Graz University of Technology
Institute for Applied Information Processing and Communications

Automata and LTL Model Checking

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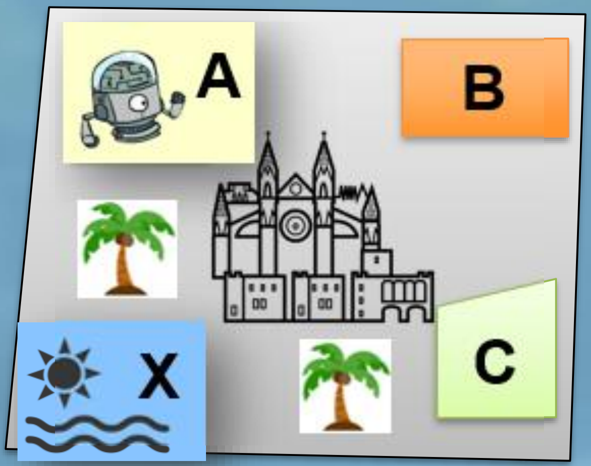

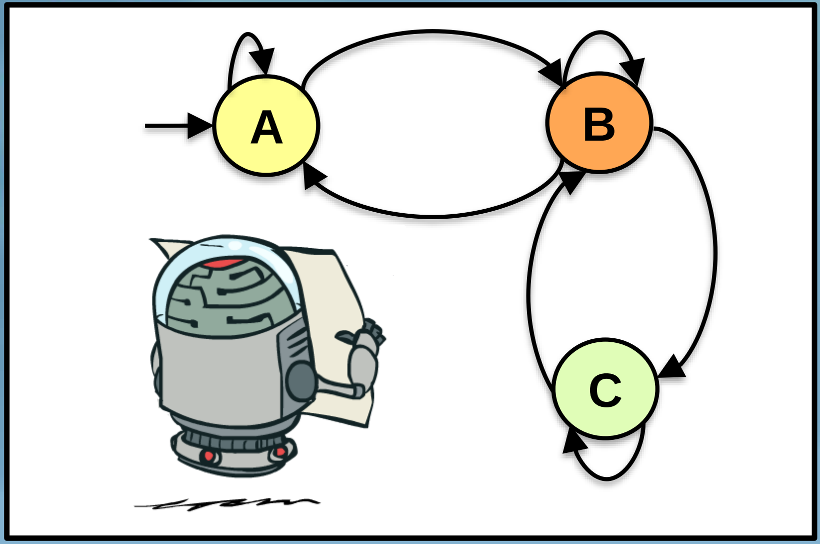

Model Checking SS23

May 25th 2023

Model Checking of LTL

given an LTL property φ and a Kripke structure M
check whether $M \models \varphi$

1. Construct $\neg\varphi$
2. Construct a **Büchi** automaton $\mathcal{S}_{\neg\varphi}$
3. **Translate M to an automaton $\mathcal{A}$.**
4. Construct the automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\mathcal{S}_{\neg\varphi})$
5. If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies φ
6. Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
   - a computation in M that does not satisfy φ

Outline

- Finite automata on finite words
- Automata on infinite words (Büchi automata)
- Deterministic vs non-deterministic Büchi automata
- Intersection of Büchi automata
- Checking emptiness of Büchi automata
- Generalized Büchi automata
- **Model checking using automata**
- Translation of LTL to Büchi automata

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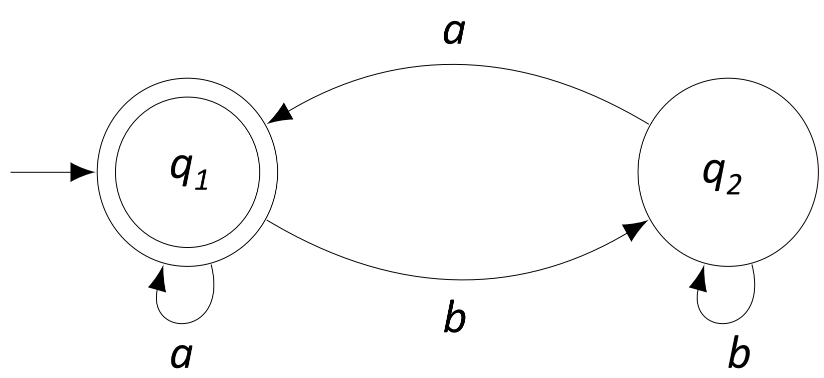
Automata on **Infinite** Words (Büchi)

$$\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$$

- An infinite run ρ is accepting ⇔ it visits an accepting state an infinite number of times.



