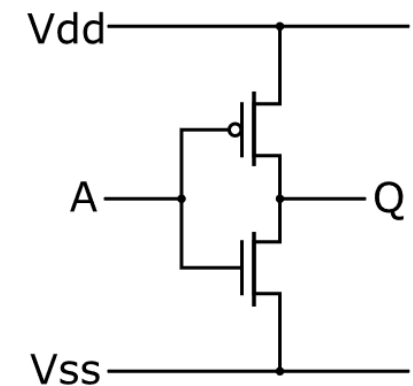


Temperature, Power Consumption

- The higher the temperature, the slower the transistors become and the lower becomes the maximum clock rate
 - The lower the temperature, the higher clock rates are possible

- Why does a CPU produce heat?

- Every time a logic gate switches, NMOS and PMOS transistors are open at the same time → there is a short current.
- Upon a switch, there is also current flowing to charge and discharge parasitics
 - The more transistors are switching, the more heat is produced



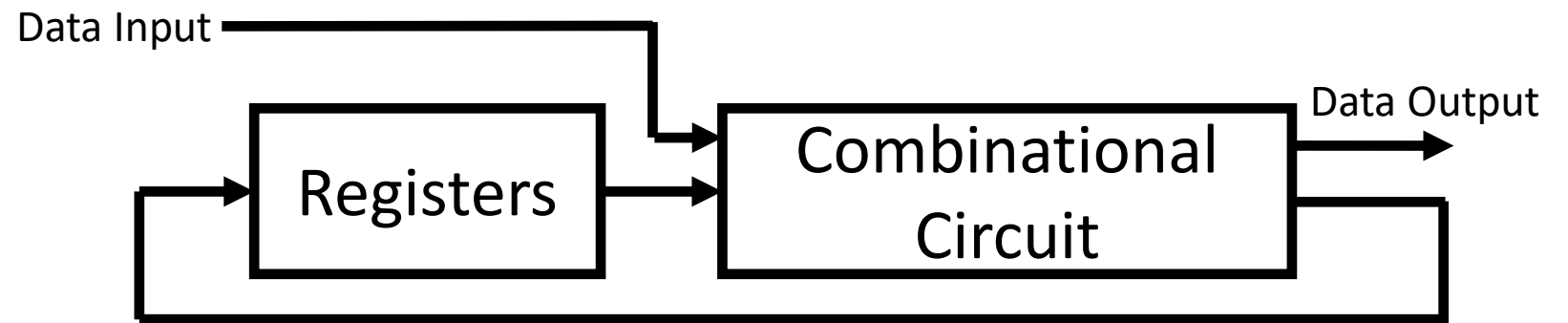
Clock Frequency Too High

What happens, if the clock frequency is too high?

- The circuit stores an intermediate state of the combinational circuit in the registers.
- The intermediate state depends on the physical layout, the temperature, fabrication details, ... → hard to predict; overclocking a processor too much typically leads to a crash

Observations

- What we have discussed the basics of combinational and sequential circuits



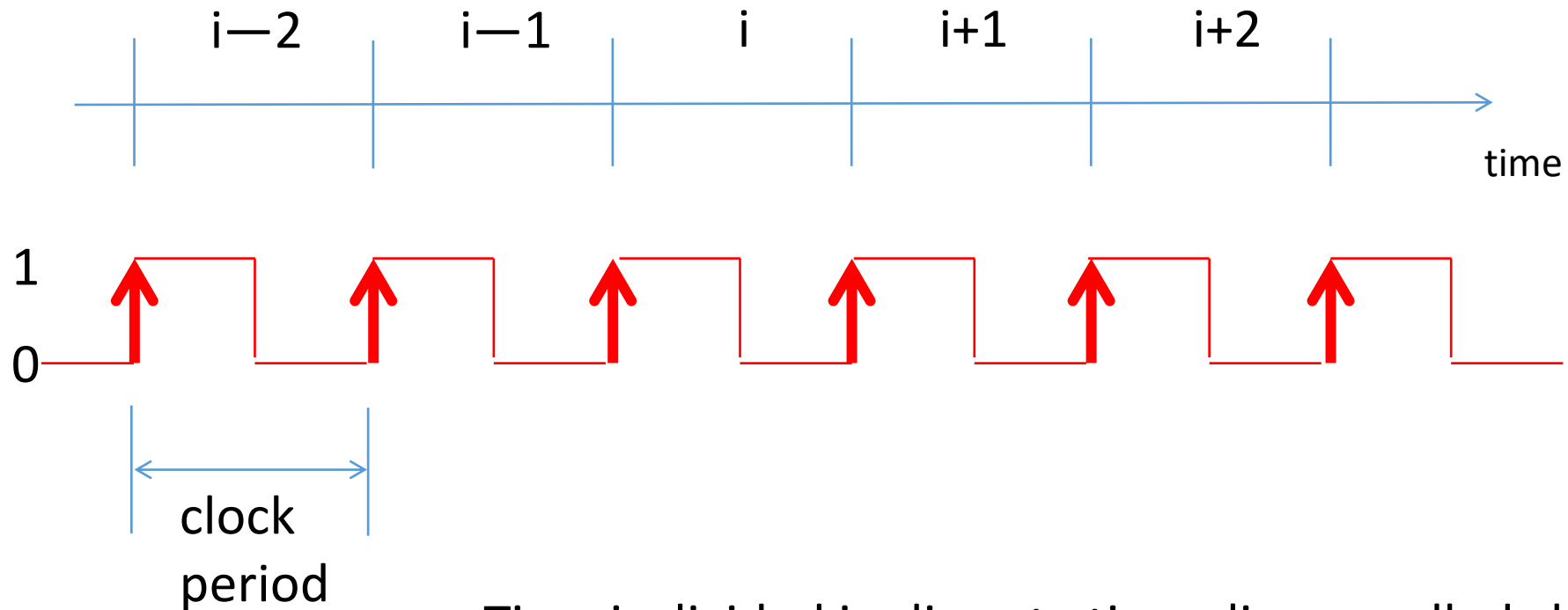
- In order to build large systems composed of registers and combinational logic, we need a structured approach and more tools and theory to describe our systems

State Machines

Finite State Machines (FSMs)

- FSMs are the “work horse” in **digital systems**.
- We look at “**synchronous**” FSMs only:
 - The “clock signal” controls the action over time
- FSMs can be described with three main “views”:
 - The **functional** view with the “state diagram”
 - The **timing** view with the “timing diagram”
 - The **structural** view with the “logic circuit diagram”

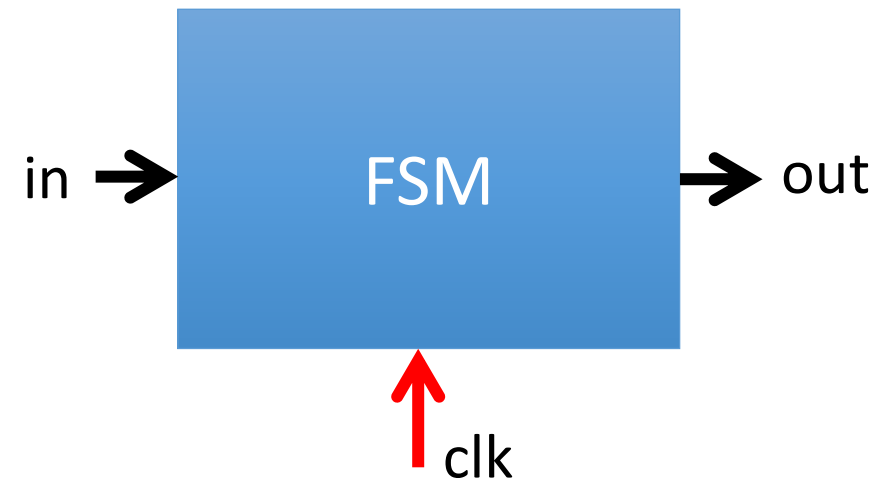
Time is Split in Discrete Slices for FSMs



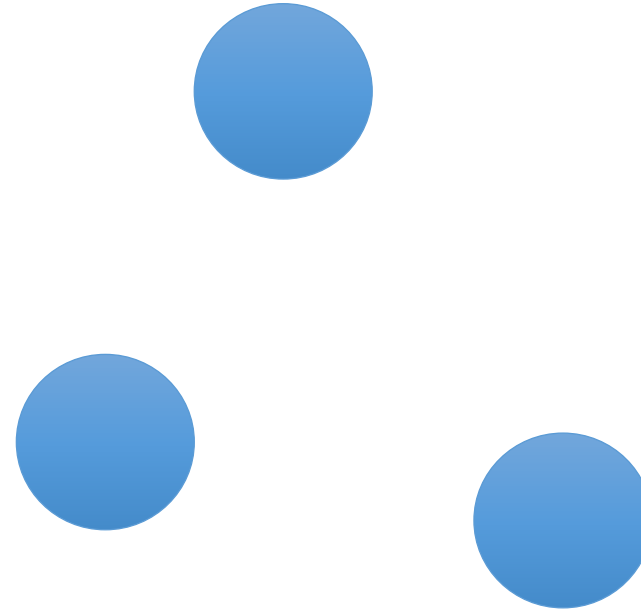
- Time is divided in discrete time slices – called clock cycles
- We call this time between two rising clock edges also “clock period”.

Finite state machine (= automaton)

- A **synchronous FSM** is clocked by a clock signal (“clk”)
- In each clock period, the machine is in a defined (current) **state**.
- With each rising edge of the clock signal, the machine advances to a defined next state.

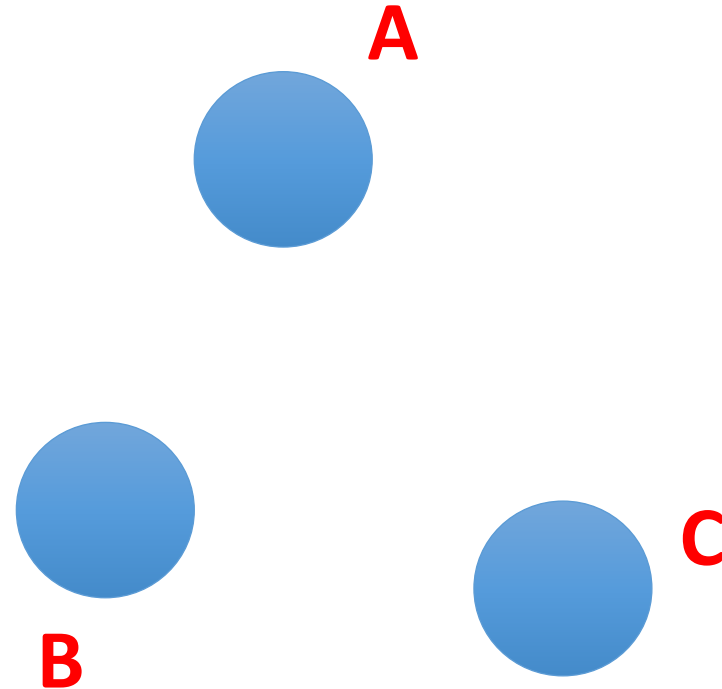


The sequence of states can be defined in a **state diagram**.

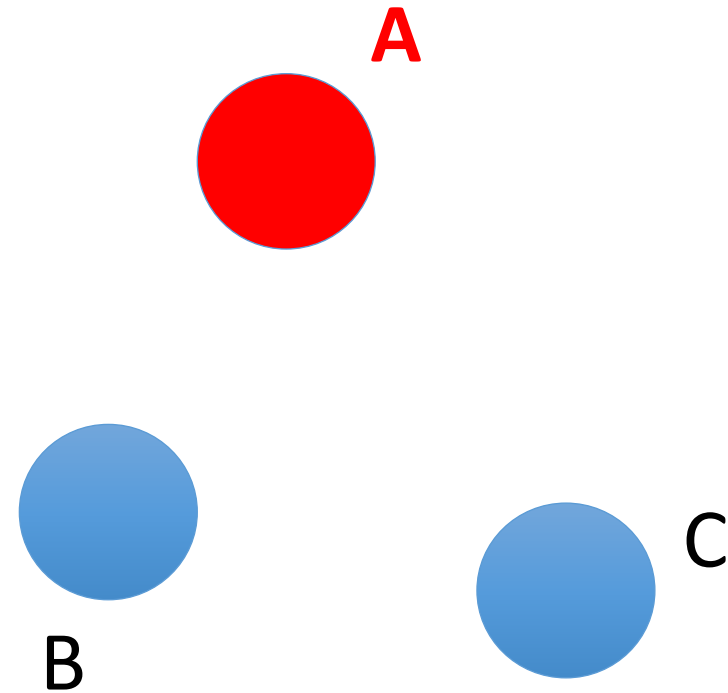


State diagram:

We denote the states with circles and give them **symbolic names**, e.g. A, B, and C.

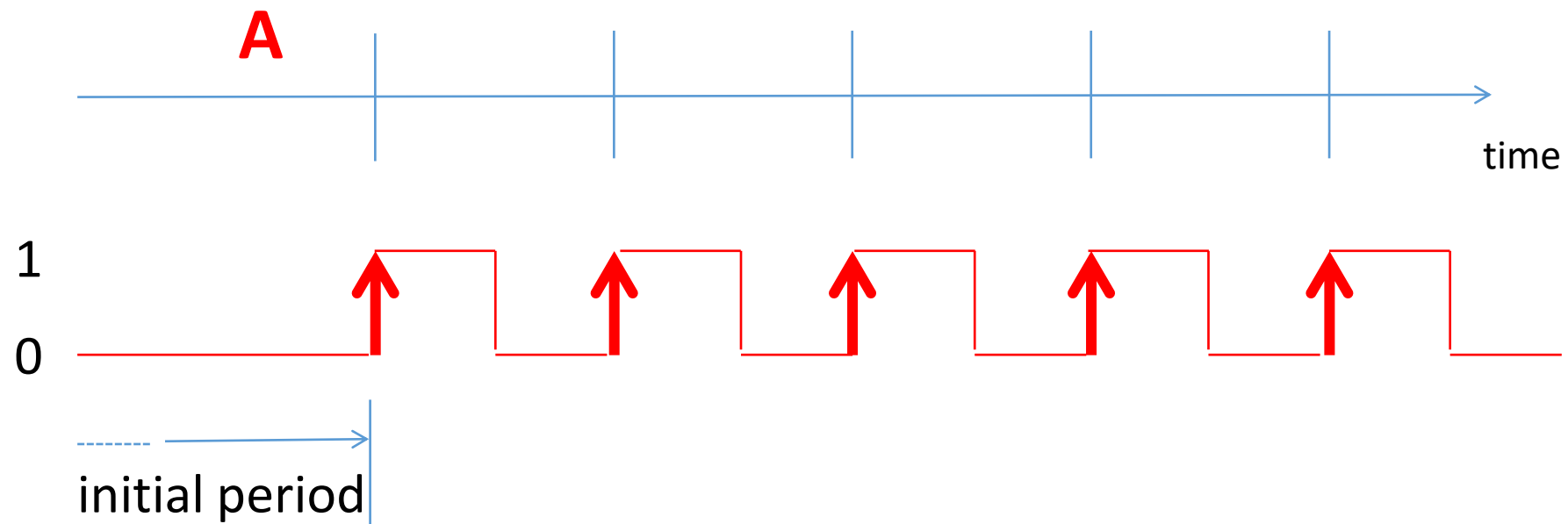


State diagram:



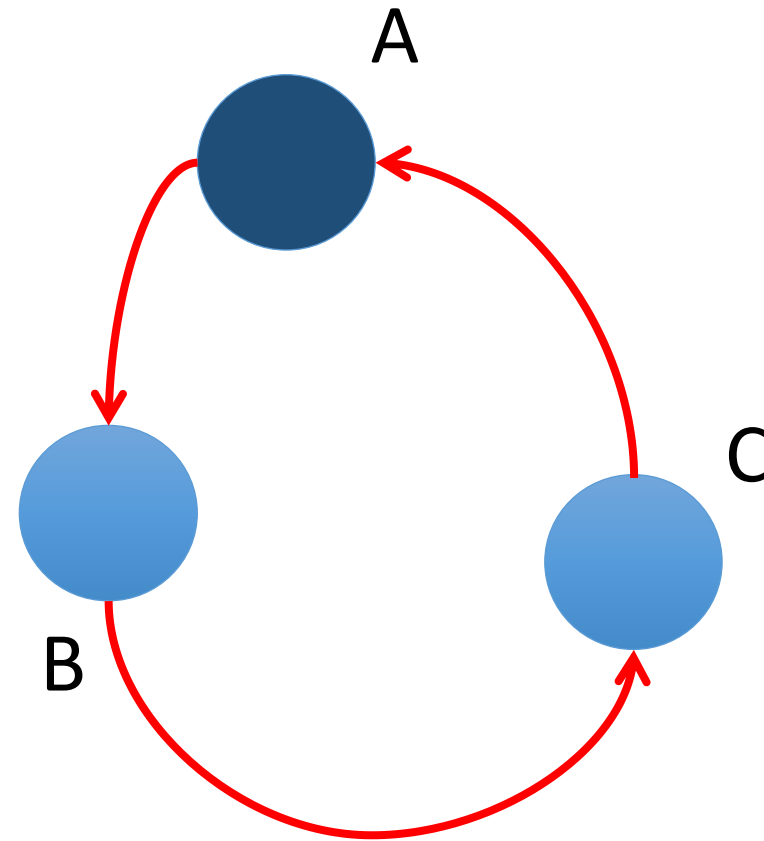
We define one of the states as the **initial state**.

In the beginning...



Initially, i.e. shortly after switching on the FSM and **before the first rising edge of clock**, there is the initial period. In this period, the FSM is in the “**initial state**”.

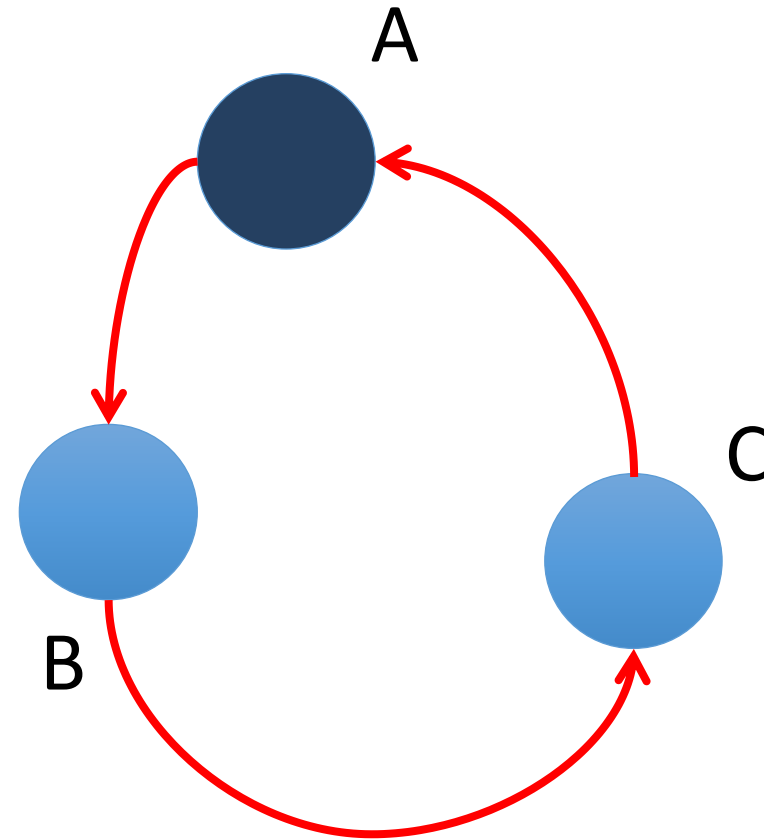
State diagram:



With arrows we
define the **sequence**
of states.

The sequence of states can also be defined in a **state transition table**.

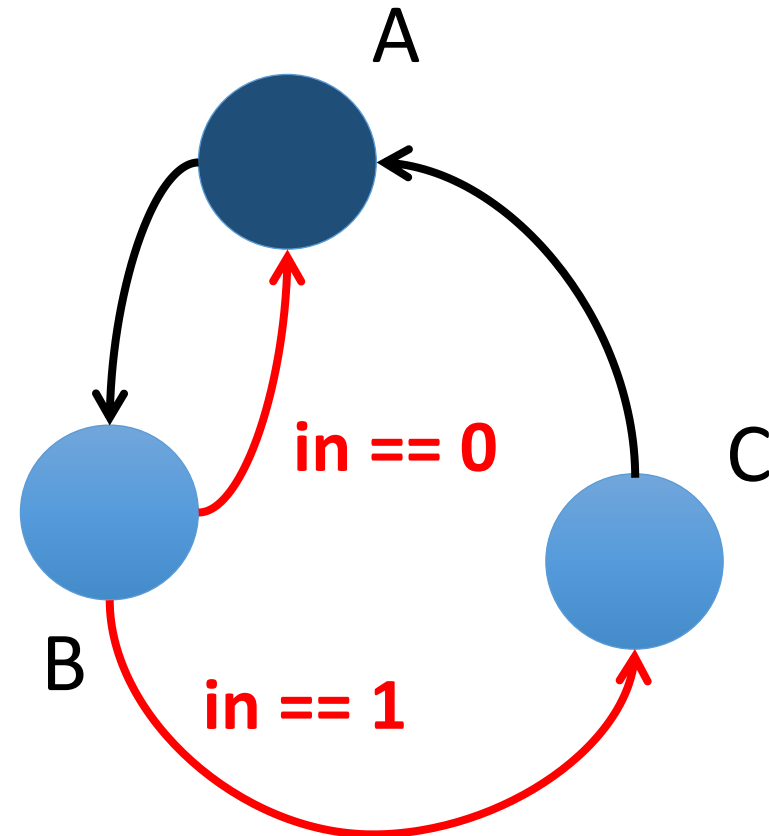
present state	next state
A	B
B	C
C	A



FSMs typically also have **inputs** influencing the transition to the next state

next state = f(state, **input**)

In this example we see that the one-bit input "in" influences the choice of the state after B.