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Det. and Non-det. Büchi Automata

- Deterministic Büchi automata are **strictly less expressive** than nondeterministic ones.
  - Not every nondeterministic Büchi automaton has an equivalent deterministic Büchi one.
- Deterministic Büchi automata are **not** closed under complementation.
- Nondeterministic Büchi automata are closed under complementation.
  - The construction is very complicated. (Safra Construction)

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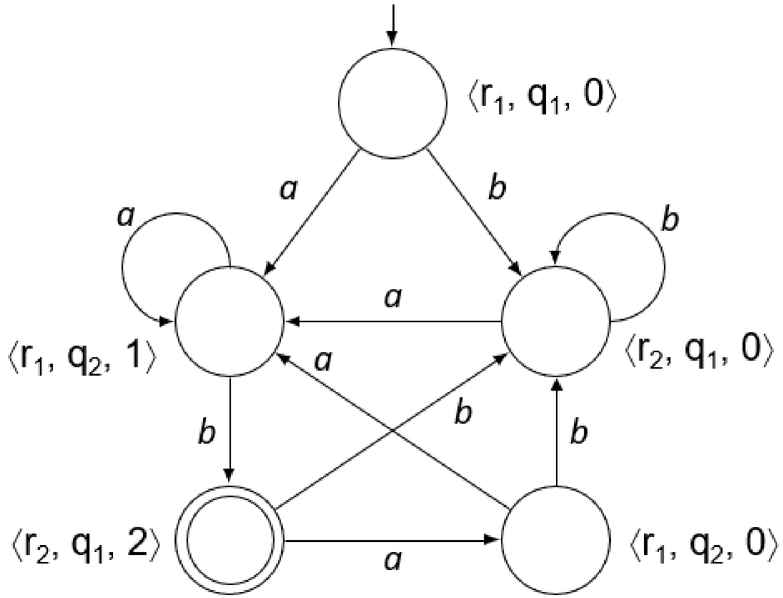
Intersection of Büchi Automata

- Given $\mathcal{B}_1 = (\Sigma, Q_1, \Delta_1, Q_1^0, F_1)$ and $\mathcal{B}_2 = (\Sigma, Q_2, \Delta_2, Q_2^0, F_2)$
- $\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$ s.t. $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{B}_1) \cap \mathcal{L}(\mathcal{B}_2)$ is defined as follows:
  - $Q = Q_1 \times Q_2 \times \{0, 1, 2\}$
  - $Q^0 = Q_1^0 \times Q_2^0 \times \{0\}$
  - $F = Q_1 \times Q_2 \times \{2\}$

$((q_1,q_2,x), a, (q'_1,q'_2,x')) \in \Delta \Leftrightarrow$

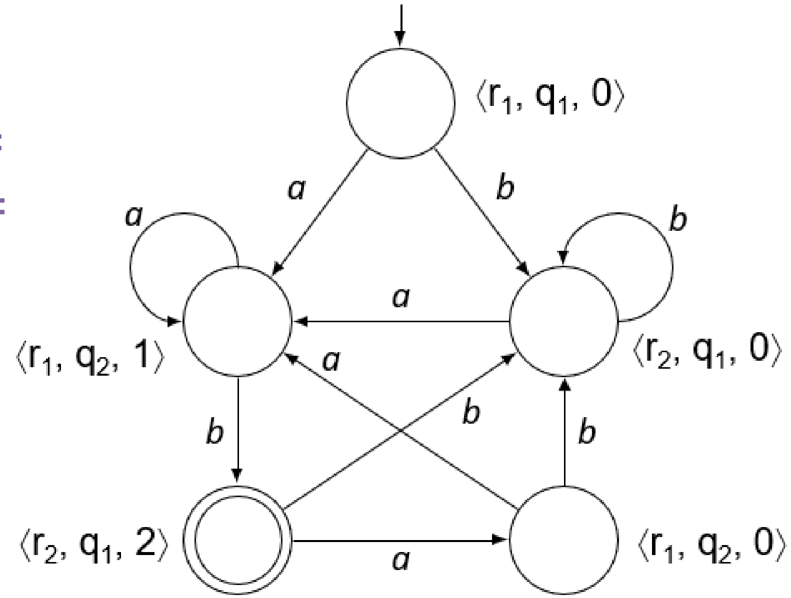
(1) $(q_1,a,q'_1) \in \Delta_1$ and $(q_2,a,q'_2) \in \Delta_2$

(2) If $x=0$ and $q'_1 \in F_1$ then $x'=1$
If $x=1$ and $q'_2 \in F_2$ then $x'=2$
If $x=2$ then $x'=0$
Else, $x'=x$