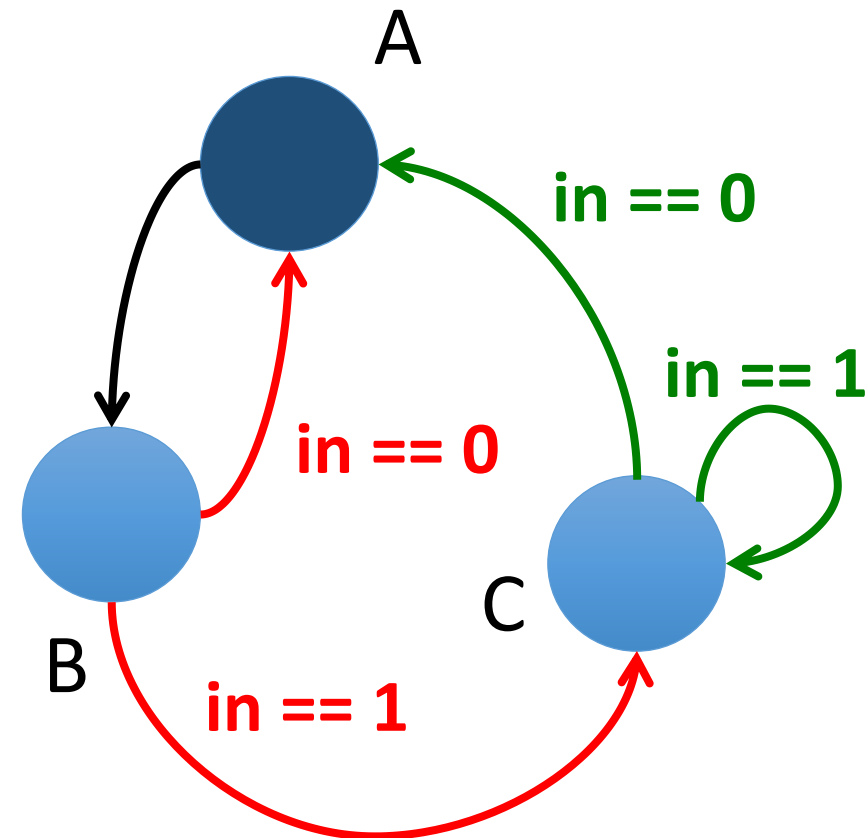


FSMs typically also have **inputs** influencing the transition to the next state

$$\text{next state} = f(\text{state}, \text{input})$$

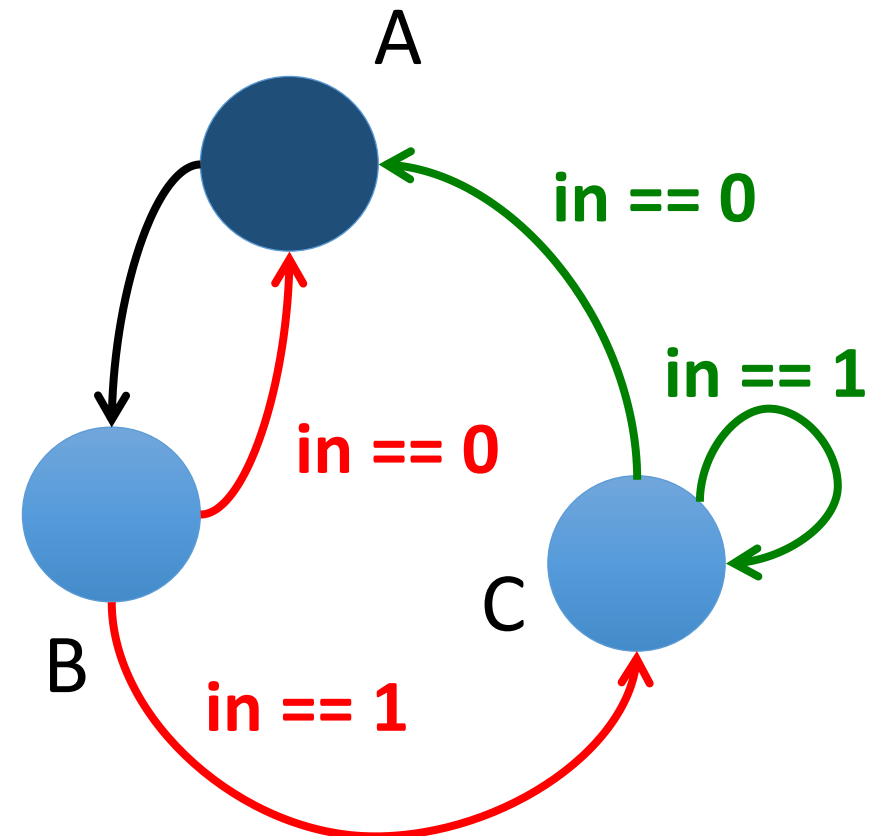
In this example we see that the one-bit input “in” influences the choice of the state after B.

The following state can also be the same as the current state.

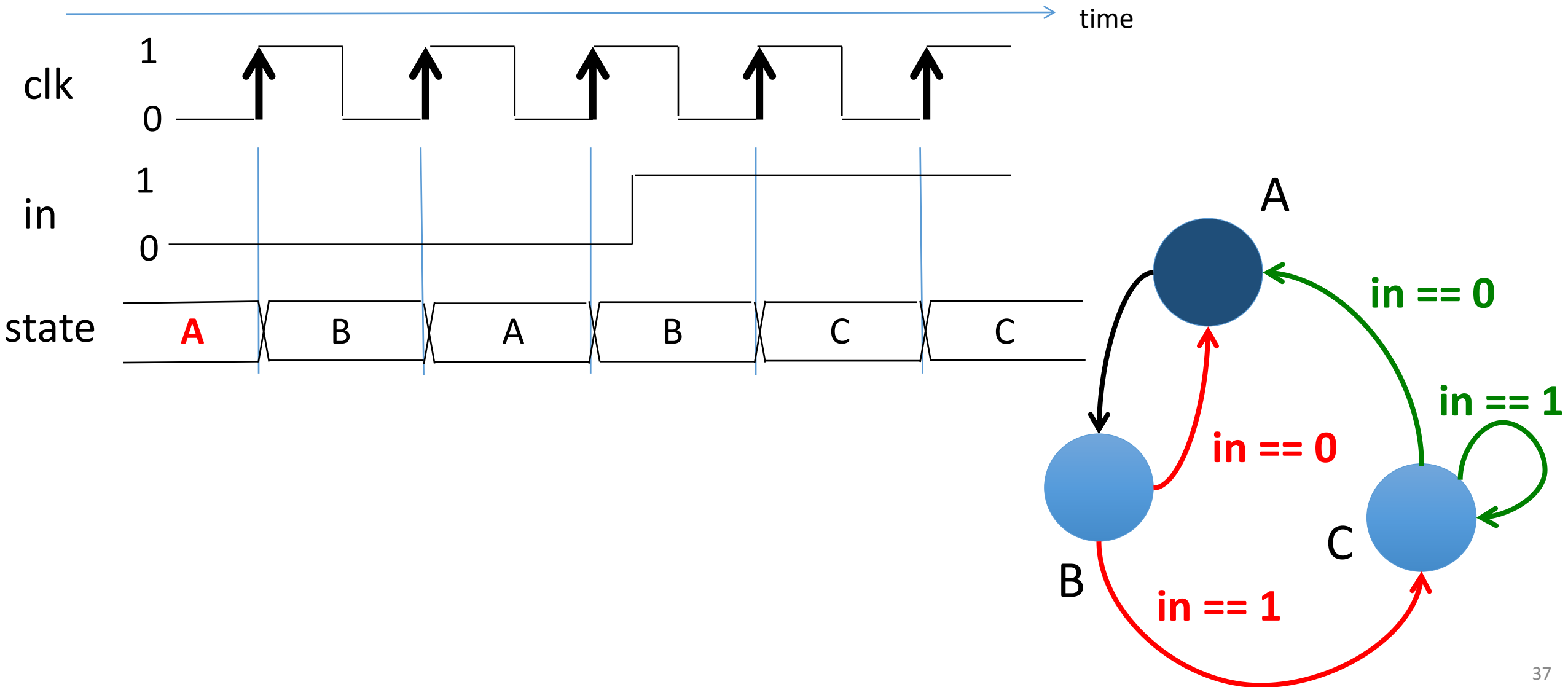


The State Transition Table

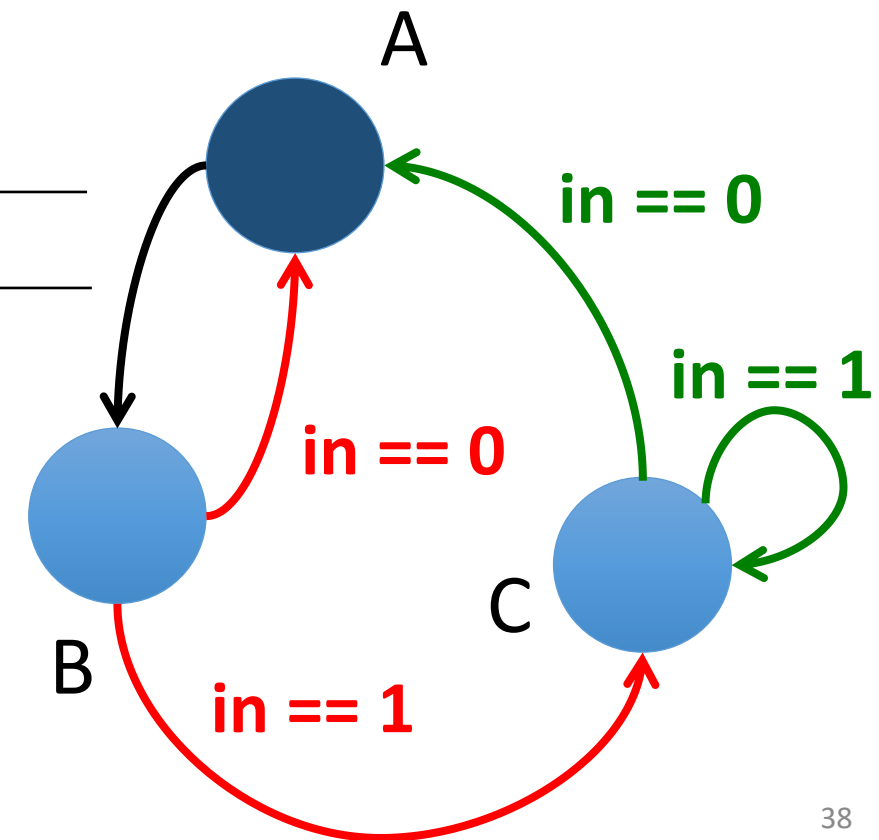
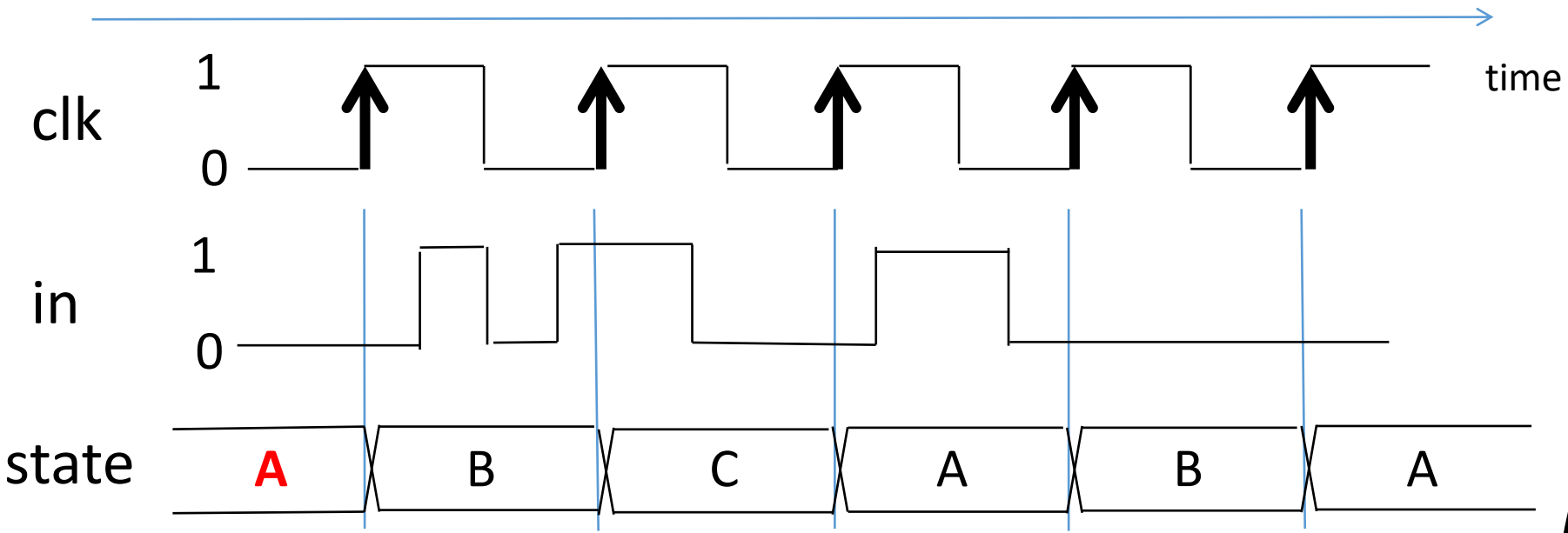
present state	in	next state
A	0	B
A	1	B
B	0	A
B	1	C
C	0	A
C	1	C



Timing Diagram – Example 1



Timing Diagram – Example 2

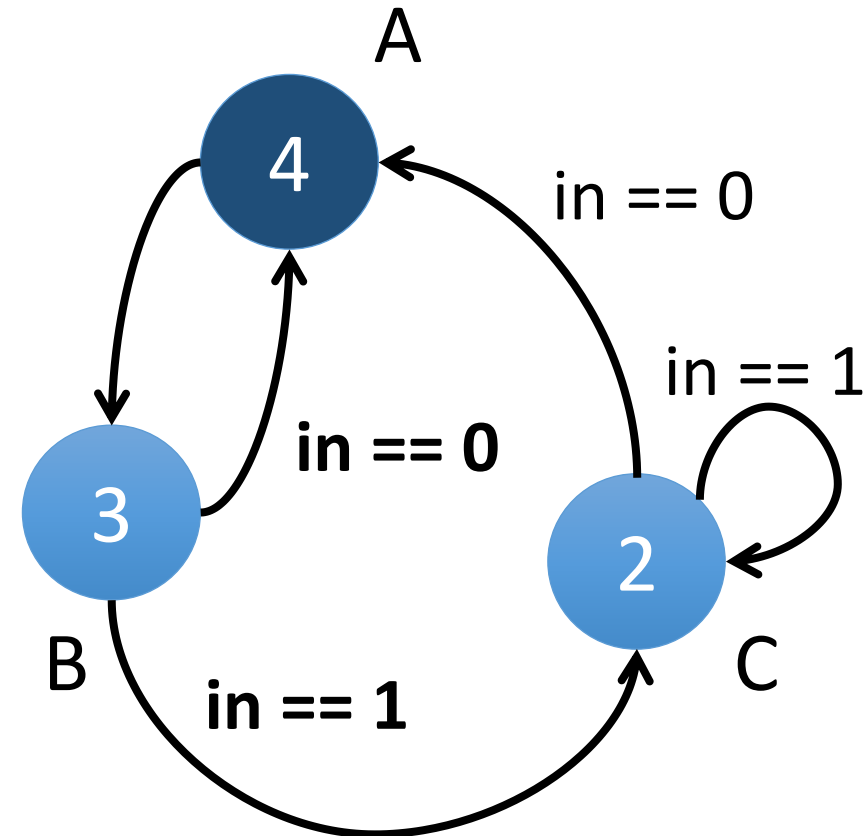


FSMs typically also have **outputs**

In this example the outputs are a function of the state. We write the output values into the circles.

We call such machines also **“Moore machines”**:

output = f(state)

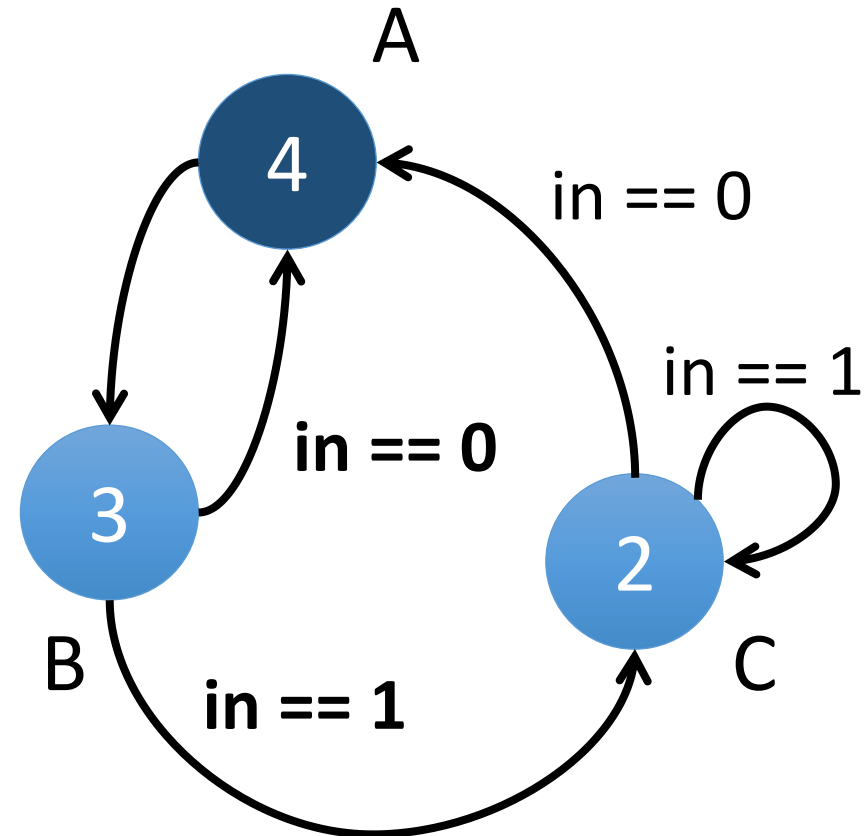


We define the outputs with the “output function”

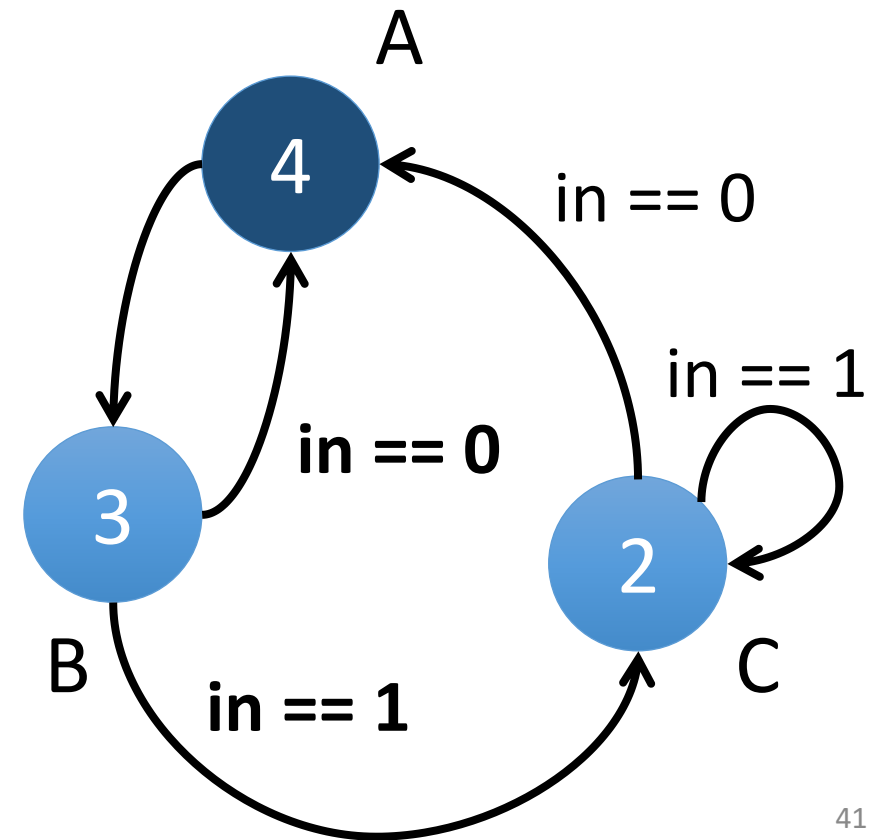
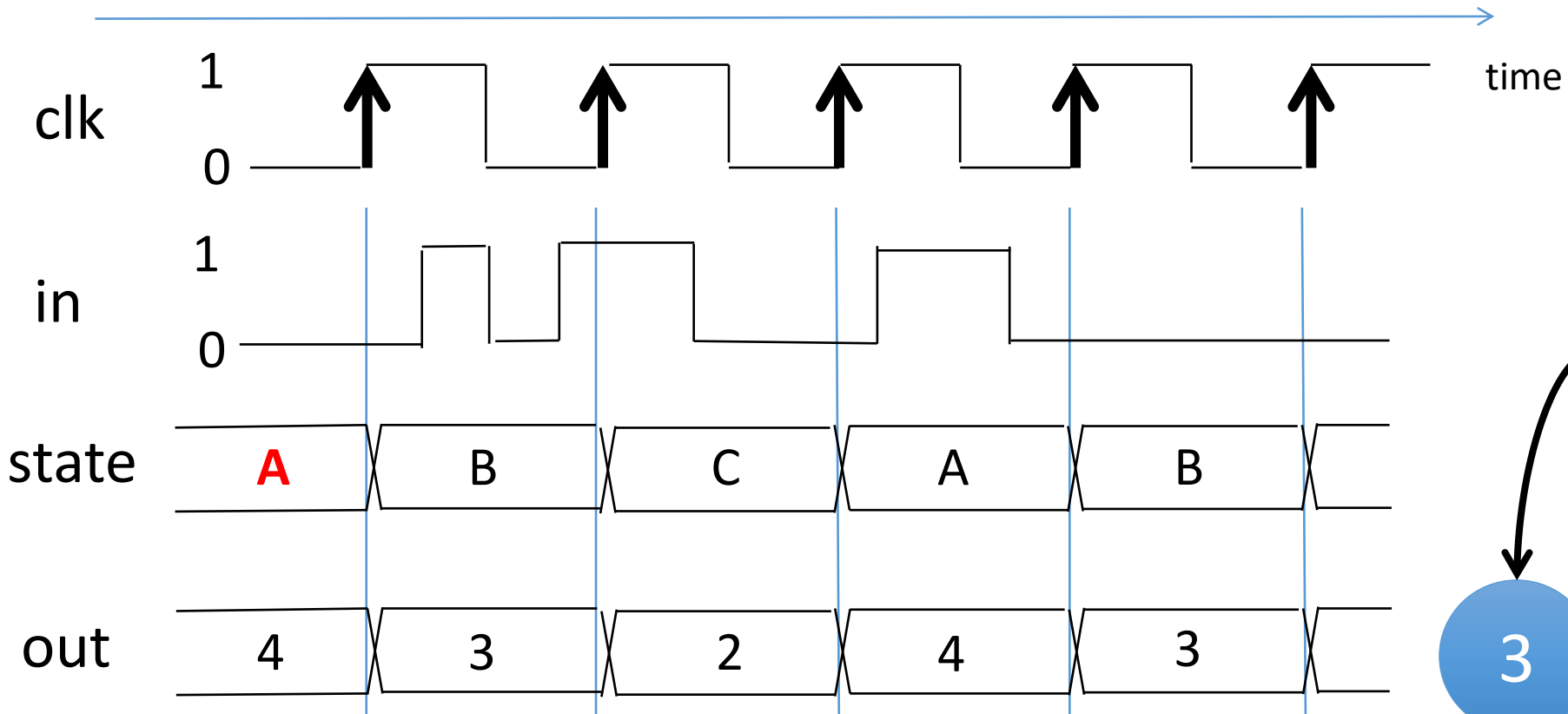
Moore machines:

output = f(state)

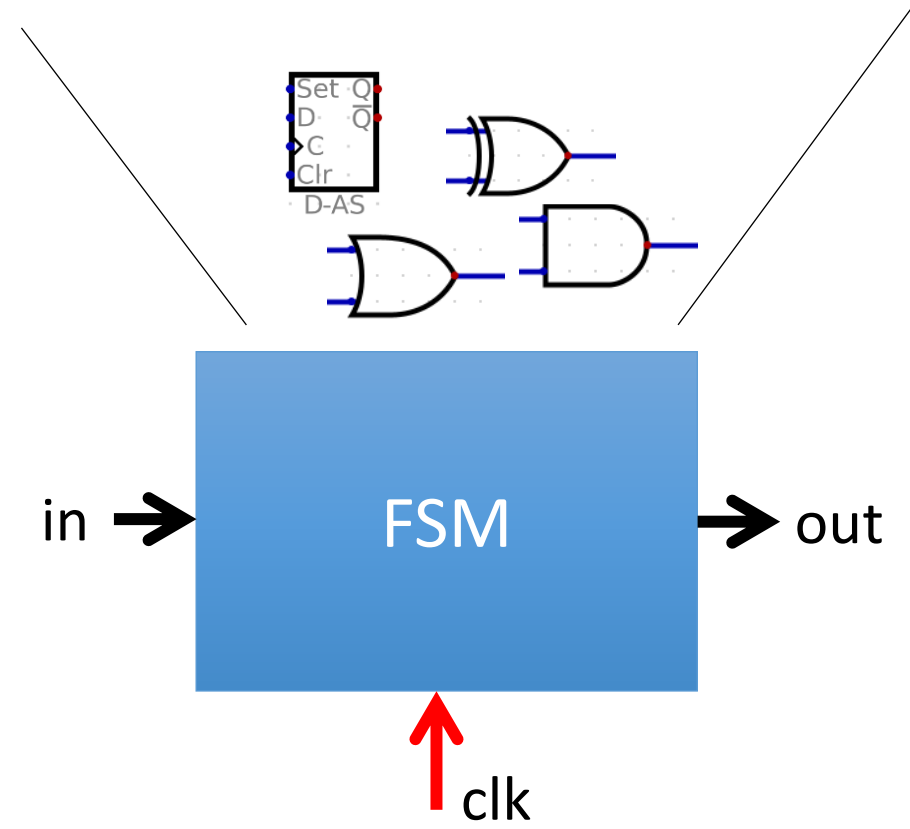
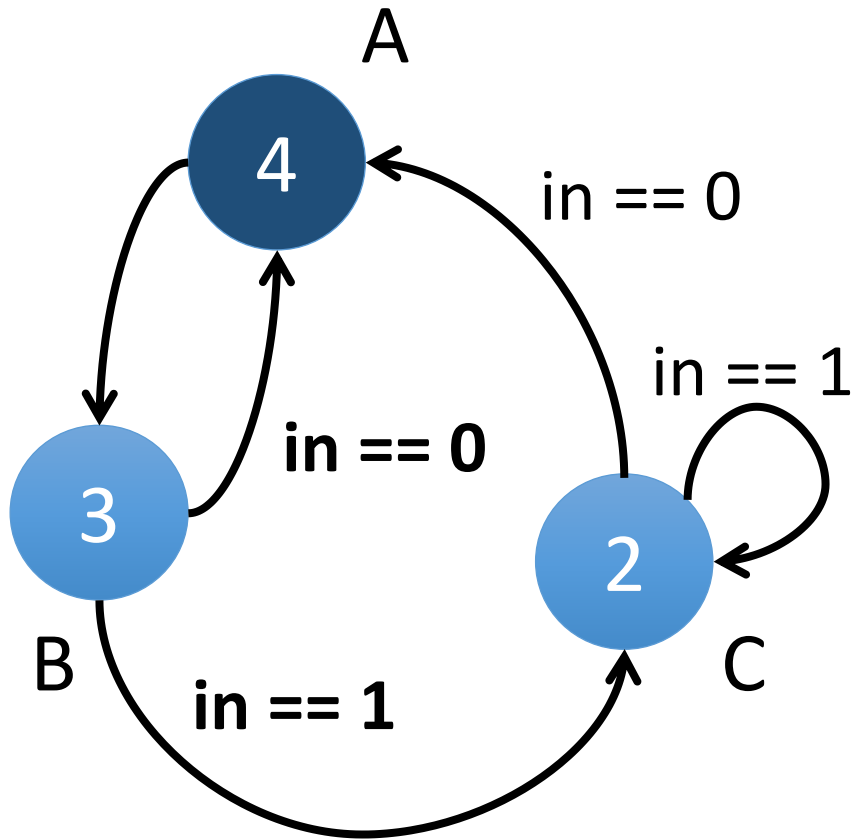
state	output
A	4
B	3
C	2



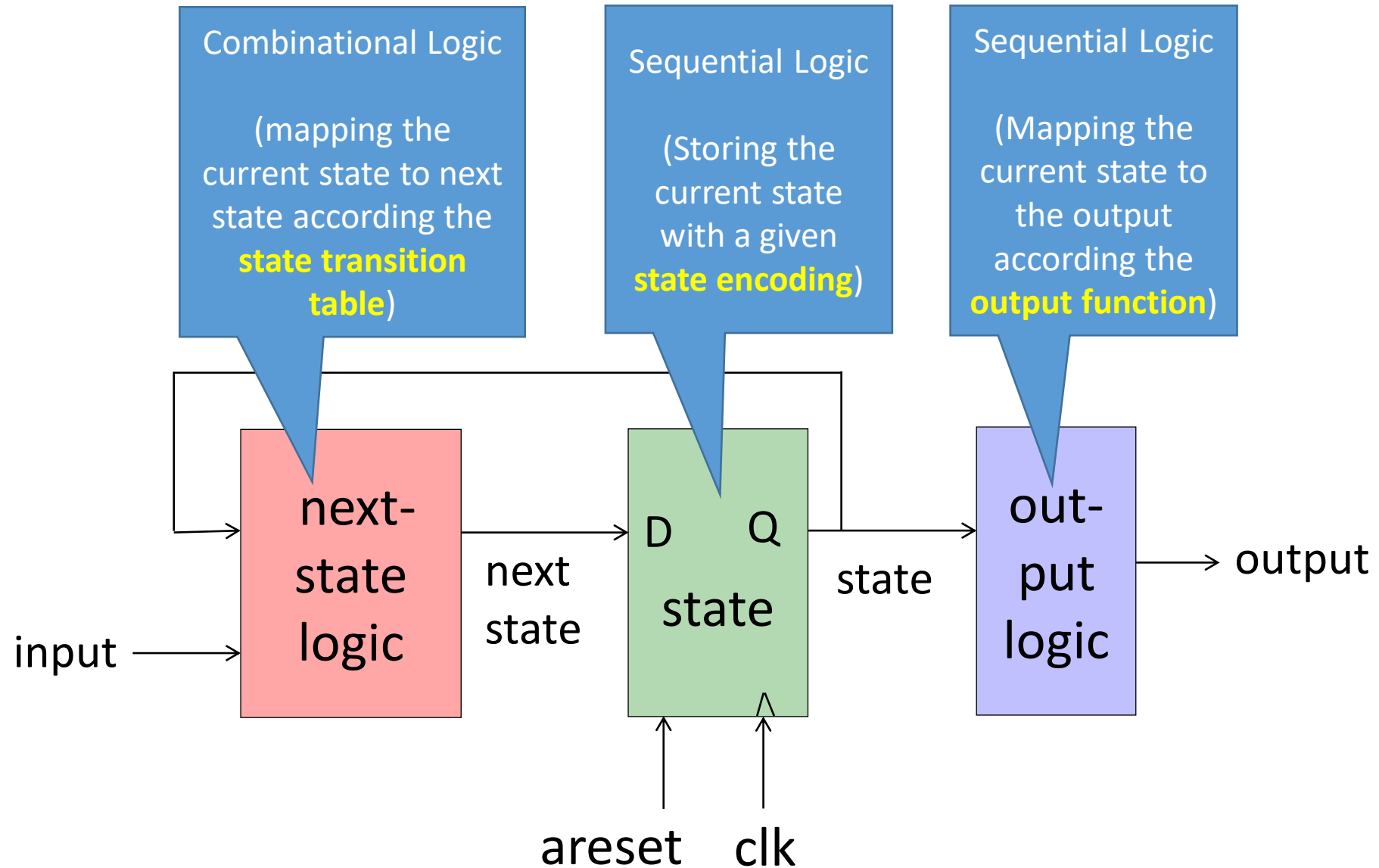
Timing Diagram – Example 3



Mapping a State Diagram to Hardware



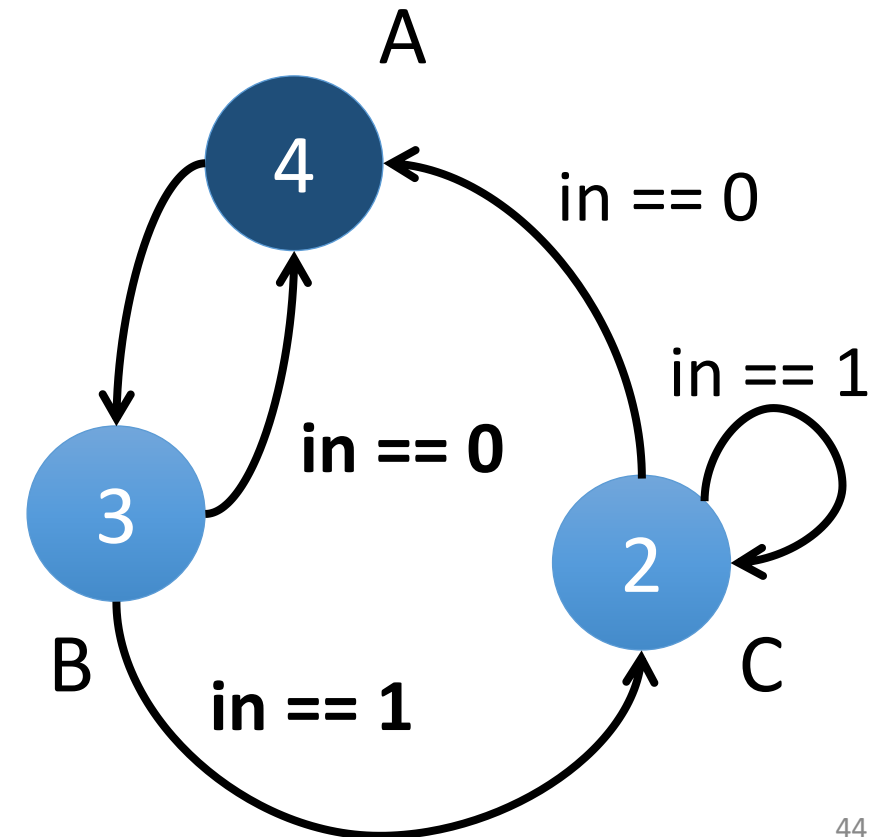
Essence of Moore Machines



State Encoding

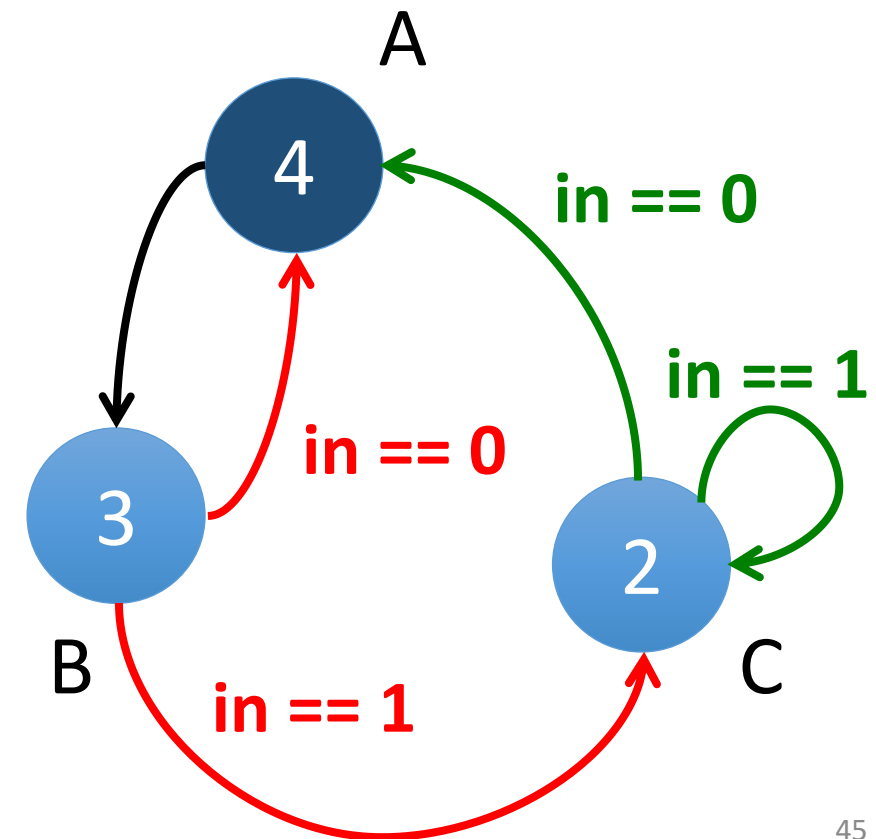
We use binary encoding \rightarrow we need two bits to encode the three states A, B, C

state	encoding
A	00
B	01
C	10



State Transition Table

present state	in	next state
A	0	B
A	1	B
B	0	A
B	1	C
C	0	A
C	1	C

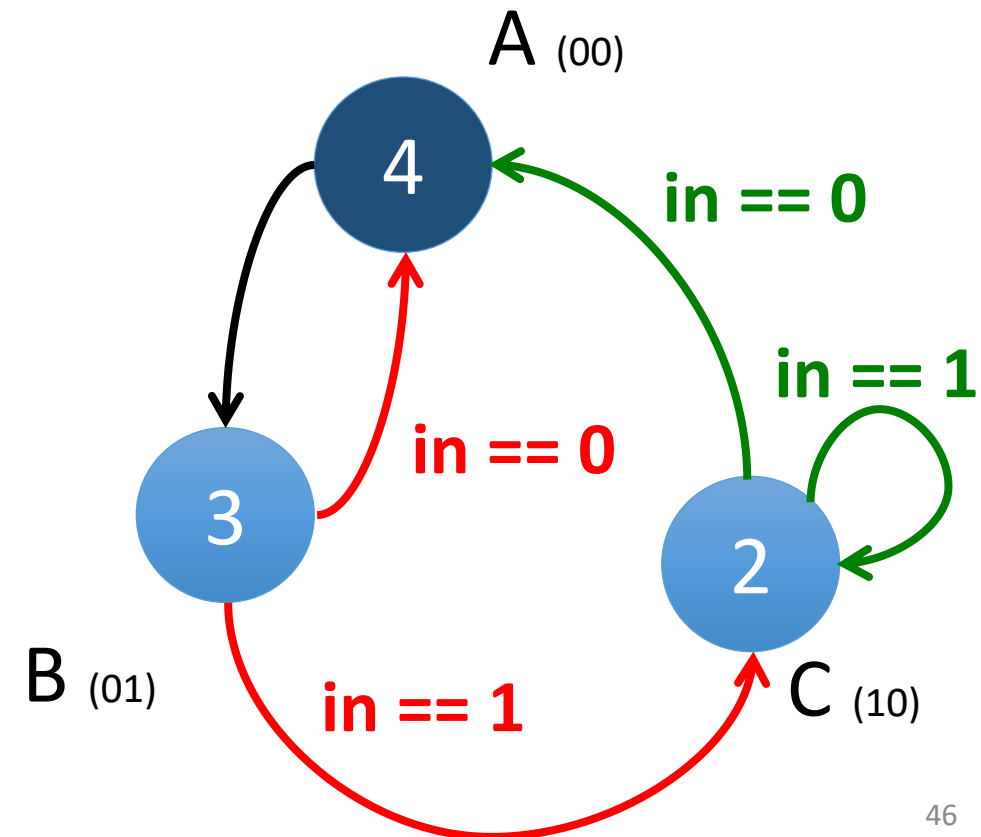


“11” does not exist: We use “Don’t Care” as the following state

present		in	next	
s1	s0		s1	s0
0	0	0	0	1
0	0	1	0	1
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	0
1	1	0	x	x
1	1	1	x	x

$$\text{next } s0 = ((\sim s1) \& (\sim s0) \& (\sim \text{in})) \mid ((\sim s1) \& (\sim s0) \& \text{in})$$

$$\text{next } s1 = ((\sim s1) \& s0 \& \text{in}) \mid (s1 \& (\sim s0) \& \text{in})$$