Intersection of Büchi Automata

- Given $\mathcal{B}_1 = (\Sigma, Q_1, \Delta_1, Q_1^0, F_1)$ and $\mathcal{B}_2 = (\Sigma, Q_2, \Delta_2, Q_2^0, F_2)$
- $\mathcal{B} = (\Sigma, Q, \Delta, Q^0, F)$ s.t. $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{B}_1) \cap \mathcal{L}(\mathcal{B}_2)$ is defined as follows:
  - $Q = Q_1 \times Q_2 \times \{0, 1\}$
  - $Q^0 = Q_1^0 \times Q_2^0 \times \{0\}$
  - For Δ
    - (2) If $x=0$ and $q_1 \in F_1$ then $x'=$
      If $x=1$ and $q_2 \in F_2$ then $x'=$
      Else, $x'=x$
  - For F
    - $F = F_1 \times Q_2 \times \{0\}$



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Intersection of Büchi Automata

- Question
  - How do we define the transition relation for $\mathcal{B}$, if x is over {0,1} only?
- Answer
  - For $\mathbf{\Delta}$
    - (2) If x=0 and $q_1 \in \mathbf{F}_1$ then x'=1
      If x=1 and $q_2 \in \mathbf{F}_2$ then x'=0
      Else, x'=x
  - For $\mathbf{F}$
    - $\mathbf{F} = \mathbf{F}_1 \times \mathbf{Q}_2 \times \{0\}$

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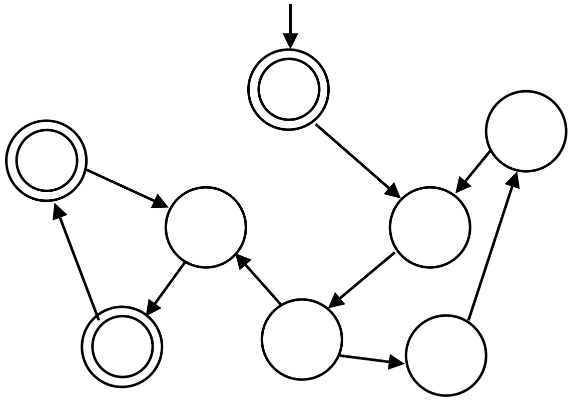
Checking for emptiness of $\mathcal{L}(\mathcal{B})$

- How to check for **L(A) = ∅?**
- Empty if there is no reachable accepting state on a cycle.



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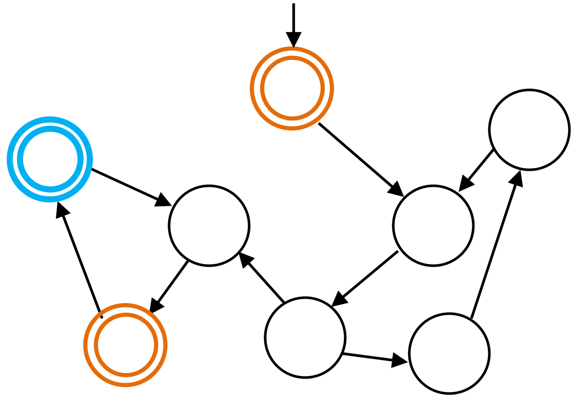
Generalized Büchi automata

- Have several sets of accepting states

- $\mathcal{B} = (\Sigma, \mathbf{Q}, \Delta, \mathbf{Q}^0, \mathbf{F})$ is a generalized Büchi automaton:
  - $\mathbf{F} = \{P_1, \ldots, P_k\}$, where for every $1 \leq i \leq k$, $P_i \subseteq Q$

- A run ρ of $\mathcal{B}$ is accepting if for each $P_i \in \mathbf{F}$, $inf(\rho) \cap P_i \neq \emptyset$



Translation from Generalized Büchi to Büchi

- Given $\mathcal{B} = (\Sigma, Q_1, \Delta_1, Q_1^0, F_1)$ with $F = \{P_1, \ldots, P_k\}$

- How can we construct a Büchi automaton $\mathcal{B}'$ that accepts the same language?

Translation from Generalized Büchi to Büchi

- $\mathcal{B} = (\Sigma, Q_1, \Delta_1, Q_1^0, F_1)$ with $F = \{P_1, \ldots, P_k\}$
- $\mathcal{B}' = (\Sigma, Q \times \{0,1,\ldots,k\}, \Delta', Q^0 x0, Q \times k)$ with:
- The transition relation Δ':
  $((q,x),a,(q',y)) \in \Delta'$ when $(q,a,q') \in \Delta$ and x and y are as follows:
  - If $q' \in P_i$ and $x=i$, then $y=i+1$ for $i<k$
  - If $x=k$, then $y=0$.
  - Otherwise, $x = y$.