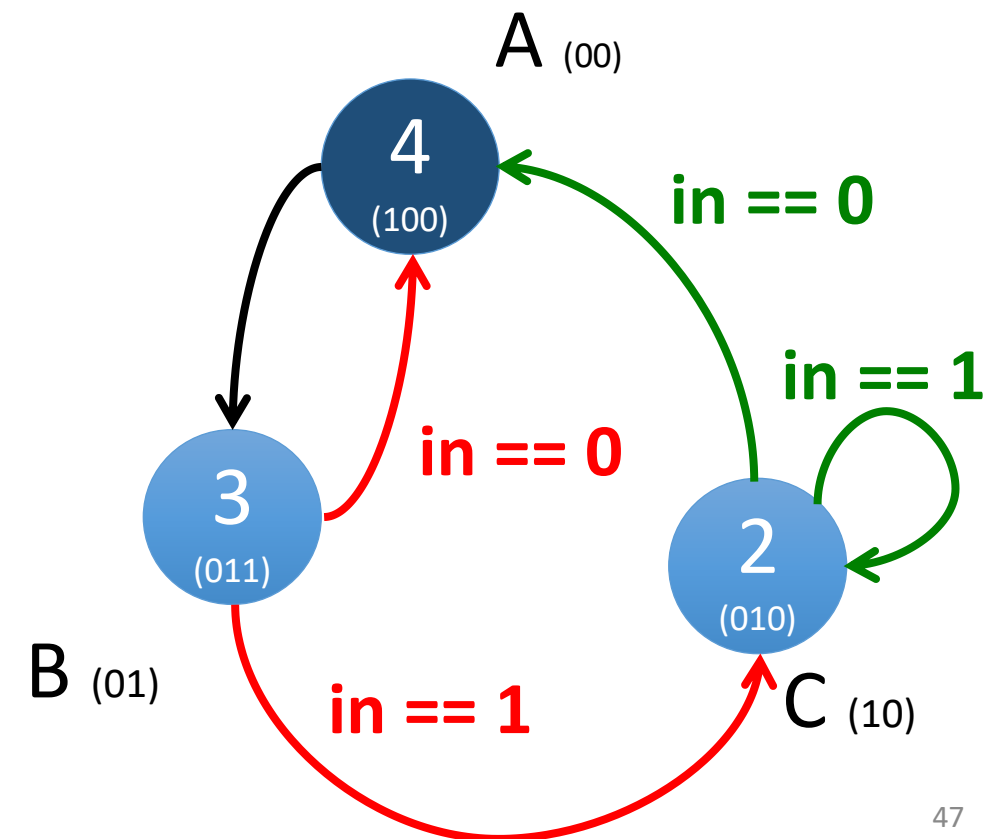
Output Function

s1	s0	o2	o1	o0
0	0	1	0	0
0	1	0	1	1
1	0	0	1	0

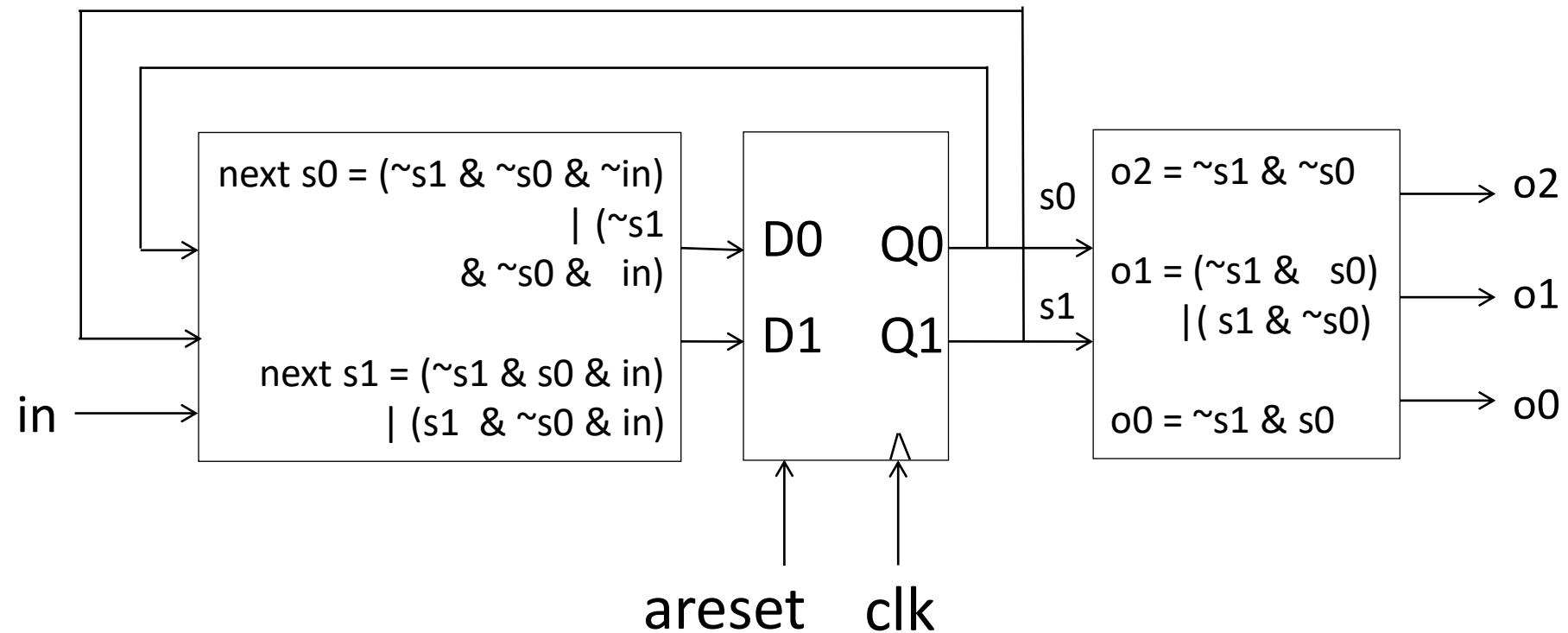
$$o2 = \sim s1 \ \& \ \sim s0$$

$$o1 = (\sim s1 \ \& \ s0) \ | \ (s1 \ \& \ \sim s0)$$

$$o0 = \sim s1 \ \& \ s0$$

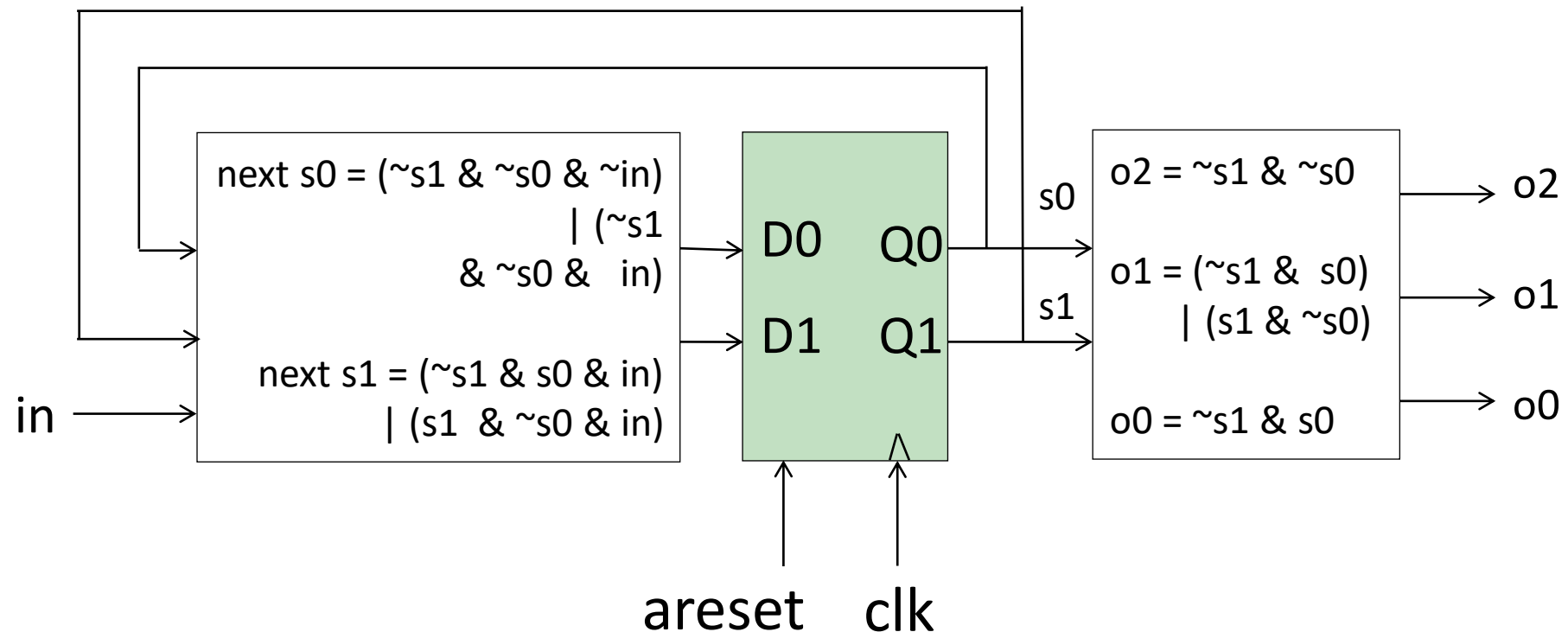


Structural diagram of the FSM



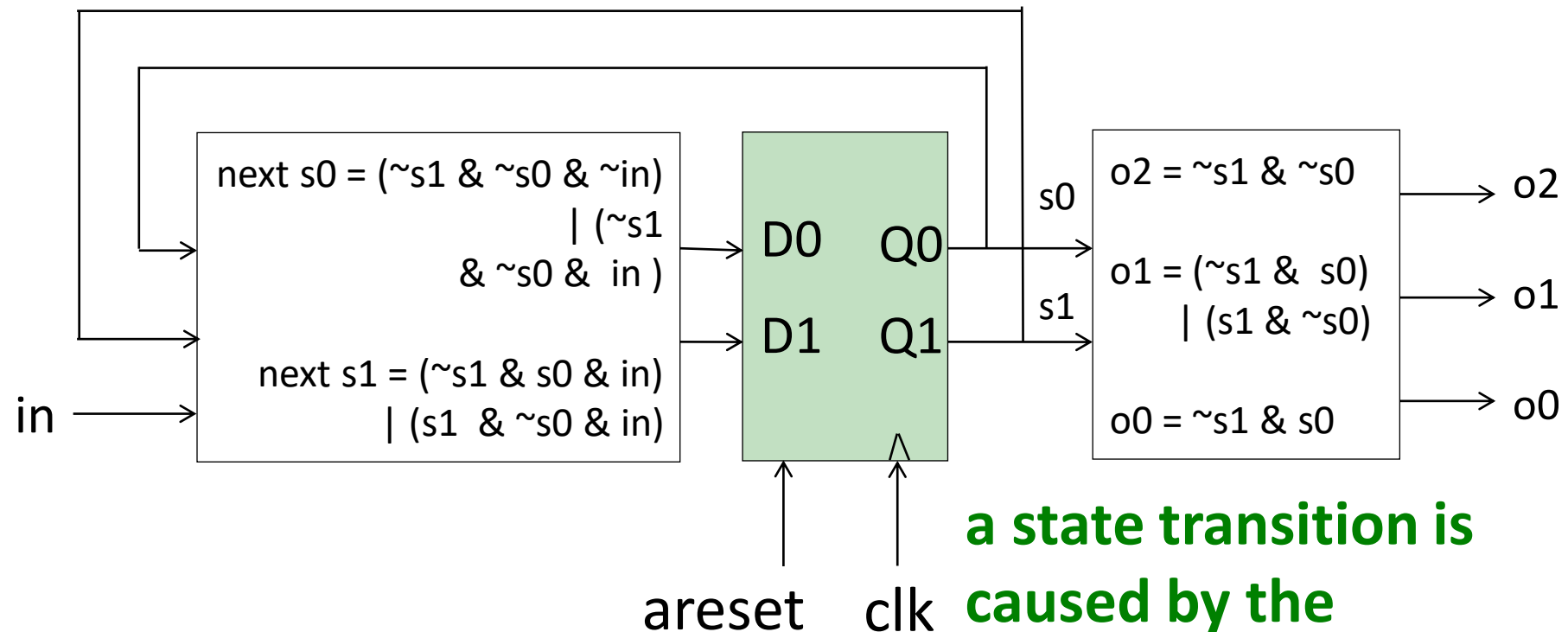
Essence of Moore Machines

the state is
stored
in a register



Essence of Moore Machines

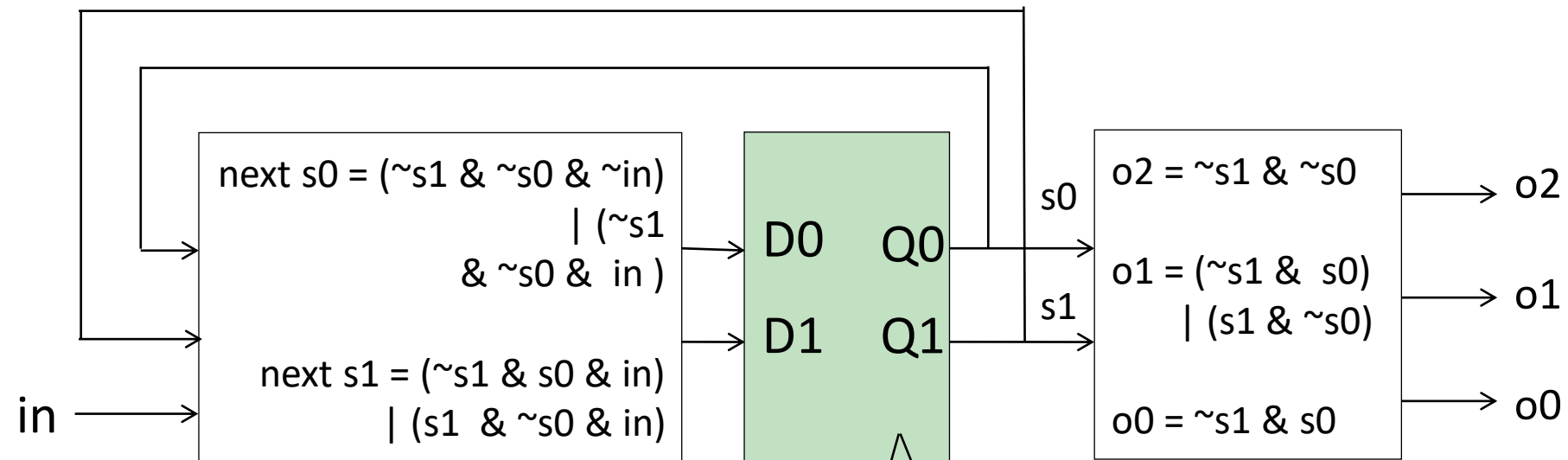
**the state is
stored
in a register**



**a state transition is
caused by the
clock signal.**

Essence of Moore Machines

**the state is
stored
in a register**



**With “areset” we can
initialize the ASM
 (“initial state”).**

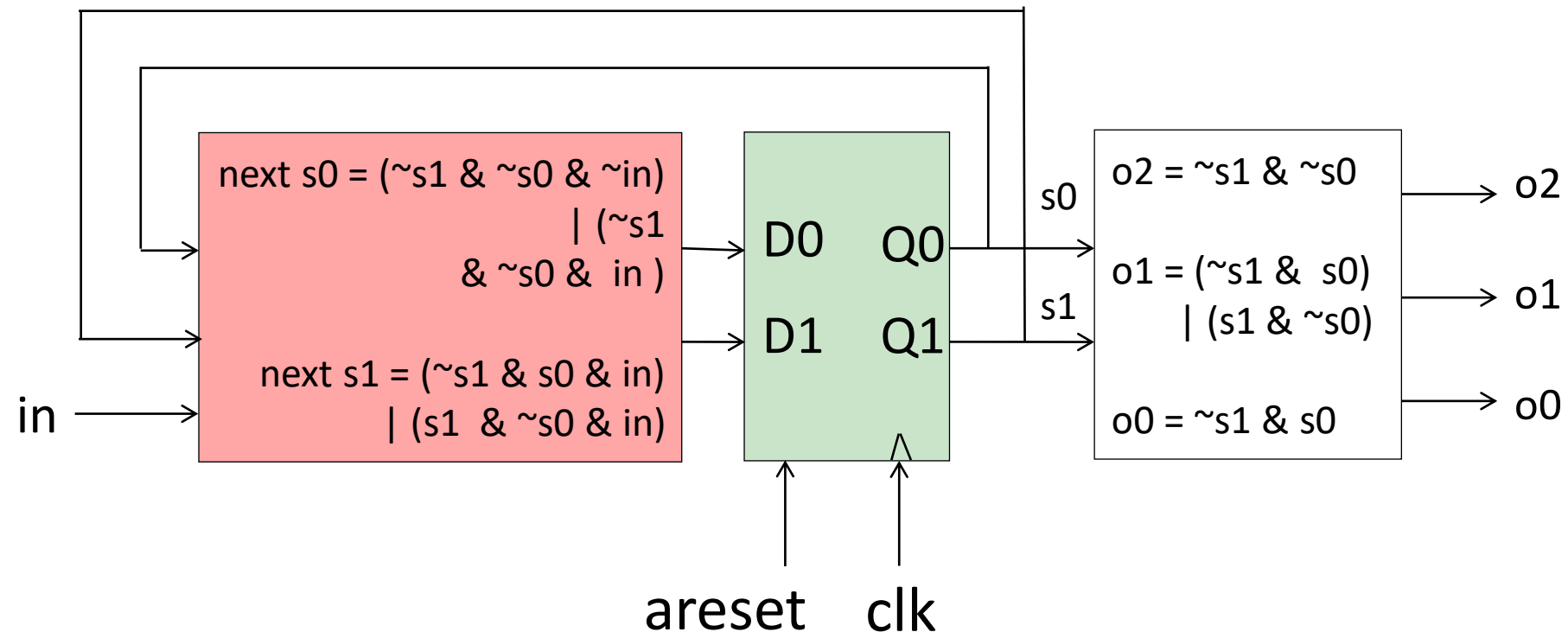
areset

clk

**a state transition is
caused by the
clock signal.**

Essence of Moore Machines

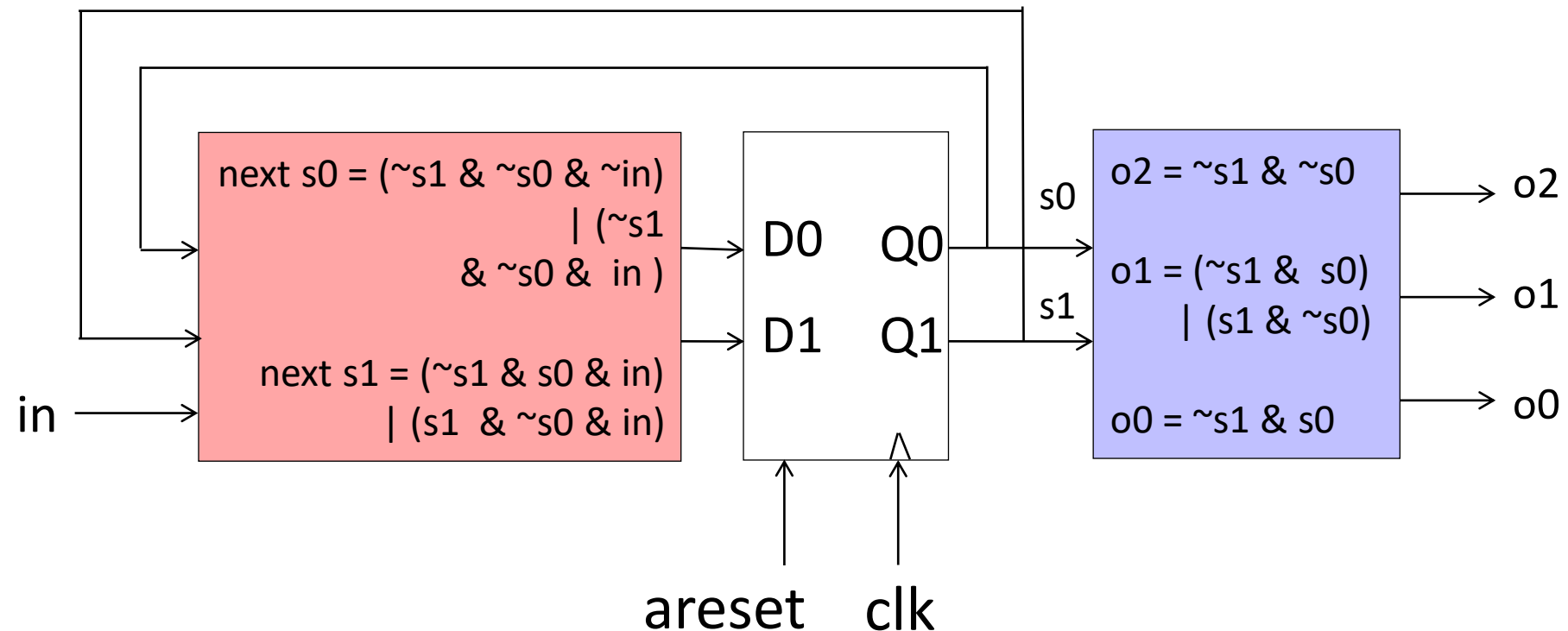
**With the next-state function f
we compute the next state:
 $\text{next state} = f(\text{state}, \text{input})$**



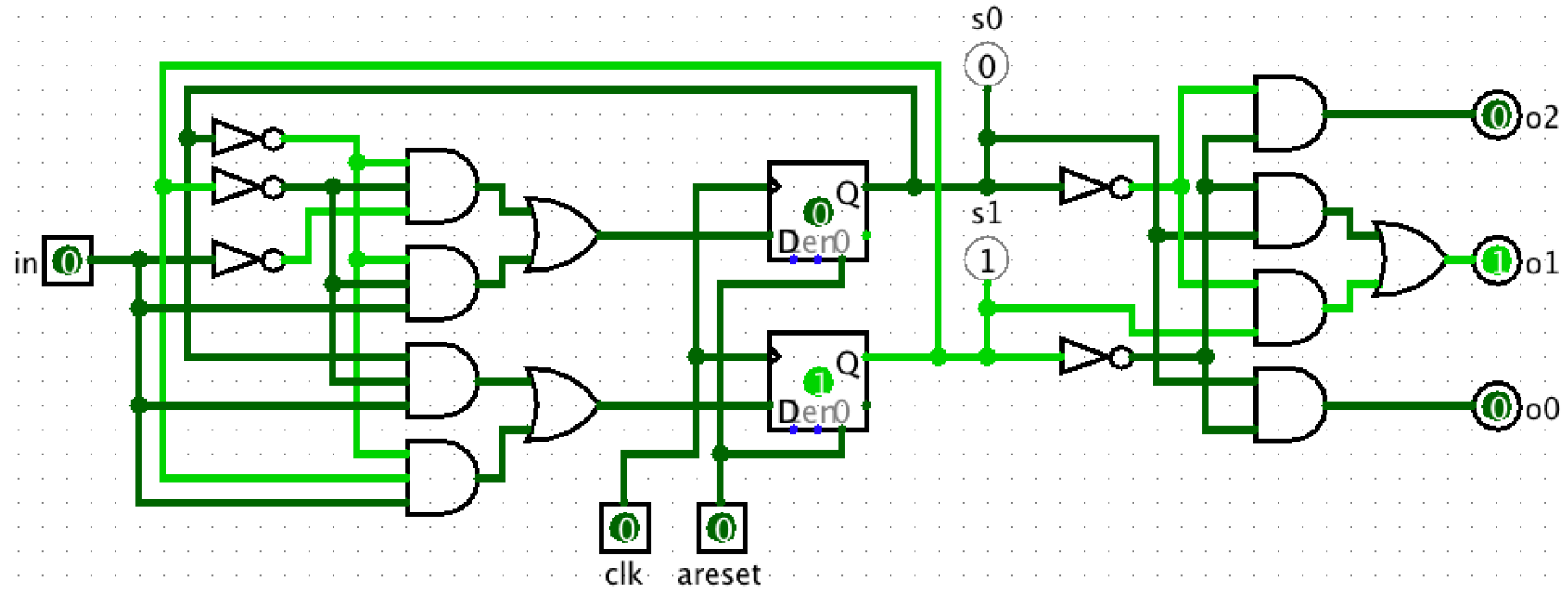
Essence of Moore Machines

With the output function we compute the output values:

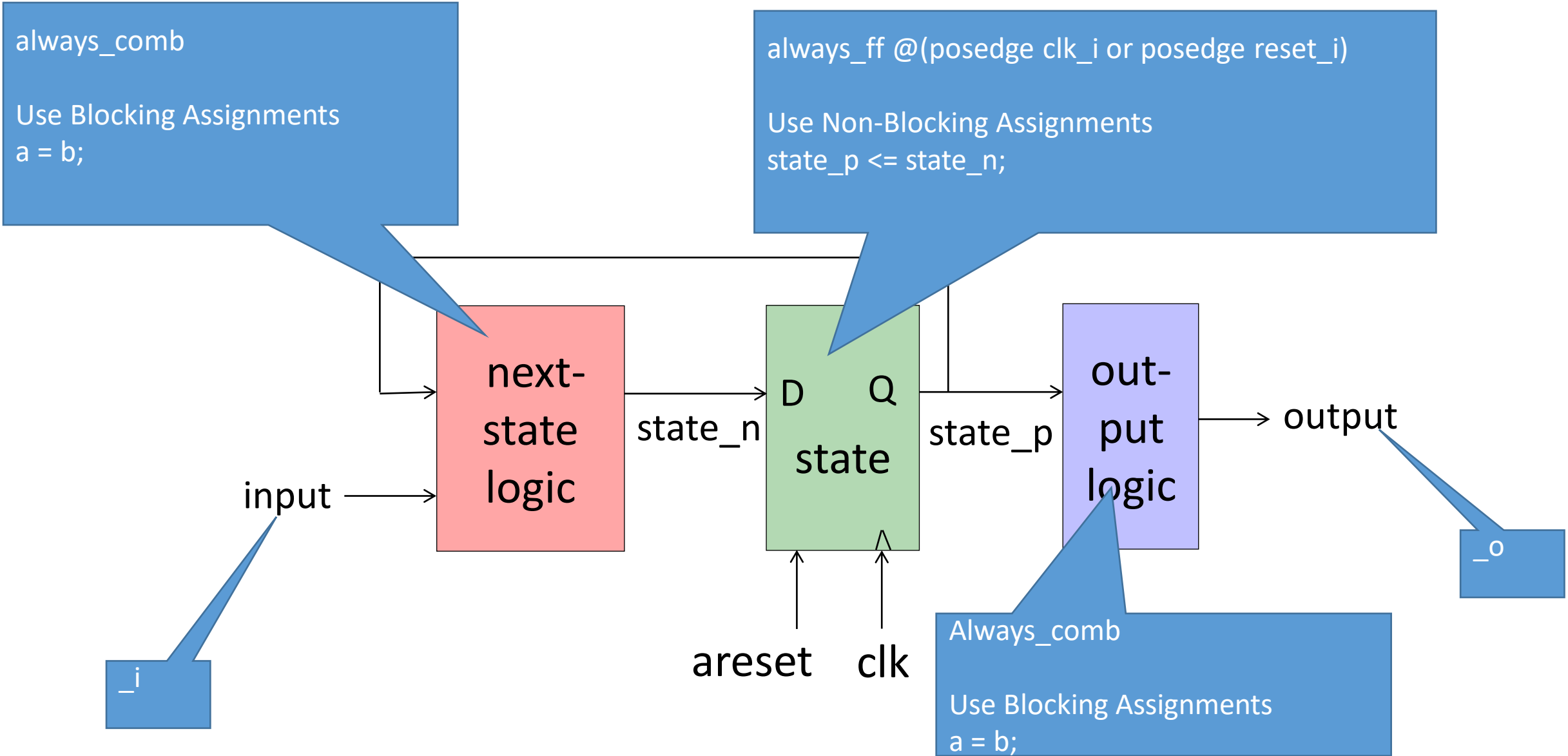
output = g(state)



Implementation with Digital

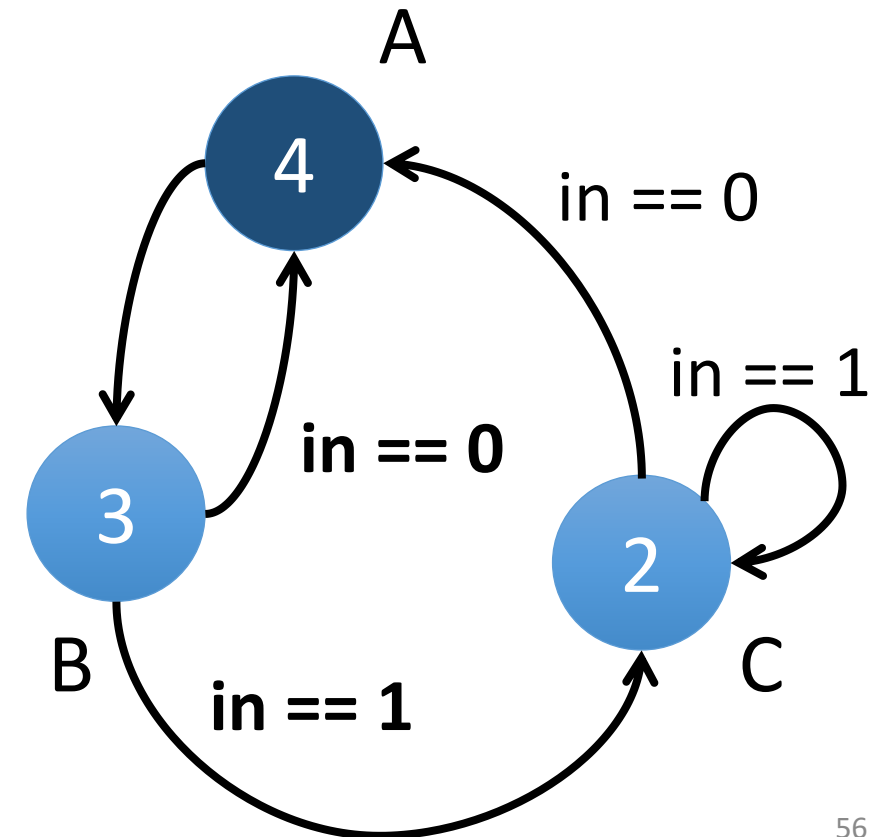


Coding Guidelines in SystemVerilog - Moore Machines



Modeling with SystemVerilog

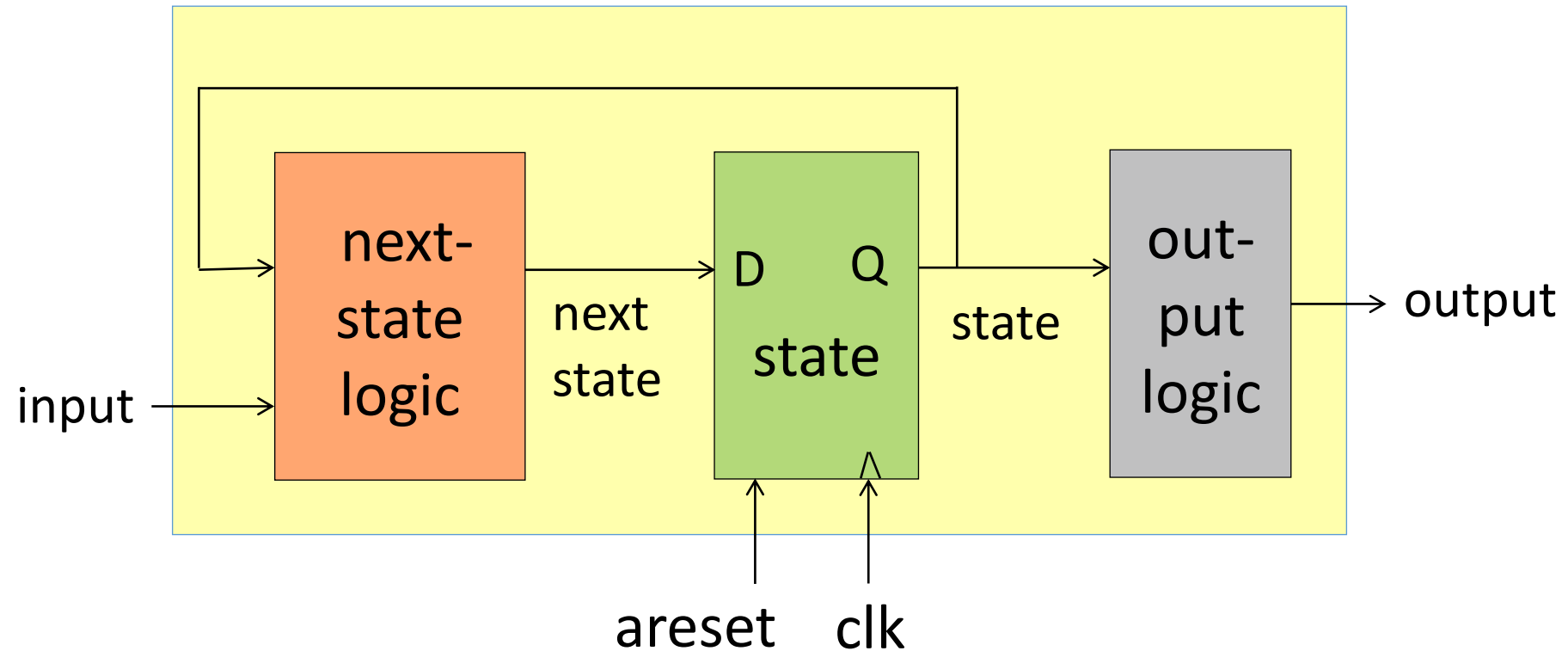
See example `con03.03_moore_fsm`



There exist 2 types of machines - check out the LITTLE but IMPORTANT difference

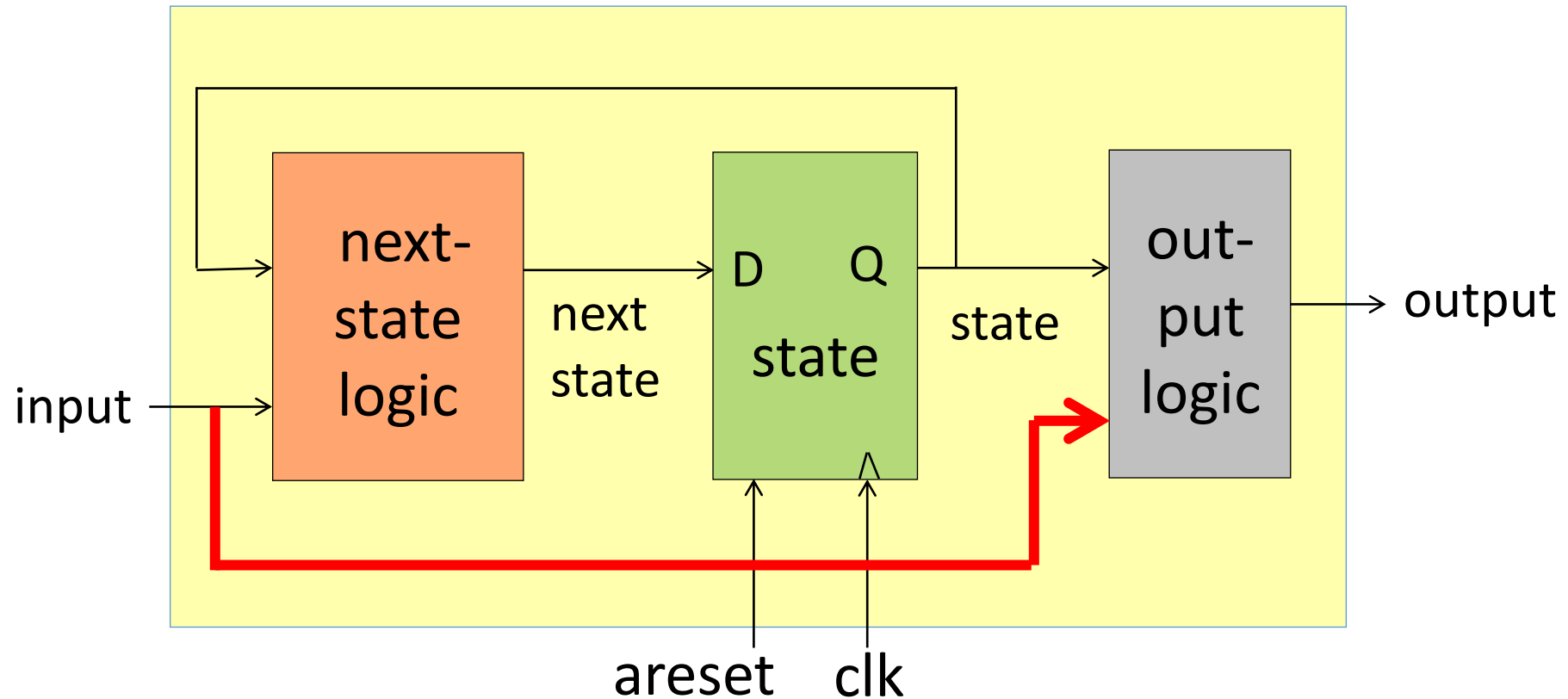
- **Moore Machines**
 - next state = function of present state and input
 - output = function of present state
- **Mealy Machines**
 - next state = function of present state and input
 - output = function of present state **and input**

Essence of **Moore** Machines



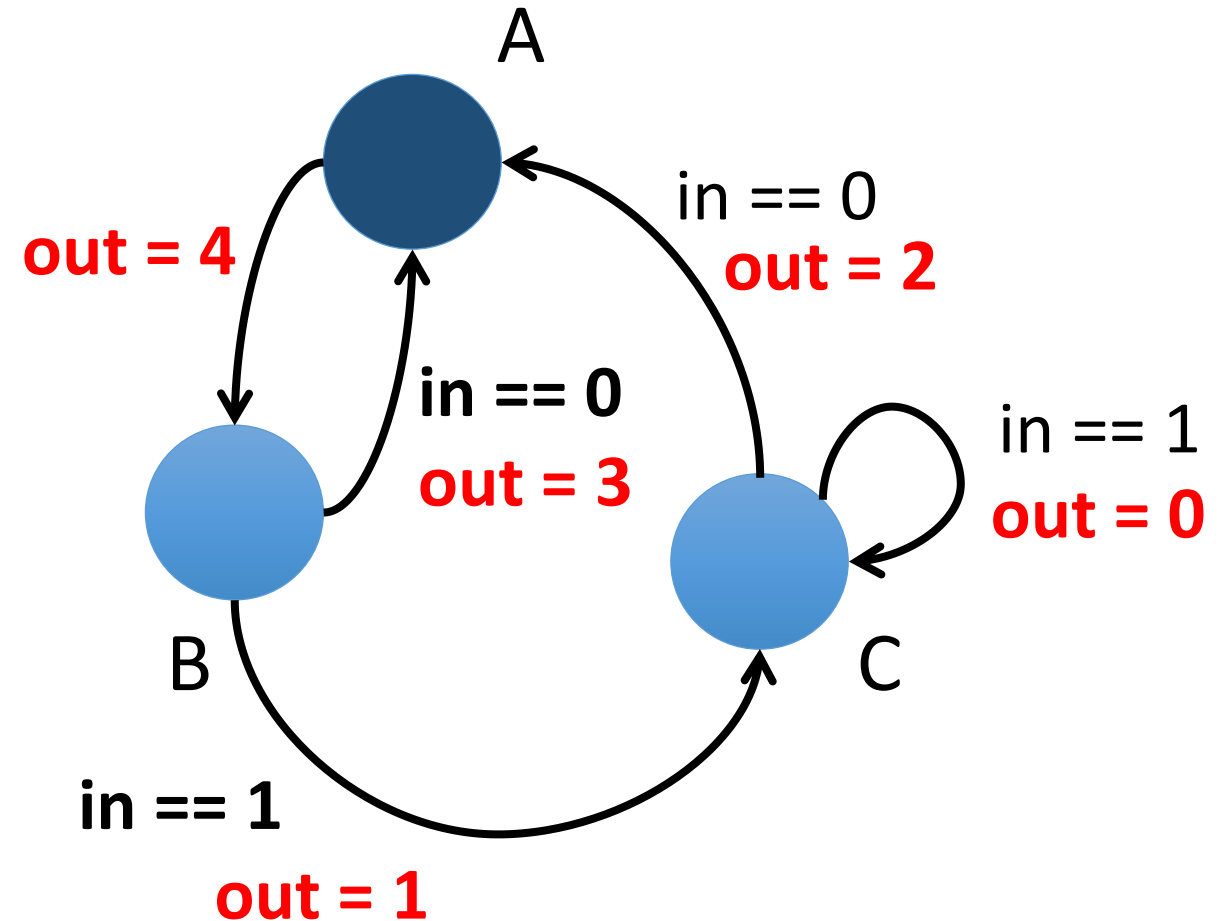
Essence of **Mealy** Machines

$$\text{output} = g(\text{state}, \text{input})$$



An example for a Mealy Machine

We write the **output values** next to the transition arrows, since the output depends not only on the state, but can also depend on the input.

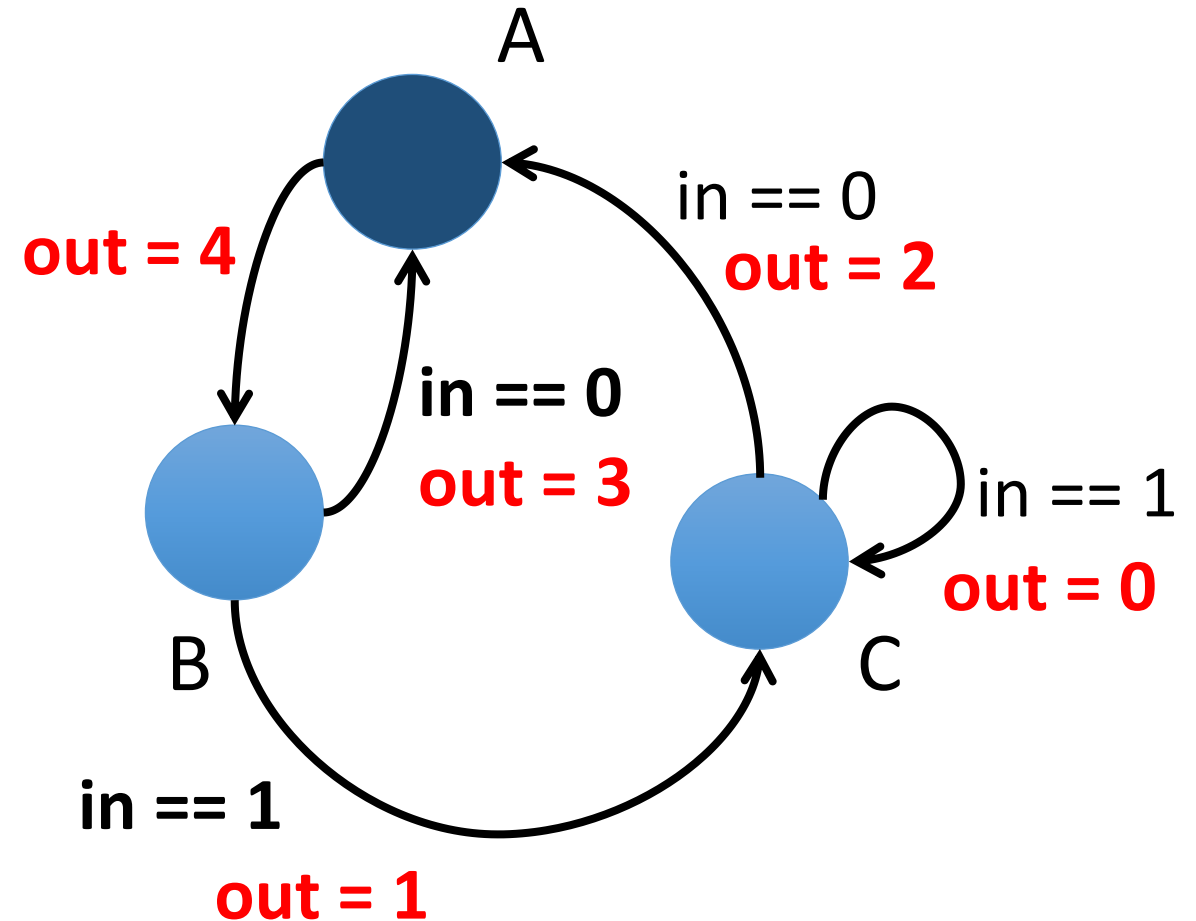


The output function

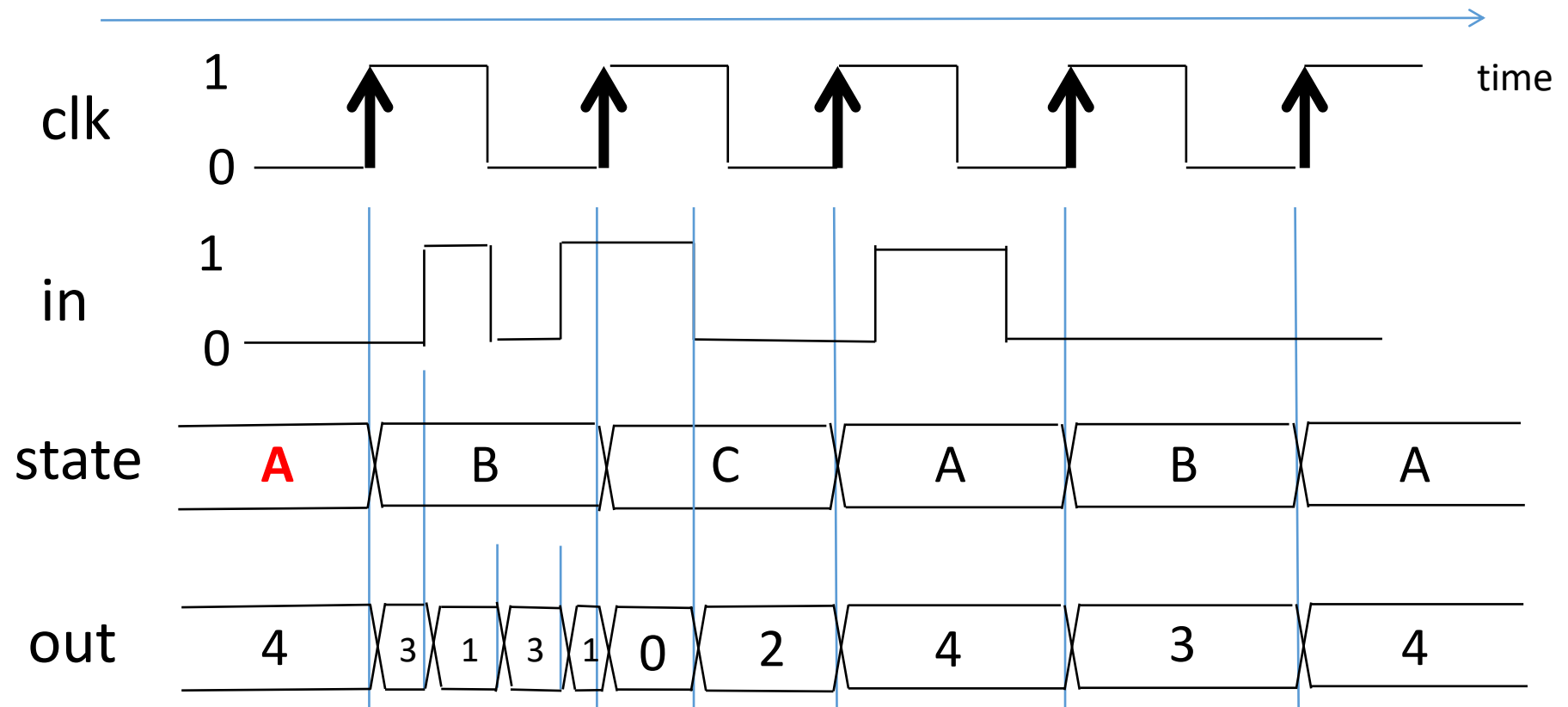
The output function can be derived from the state diagram.

output = g(state, input)

state	in	output
A	0	4
A	1	4
B	0	3
B	1	1
C	0	2
C	1	0



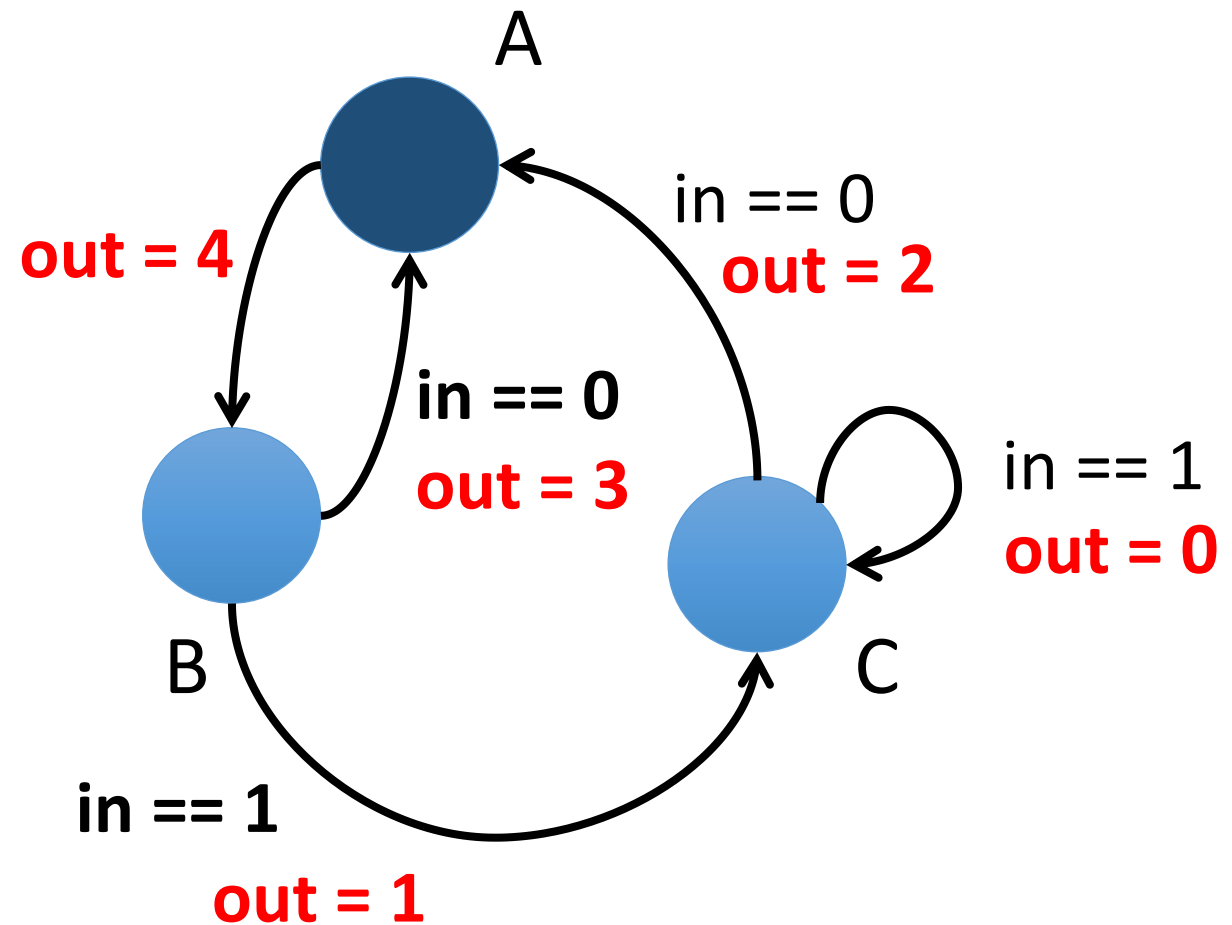
Timing diagram



Note how the value of “in” immediately influences the value of “out”.

Modeling with SystemVerilog

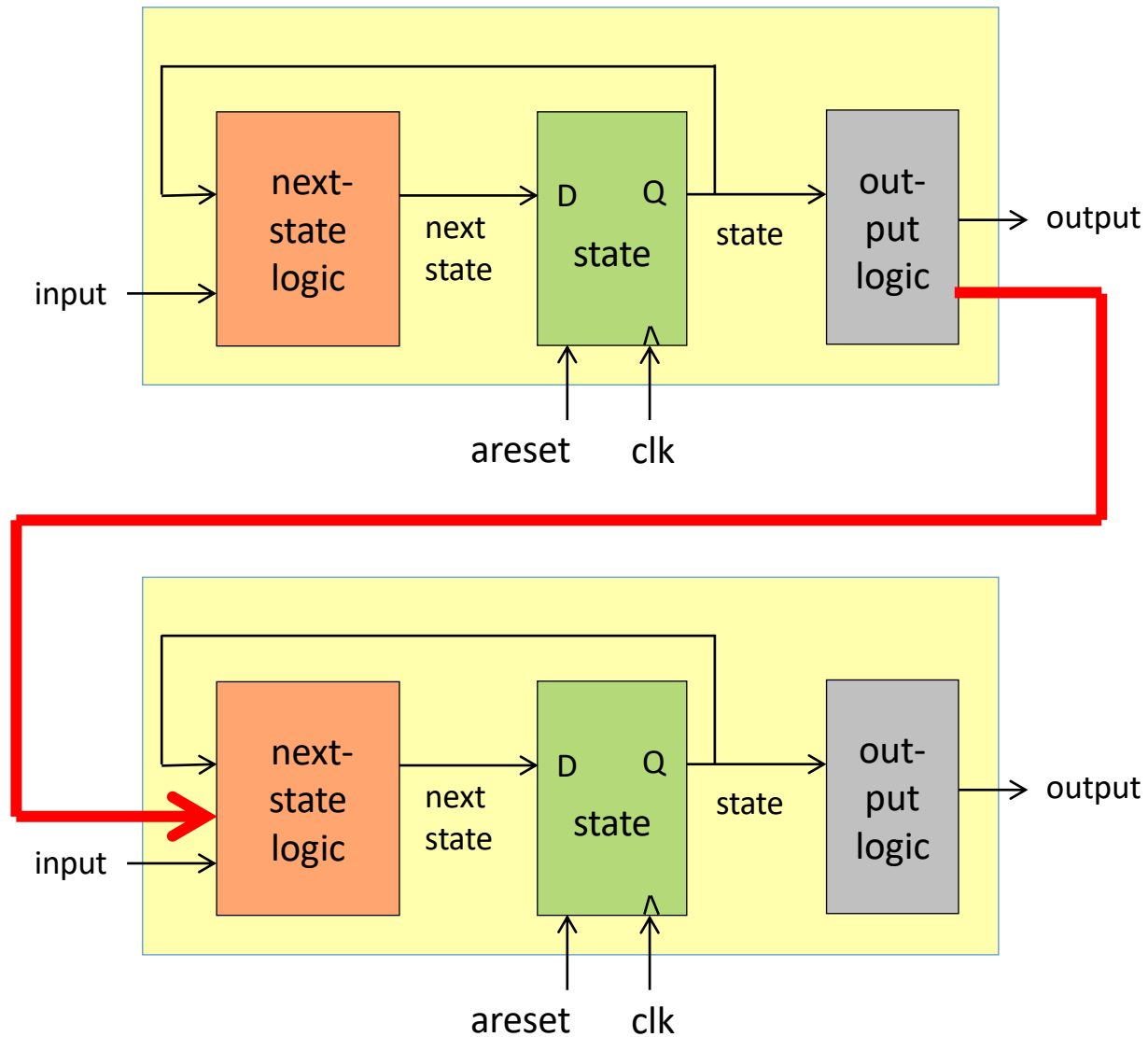
See example con03.04_mealy_fsm



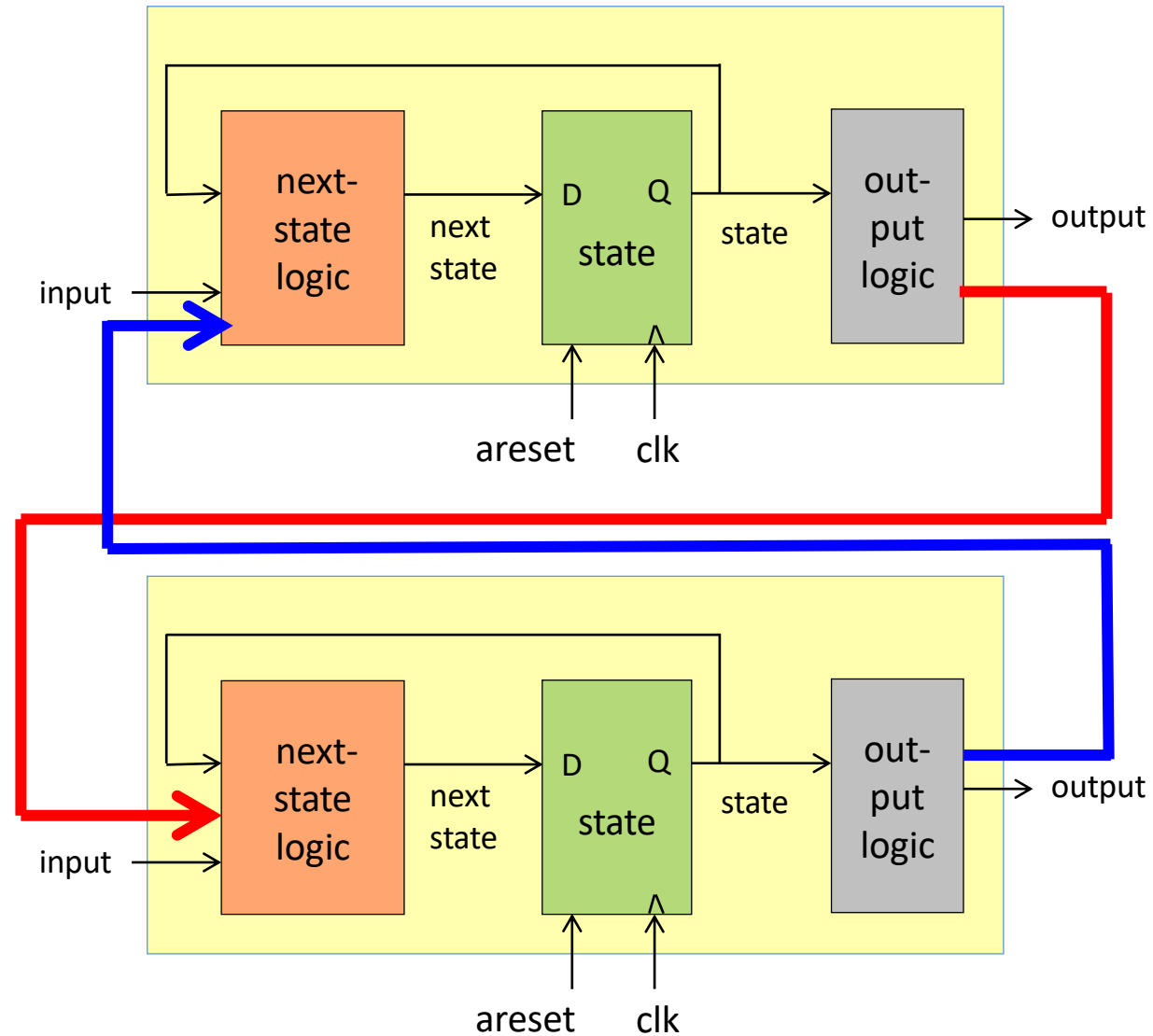
We can combine machines

- Combining Moore Machines causes no problem. We get another Moore Machine.
- Combining a Moore Machine with a Mealy Machine causes also no problem. We get a Moore Machine or a Mealy Machine.
- Combining two Mealy Machines can cause troubles: **One needs to avoid combinational loops!**