Size of $\mathcal{B}'$ = (size of $\mathcal{B}$) $\times$ (k+1)

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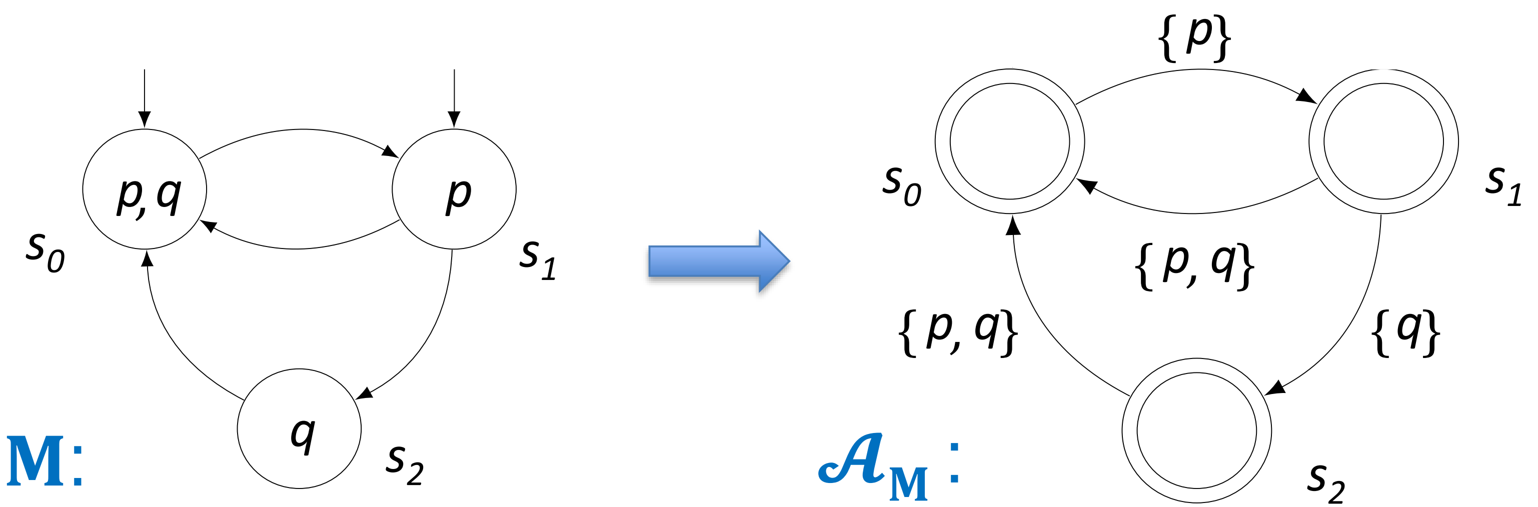
Kripke Structure $\mathbf{M}$ to Büchi Automaton $\mathcal{A}_{\mathbf{M}}$

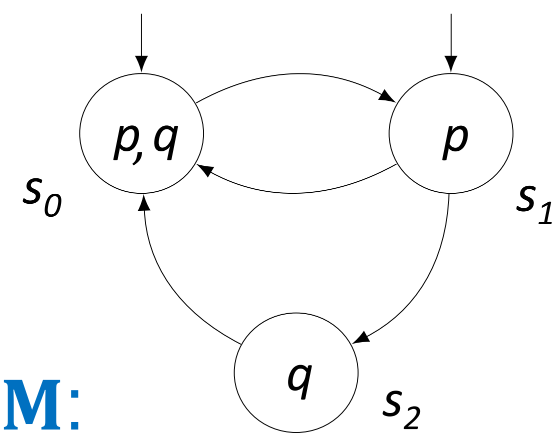
- Move labels to incoming transitions
  - Push labels backwards
- All states are accepting
- ToDo: What about initial states?