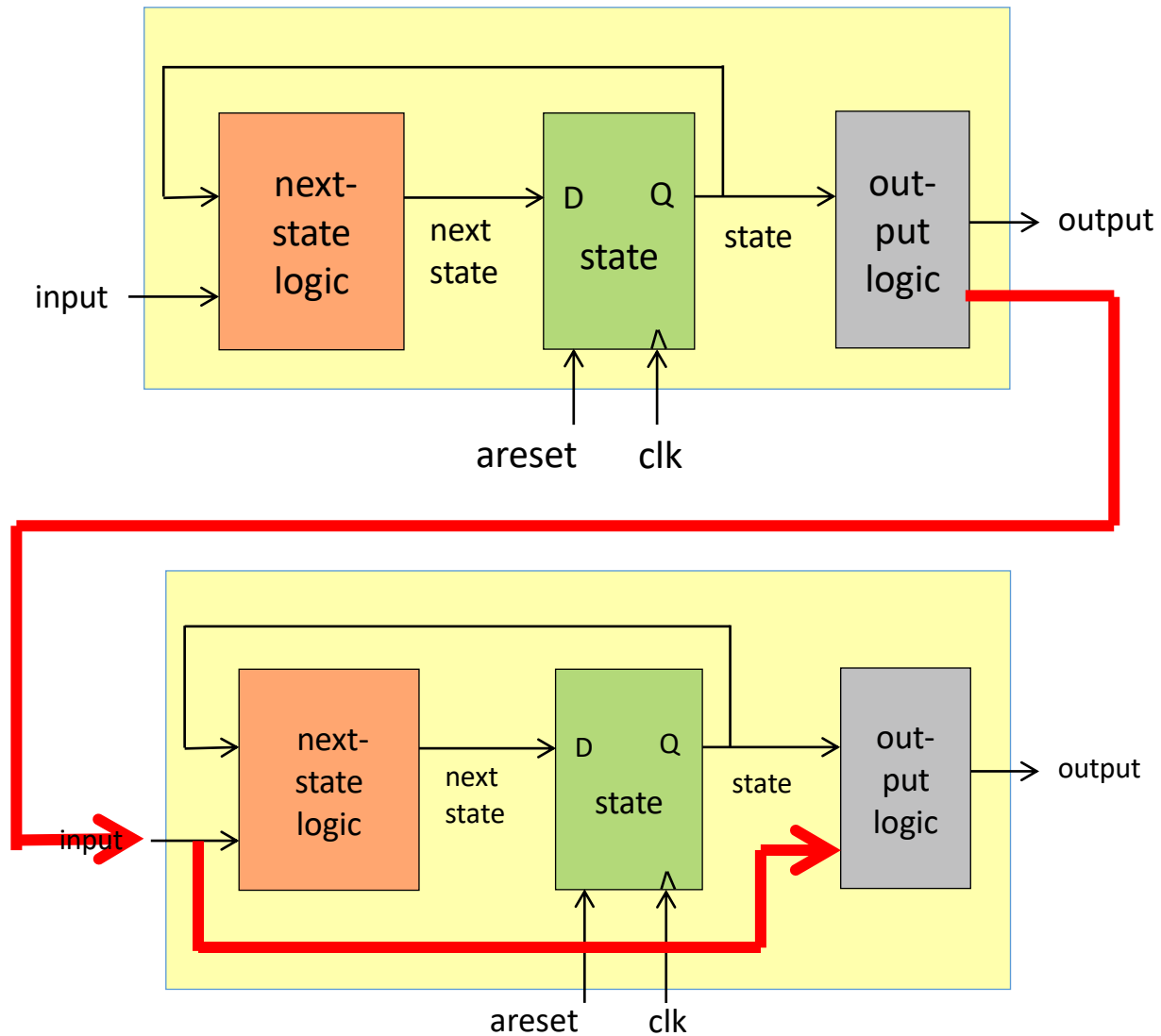
The combination of two Moore Machines creates again a (more complex) Moore Machine



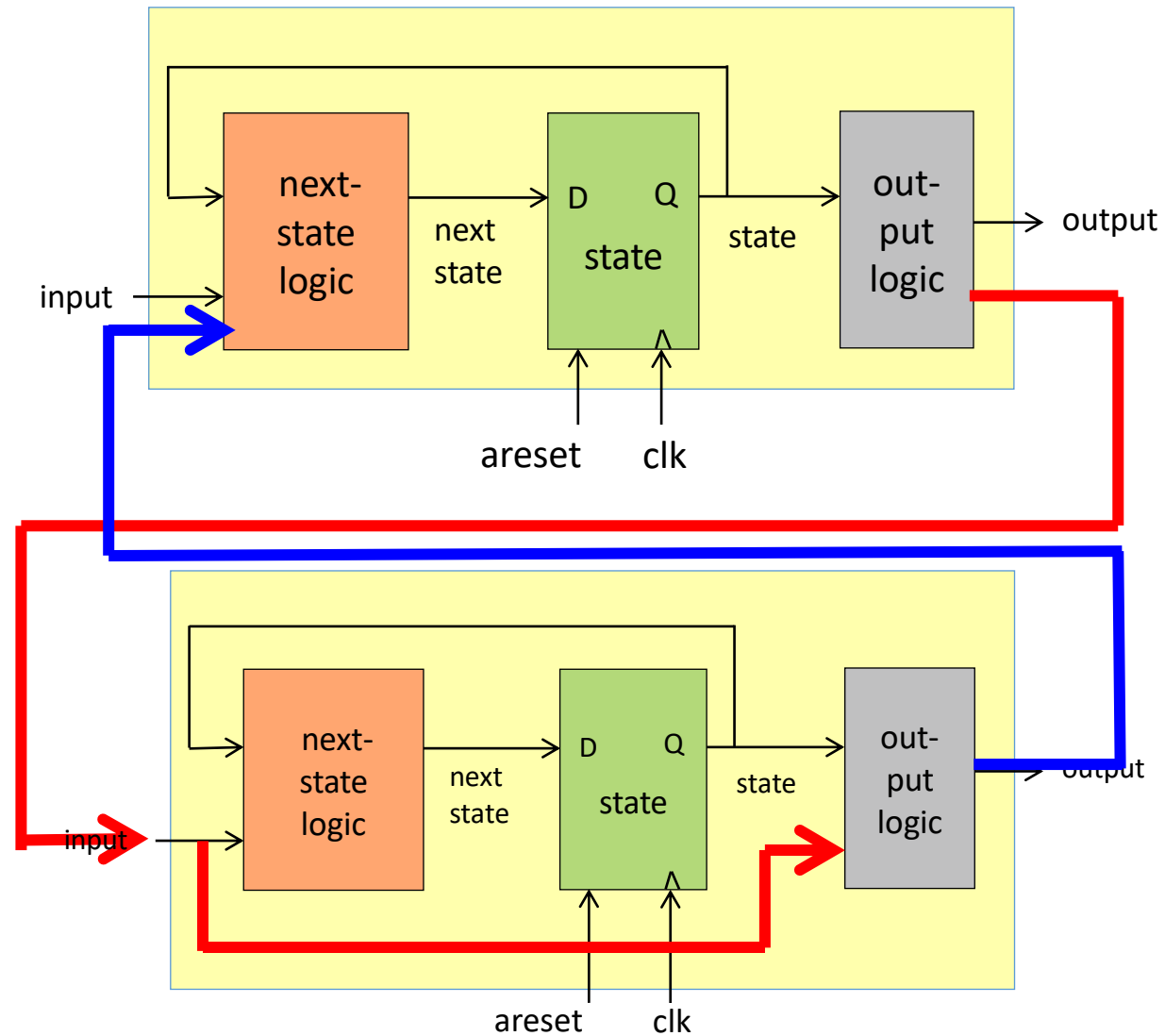
We can even connect More Machines in a loop-like fashion



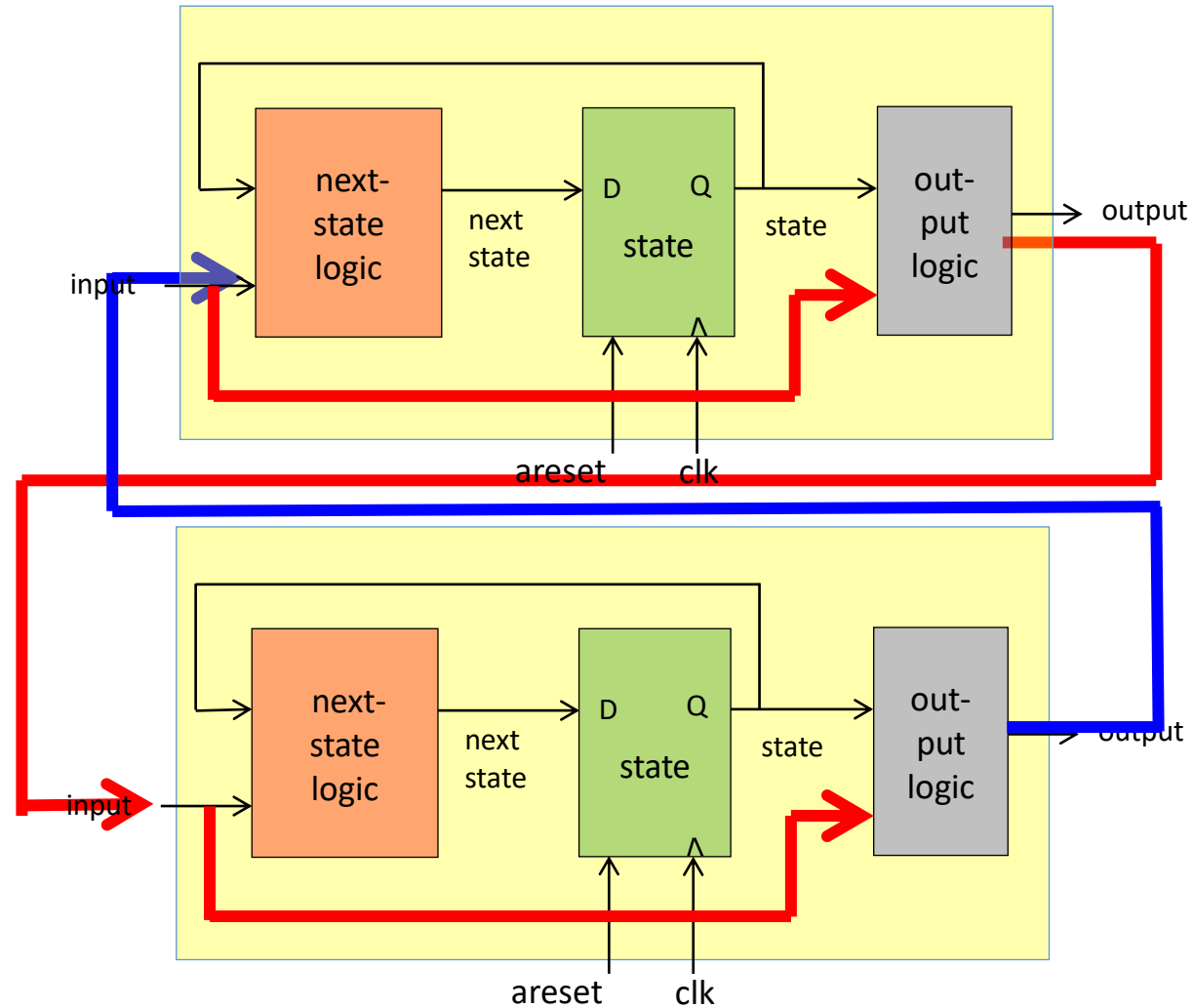
The combination of a Moore Machine with a Mealy Machine creates a Moore Machine or a Mealy Machine



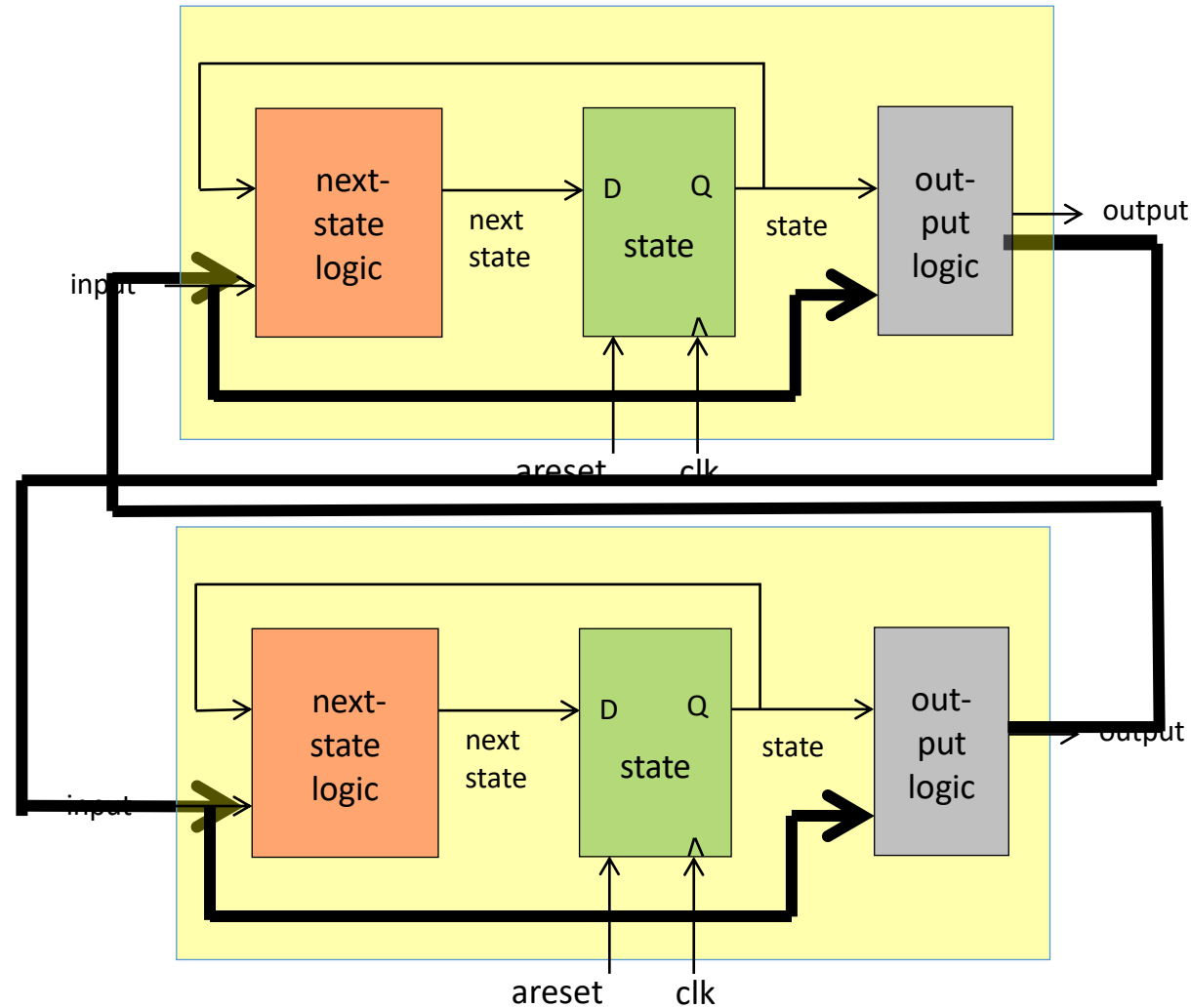
The combination of a Moore Machine with a Mealy Machine creates a Moore Machine or a Mealy Machine



The combination of **two Mealy Machines** is “dangerous”:
You need to avoid “combinational loops”



The combination of two Mealy Machines is “dangerous”: You need to avoid “**combinational loops**”



Summary

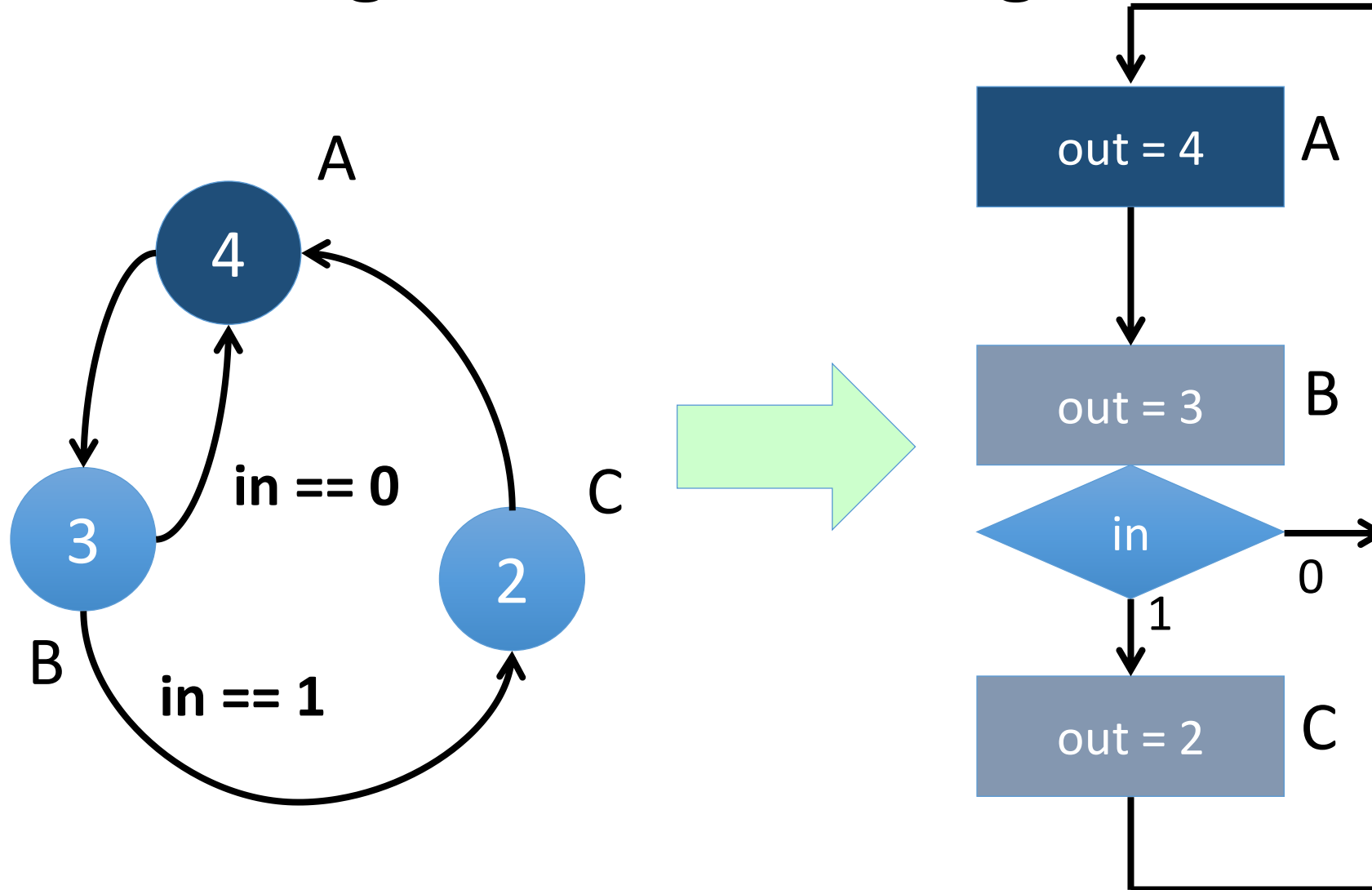
- All digital logic can in principle be built with Moore Machines and Mealy Machines.
- You always start by defining the function with a state diagram.
- If you choose values for the input signal(s), then you can derive the timing diagram by using the state diagram.
- From a state diagram, you can always derive a circuit diagram.

Algorithmic State Machines

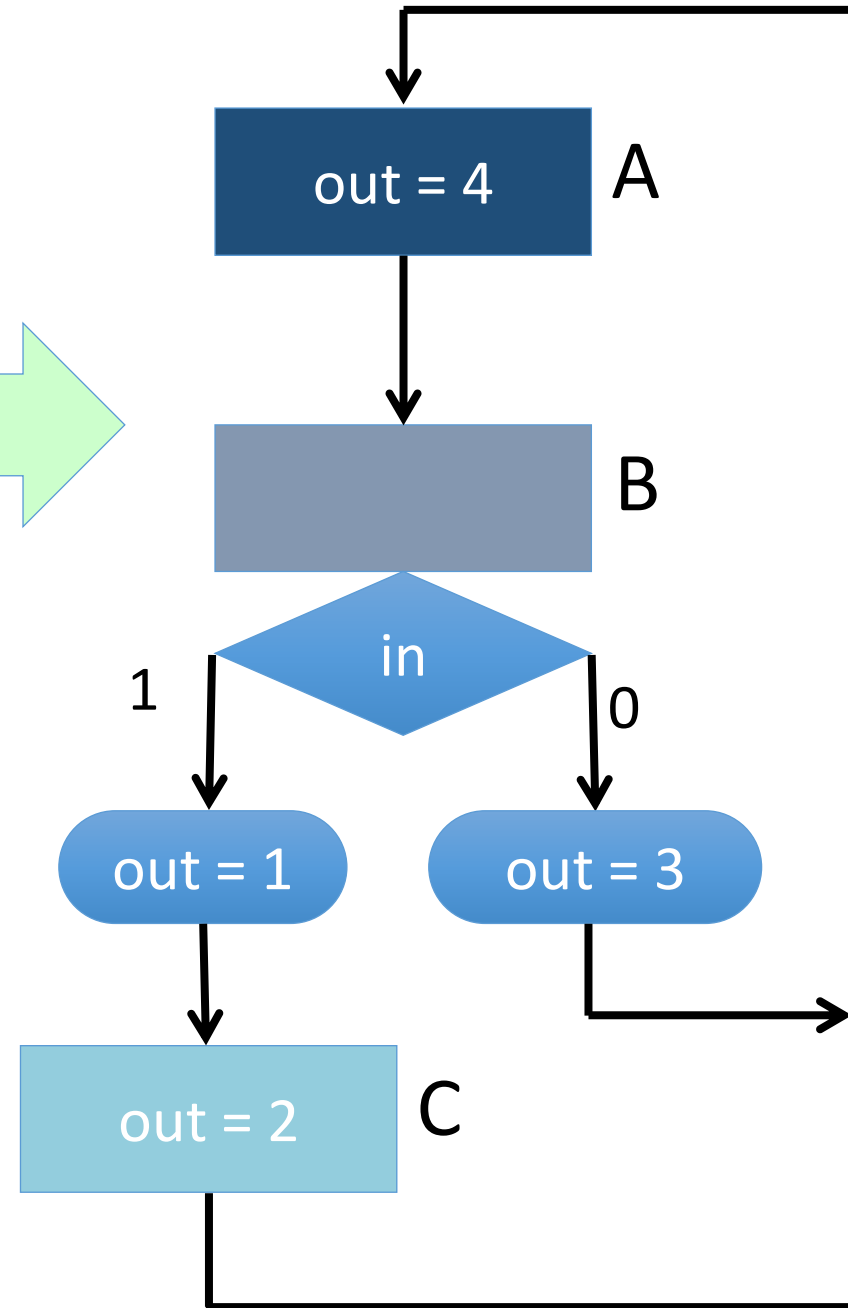
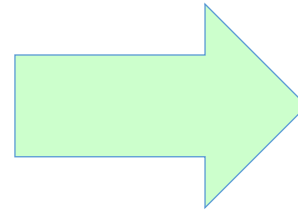
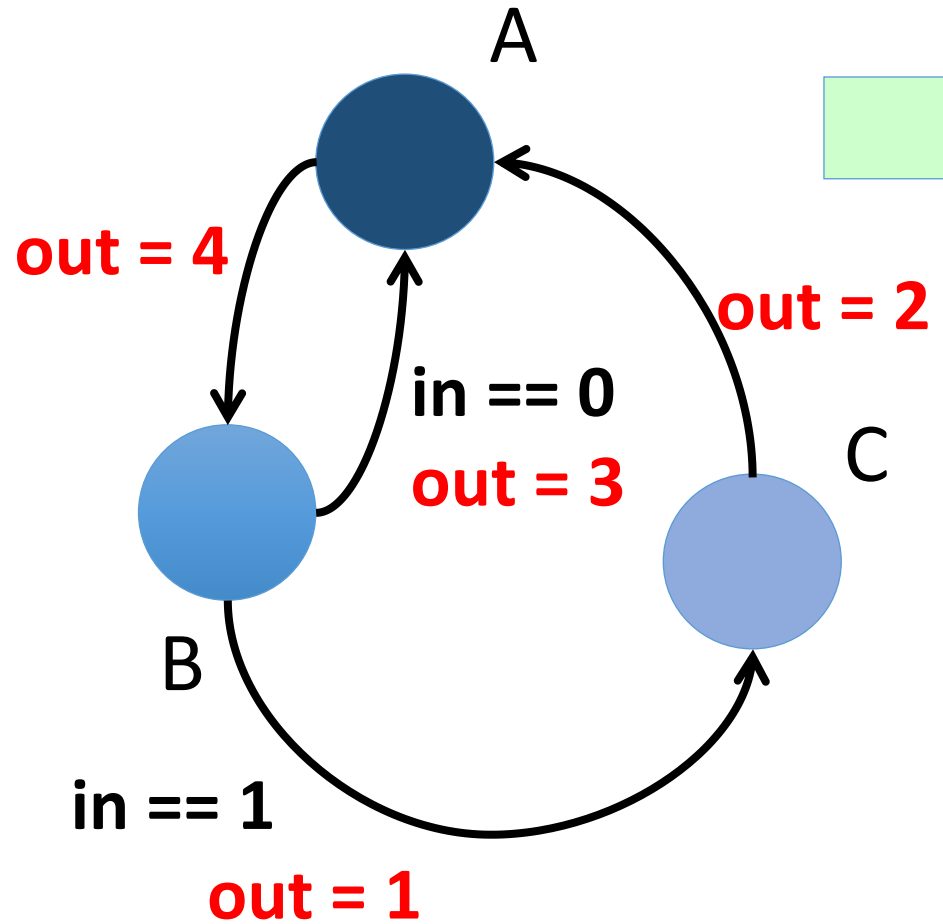
Algorithmic State Machines (ASMs)

- ASMs are a useful extension to finite state machines
- ASMs allow to specify a system consisting of a data path together with its control logic
- All FSM state diagrams have an equivalent ASM diagram

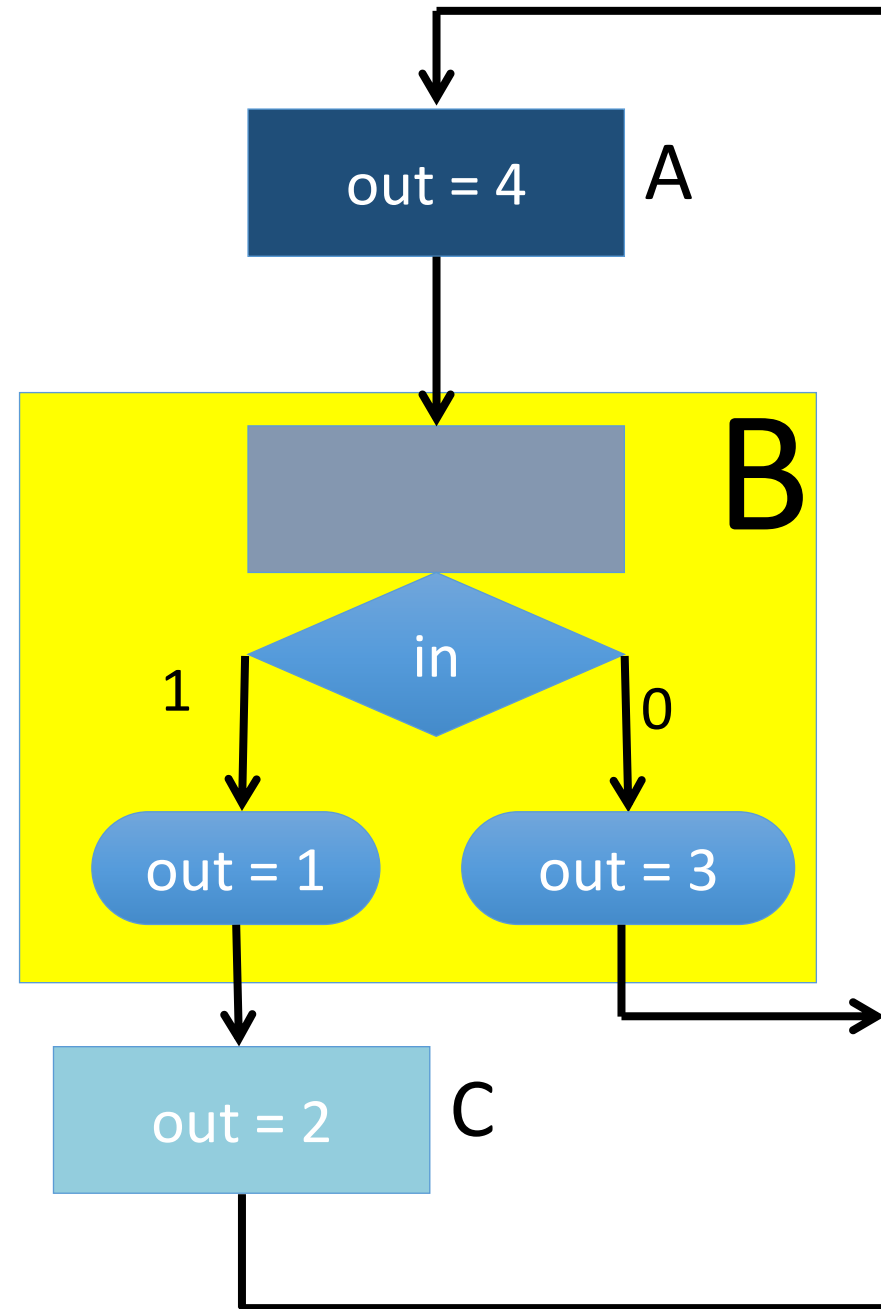
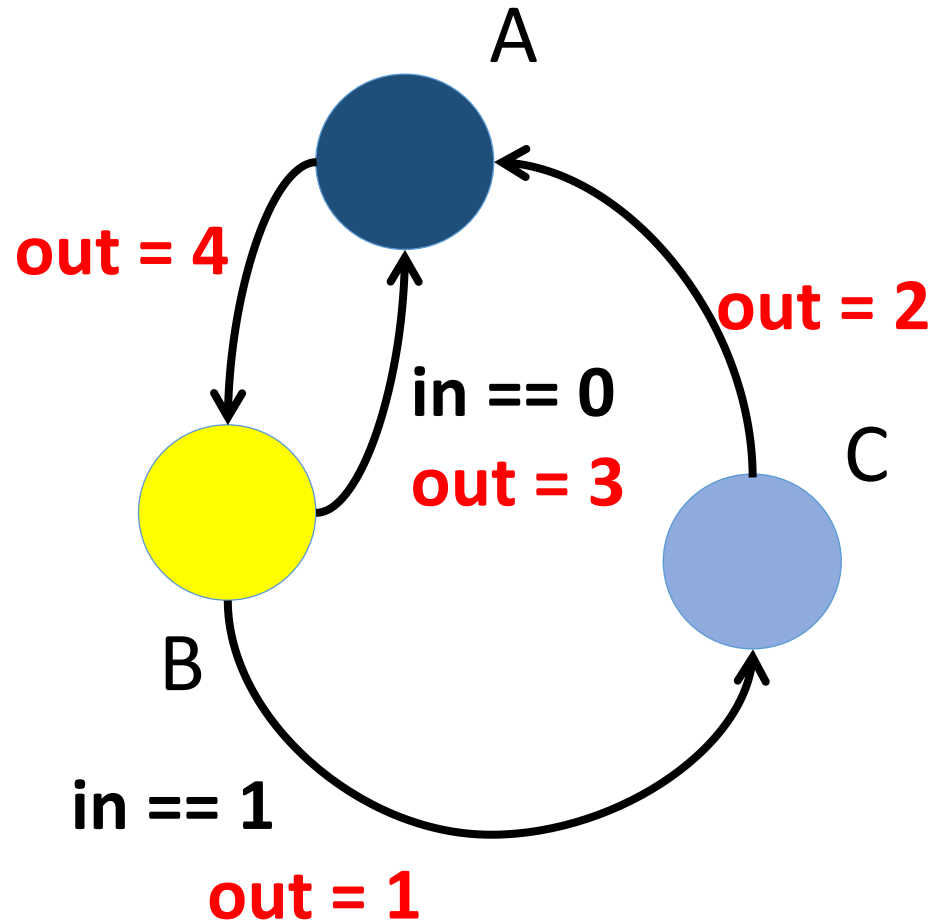
FSM state diagram \rightarrow ASM diagrams



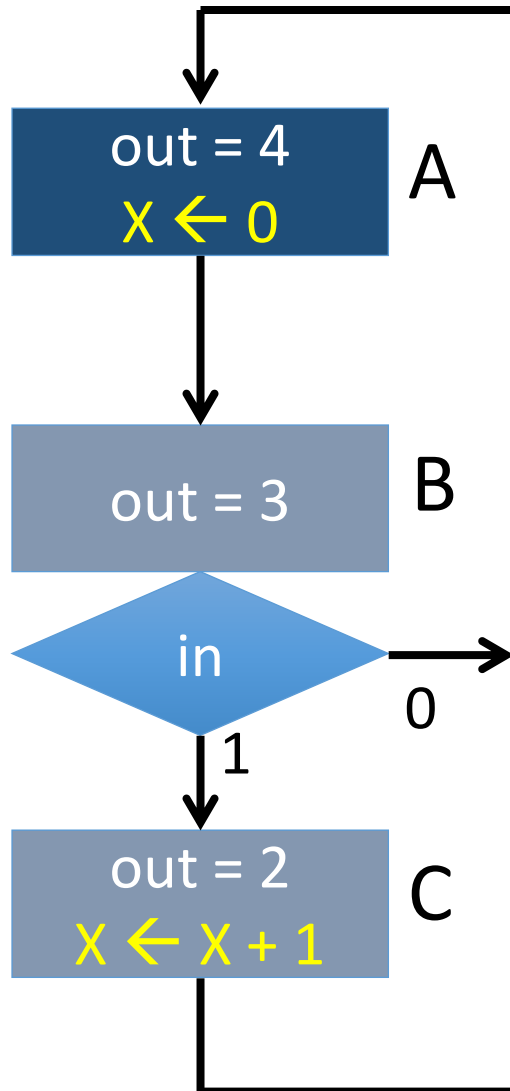
Mealy Machines



Mealy Machines



ASM diagram with two register-transfer statements



← The value stored in register X gets 0 at the state transition from state A to state B.

← The value in register X does not change upon leaving state B.

← The value in register X gets incremented at the state transition following state C.

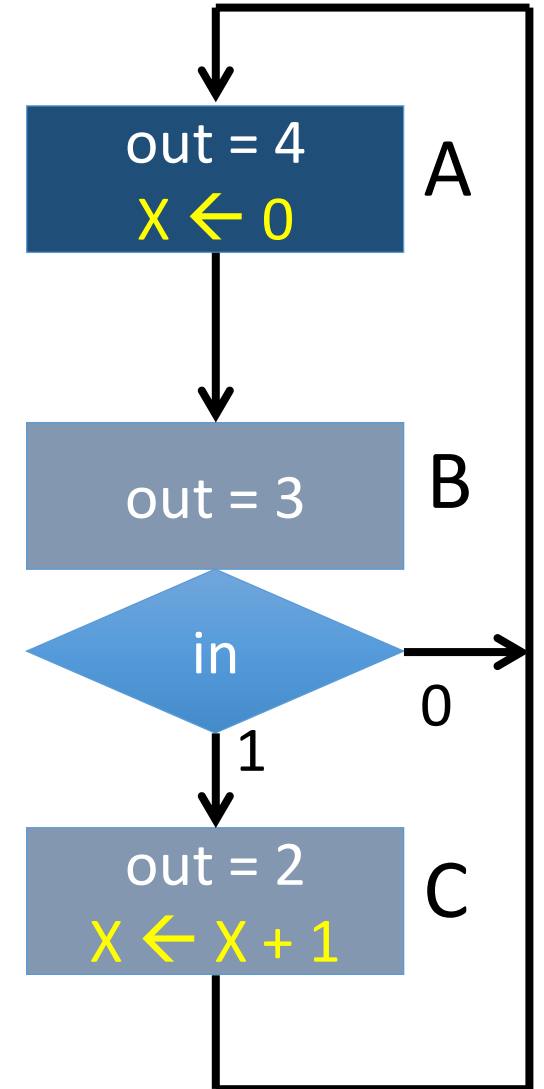
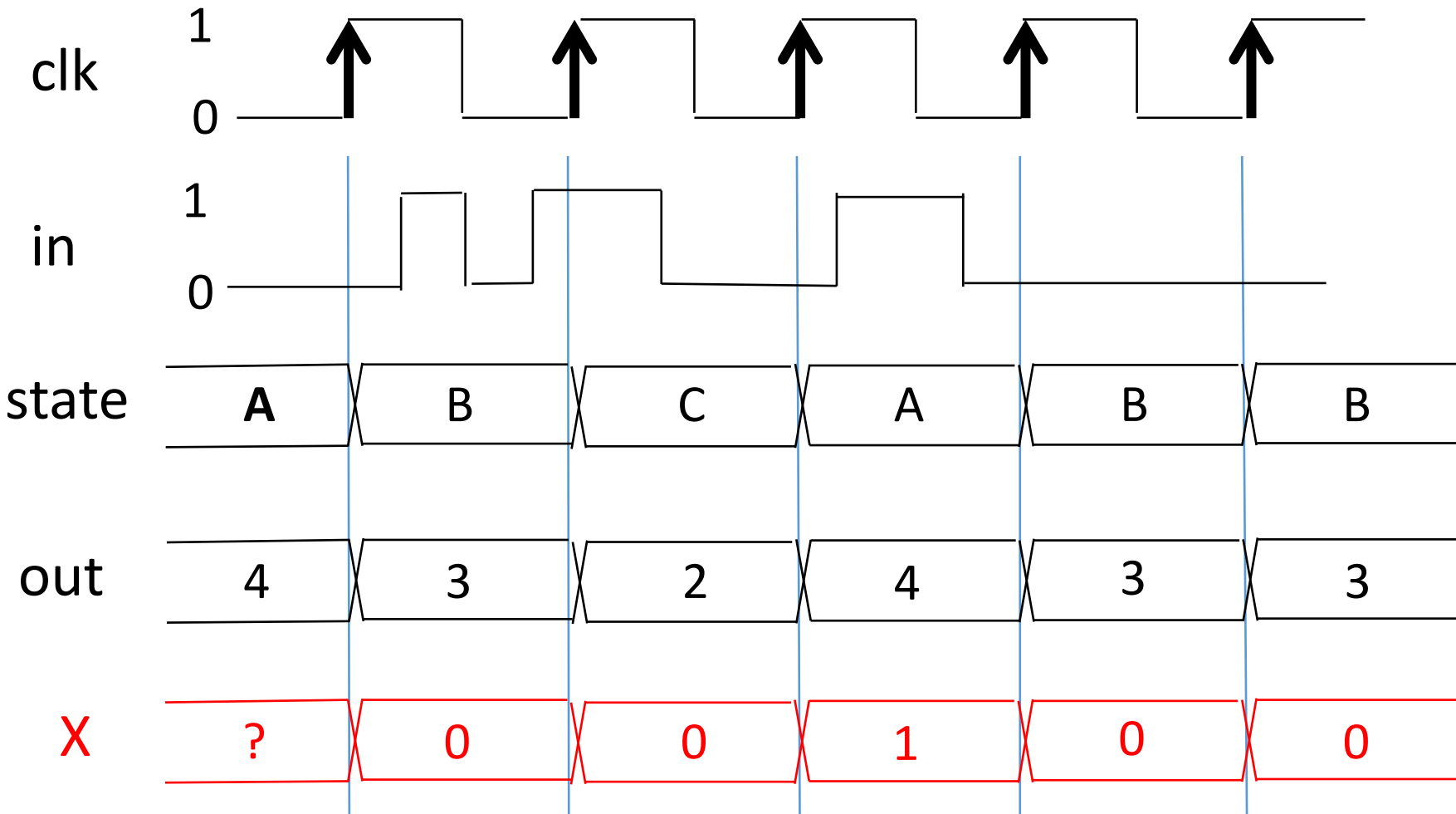
Register-Transfer Statements

- Register-transfer statements define the change of a value stored in a register.
- Values in registers can only change at the active (= rising) edge of clock.
- We denote “register-transfer statements” with a “left arrow” (“ \leftarrow ”)
- Example: “ $a \leftarrow x$ ” means that the value in the register “a” gets the value of “x” at the “next” active (= rising) edge of clock.
- We can specify register-transfer statement in an ASM diagram.

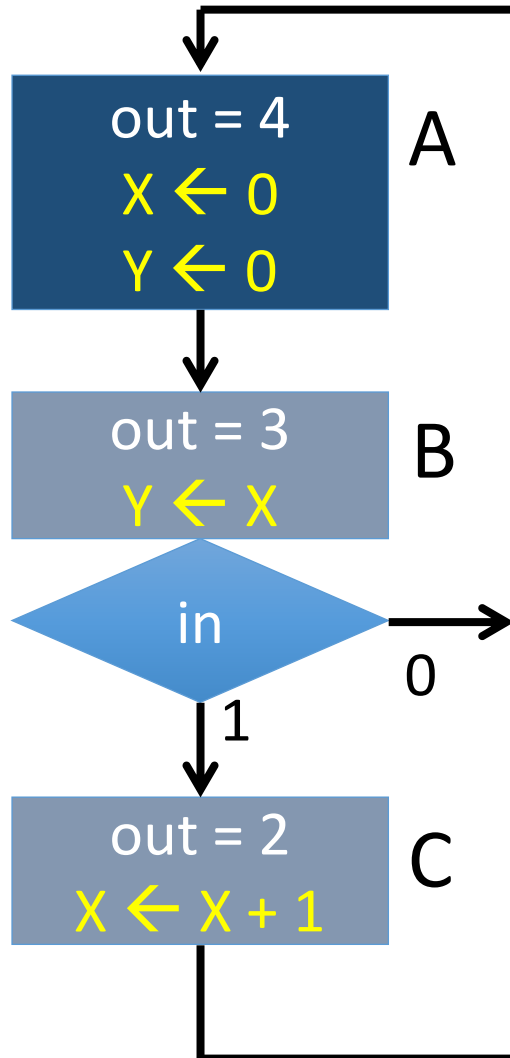
“=” versus “←”

- With the equal sign (“=”) we denote that the output of the FSM has a certain value during a particular state.
- With the left-arrow (“←”) we denote a register-transfer statement: The register value left of the arrow changes to whatever is defined right of the arrow upon the next active (= rising) edge of clock.

Timing diagram



Several register-transfer statements can be specified within one state

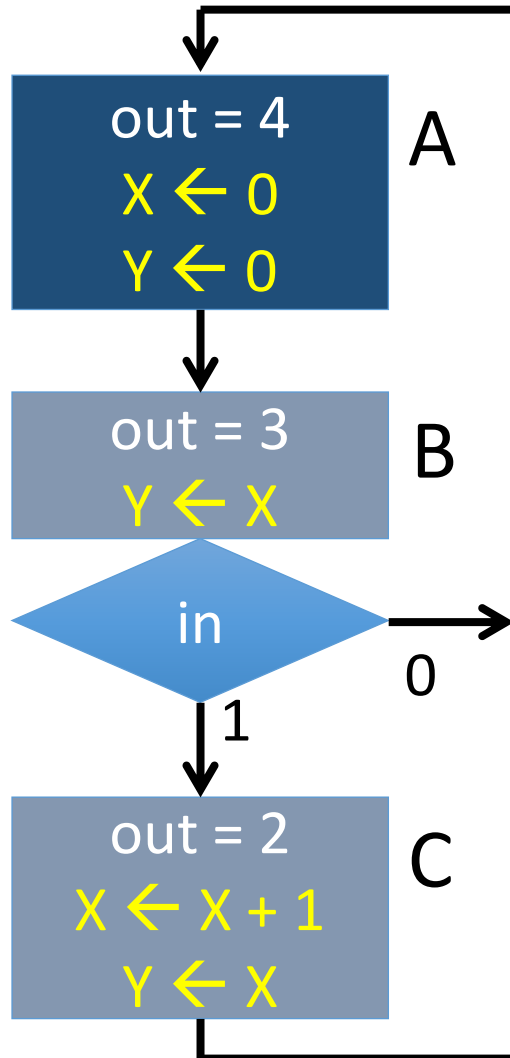


The values stored in register X and register Y become 0 at the state transition from state A to state B.

The value in register X does not change upon leaving state B. The value stored in register Y gets the value of register X upon leaving state B.

The value in register X gets incremented at the state transition following state C.

Several register-transfer statements can be specified within one state



The values stored in register X and register Y become 0 at the state transition from state A to state B.

The value in register X does not change upon leaving state B. The value stored in register Y gets the value of register X upon leaving state B.

The value in register X gets incremented at the state transition following state C. Register Y gets the “old” value from X; i.e. the value before X gets incremented.

Separating Control and Data Path

Control Unit

- State machine generating control signals for the data path



“Piano Player”

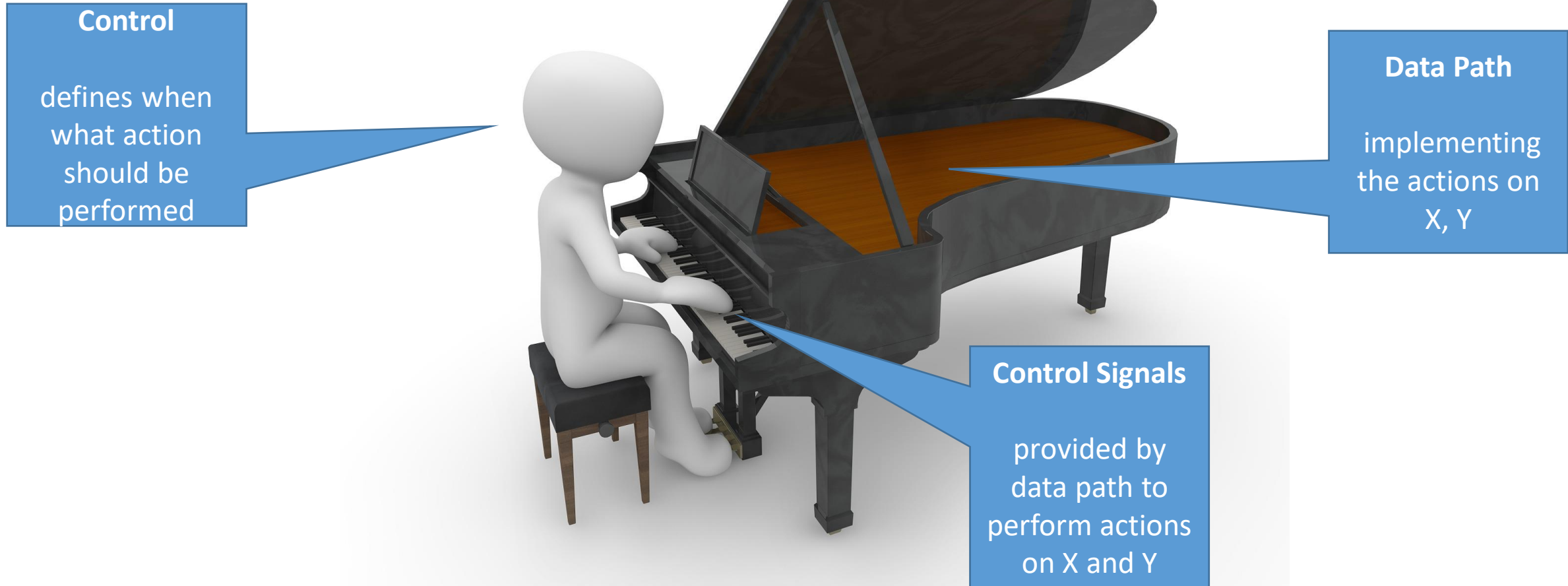
Data Path

- Contains all functional units and registers related to data processing
- Receives control signals to perform operations on the data.
- Provides status signals to the control-related data to the control unit

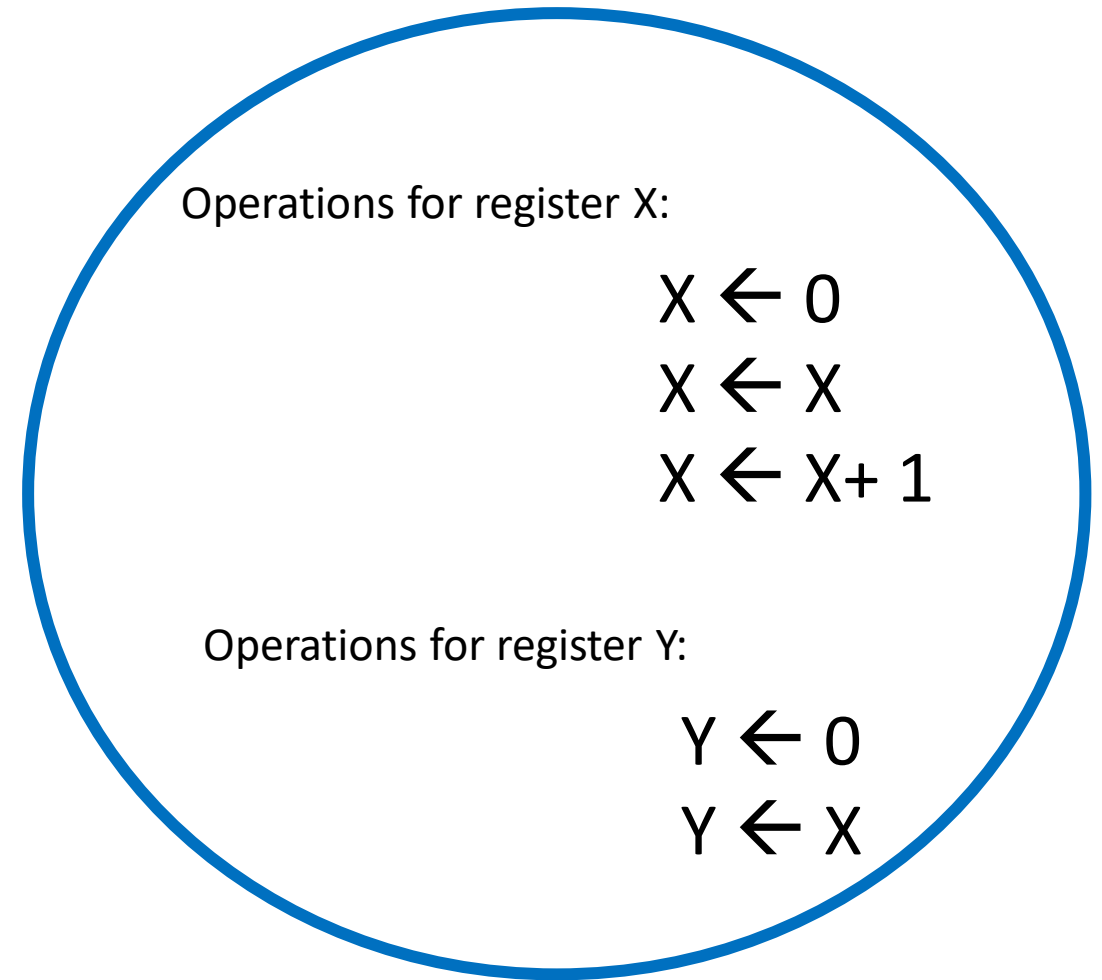
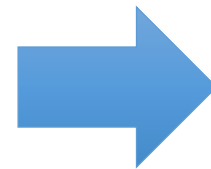
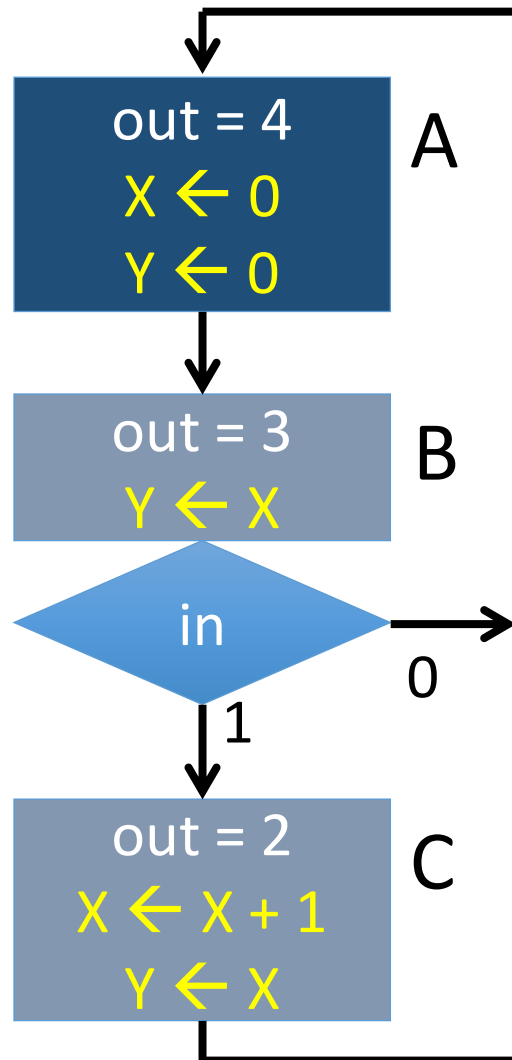


“Piano”

Music

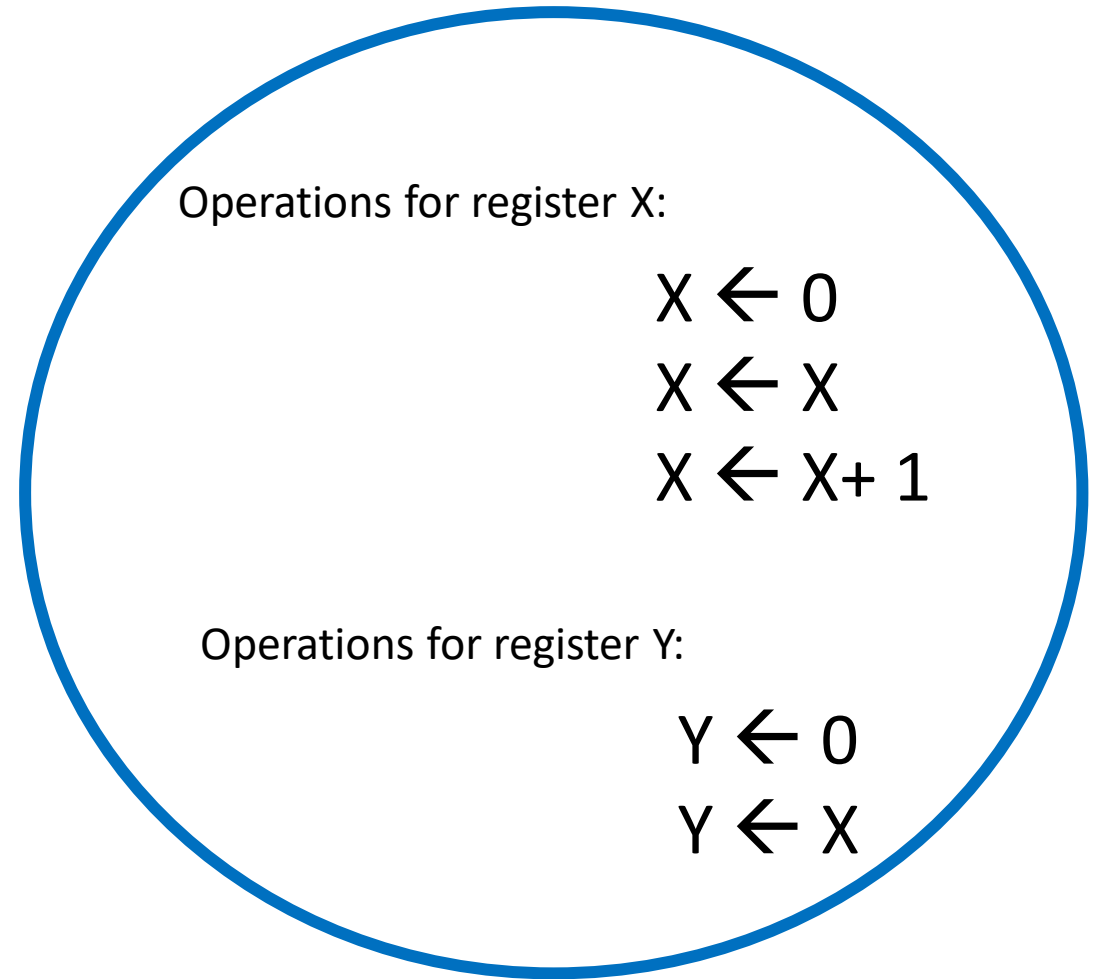
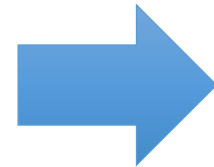
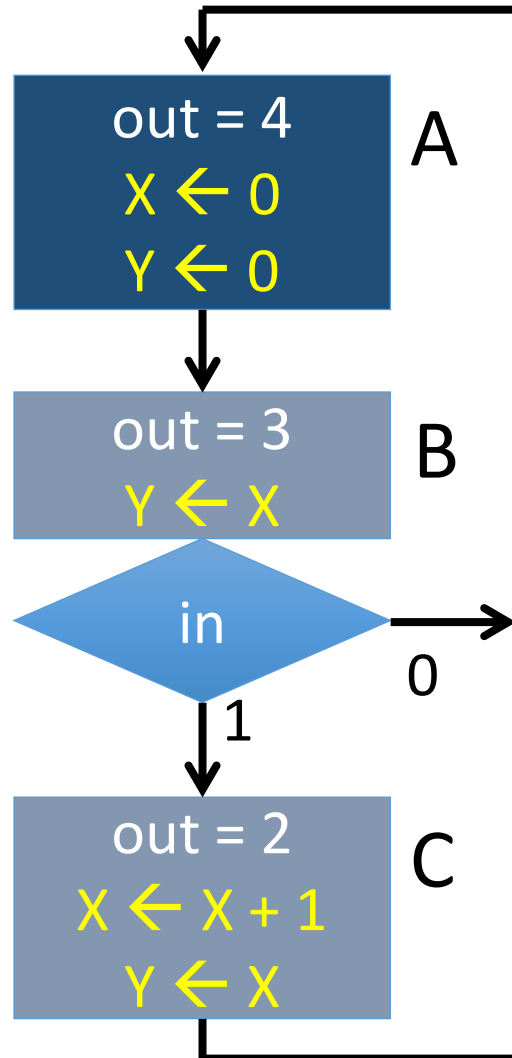


Register-Transfer Statements Define the Data Path



These are the actions that our system is able to perform on
The data registers X and Y

Register-Transfer Statements Define the Data Path



These are the operations that our data path implements

Operations for register X

Case 0: $X \leftarrow X$

Case 1: $X \leftarrow X + 1$

Case 2: $X \leftarrow 0$

We need to distinguish
between 3 cases.

→ A one bit control signal is not enough. We need two control signals.

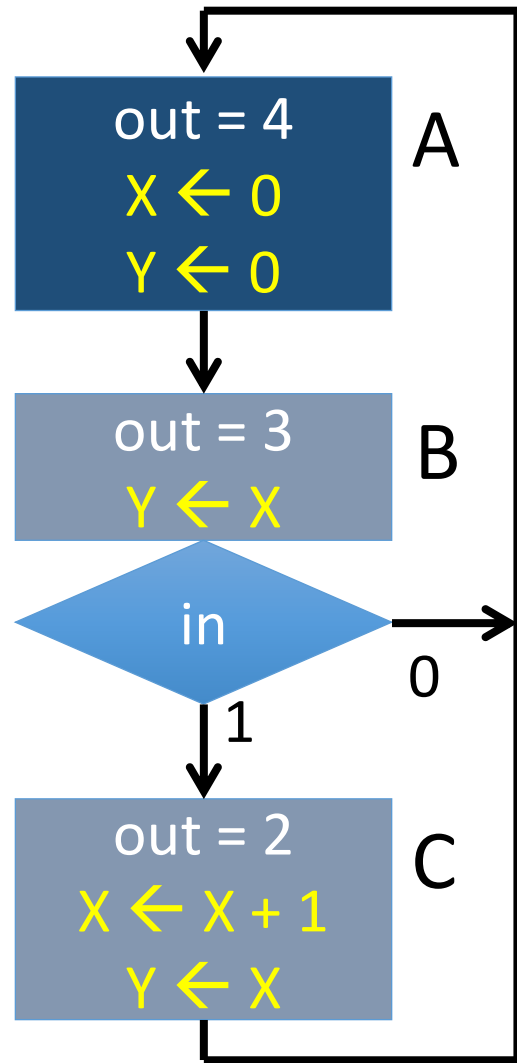
Control Signals and Datapath for the Actions on Register X

clrx	incx	action
0	0	$X \leftarrow X$
0	1	$X \leftarrow X + 1$
1	0	$X \leftarrow 0$

Control Signals and Datapath for the Actions on Register Y

clry	ldy	action
0	0	$Y \leftarrow Y$
0	1	$Y \leftarrow X$
1	0	$Y \leftarrow 0$

The Datapath



```

// registers of the data path
always_ff @(posedge clk_i or posedge reset_i) begin
  if (reset_i) begin
    x_p <= 3'b000;
    y_p <= 3'b000;
  end else begin
    x_p <= x_n;
    y_p <= y_n;
  end
end
end
  
```

```

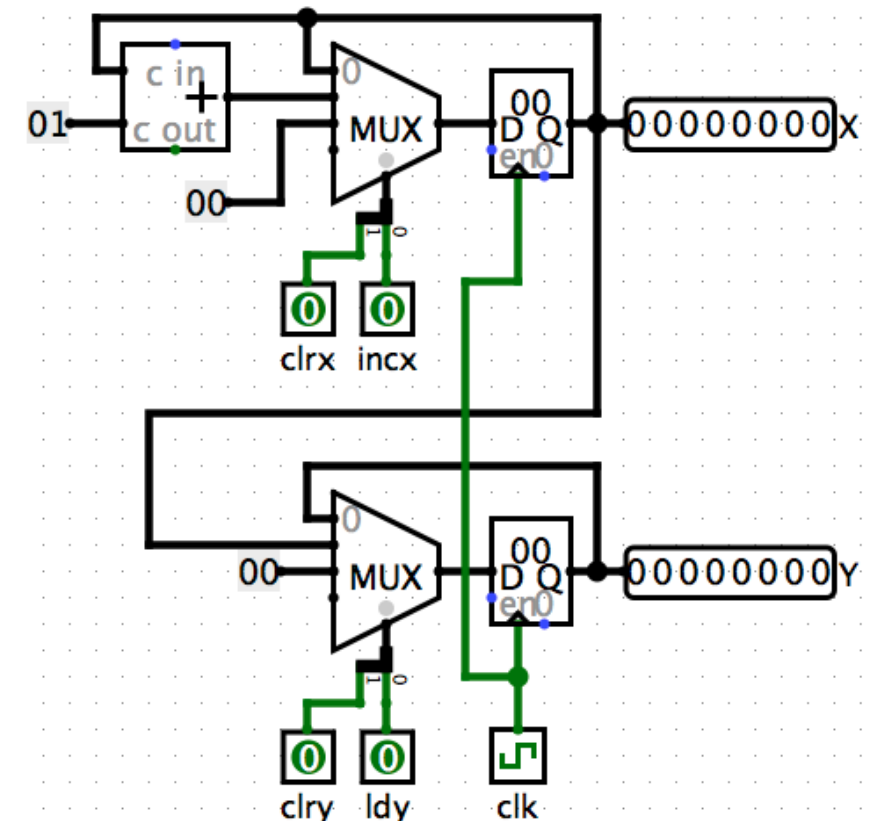
// Combinational logic of the data path
always_comb begin
  x_n = x_p;
  y_n = y_p;

  // operations for register X
  if (incx)
    x_n = x_p + 1;

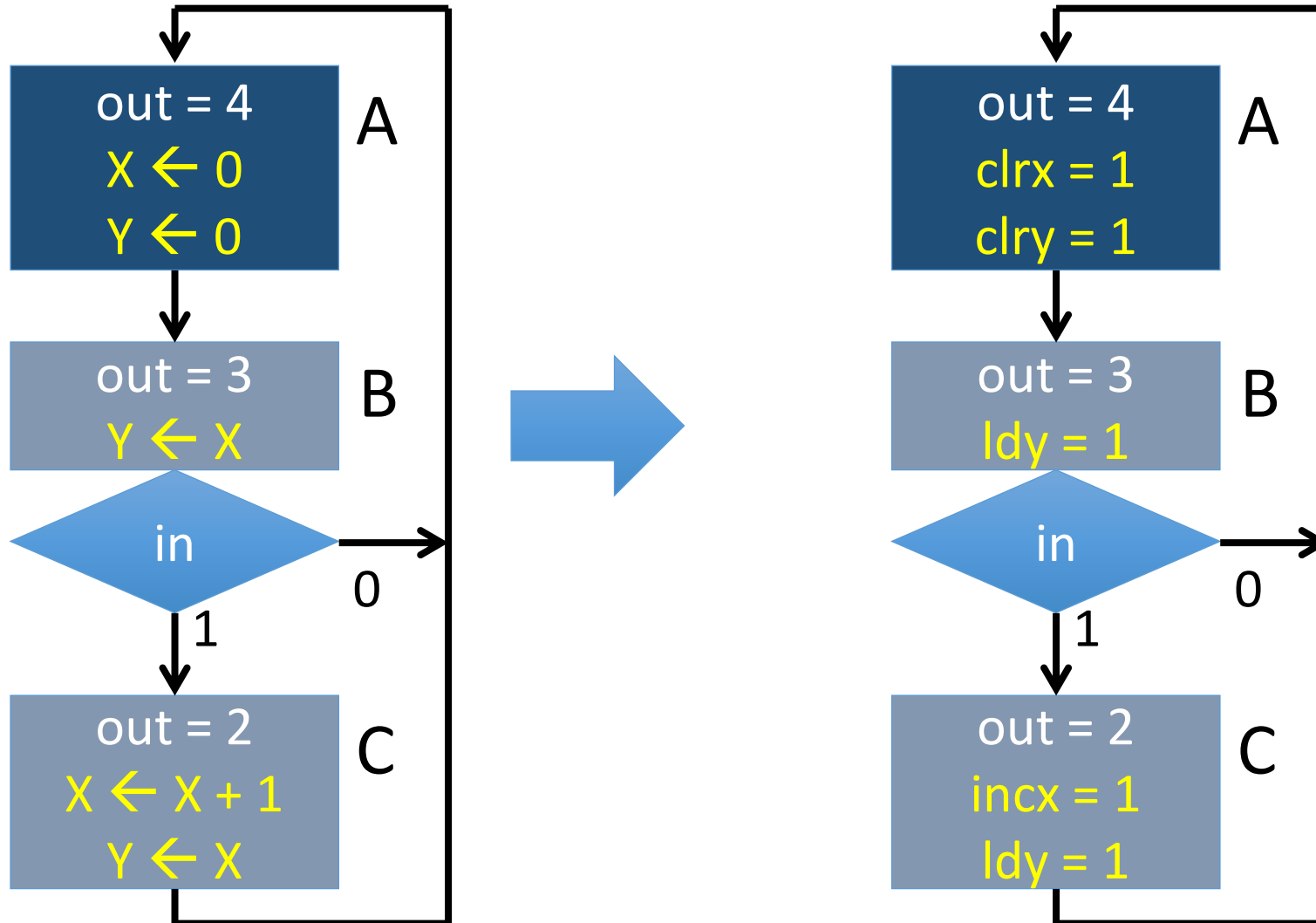
  if (clrx)
    x_n = 3'b000;

  // operations for register Y
  if (ldy)
    y_n = x_p;

  if (clry)
    y_n = 3'b000;
end
end
  
```



Register-transfer statements become assignment of control signals in the controller



The Control Logic in SystemVerilog

See example `con03.05_asm_example_with_separate_datapath`

Appendix - SystemVerilog Coding Style

- Suffix `_o` for module outputs, `_i` for module inputs
- Register variables with suffix `_p` for previous and `_n` for next value
- Array range with `[MSB:LSB]`, like e.g. `[31:0]`
- Clocked processes use non-blocking (`<=`) others use blocking assignments (`=`)
- Clocked processes only update registers, everything else has to be done in combinational blocks
- Filename corresponds to module name: module `MyDesign` in file `mydesign.sv`
- Module instantiation always with named assignments (`.A(C)`)
- **With significant implications beyond style:**
 - Always use default assignments (e.g. `state_n = state_p`)
 - Always use default branches (`default:`) in case statements
 - If you do not assign the output of a combinational block for all input conditions, latches are created for data storage!