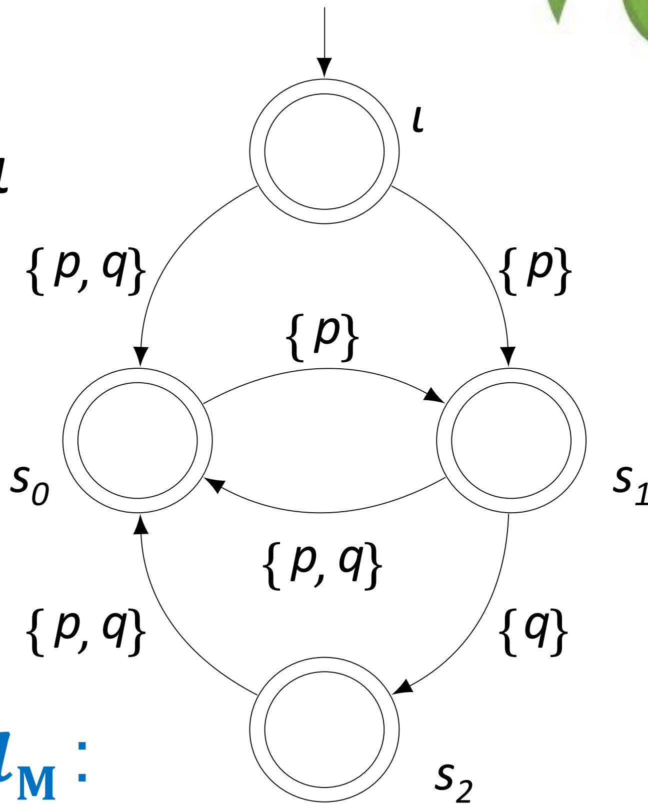
Kripke Structure **M** to Büchi Automaton $\mathcal{A}_{\mathbf{M}}$

- Move labels to incoming transitions
- All states are accepting
- Introduce new initial state ι

M:



$\mathcal{A}_{\mathbf{M}}$:



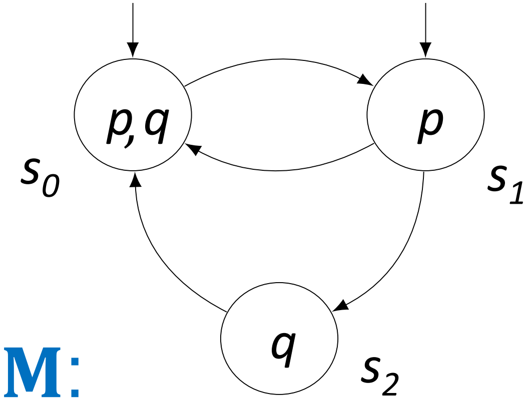
Automata and Kripke Structures

$\mathbf{M} = (\mathbf{S}, \mathbf{S}_0, \mathbf{R}, AP, \mathbf{L}) \Rightarrow \mathcal{A}_{\mathbf{M}} = (\Sigma, \mathbf{S} \cup \{\iota\}, \Delta, \{\iota\}, \mathbf{S} \cup \{\iota\})$,

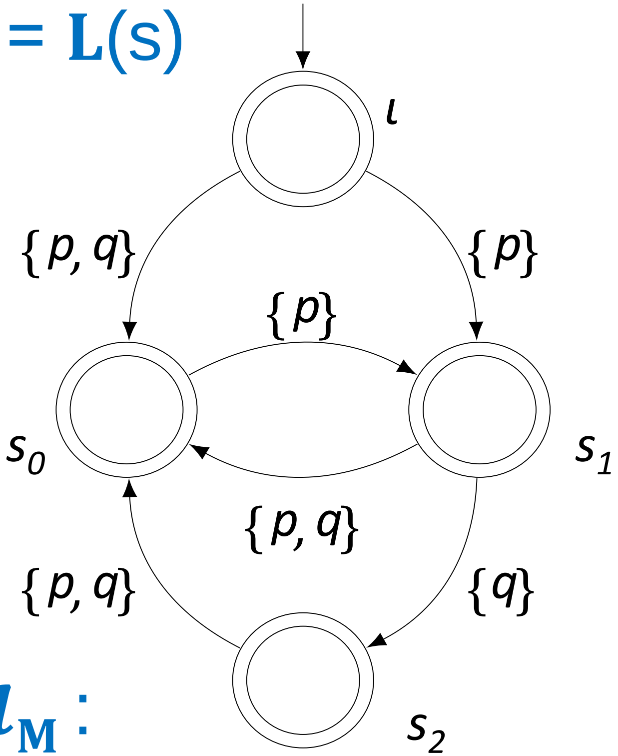
where $\Sigma = P(AP)$.

- $(s,\alpha,s') \in \Delta$ for $s,s' \in \mathbf{S} \Leftrightarrow (s,s') \in R$ and $\alpha = \mathbf{L}(s')$
- $(\iota,\alpha,s) \in \Delta \Leftrightarrow s \in \mathbf{S}_0$ and $\alpha = \mathbf{L}(s)$

M:



$\mathcal{A}_{\mathbf{M}}$:



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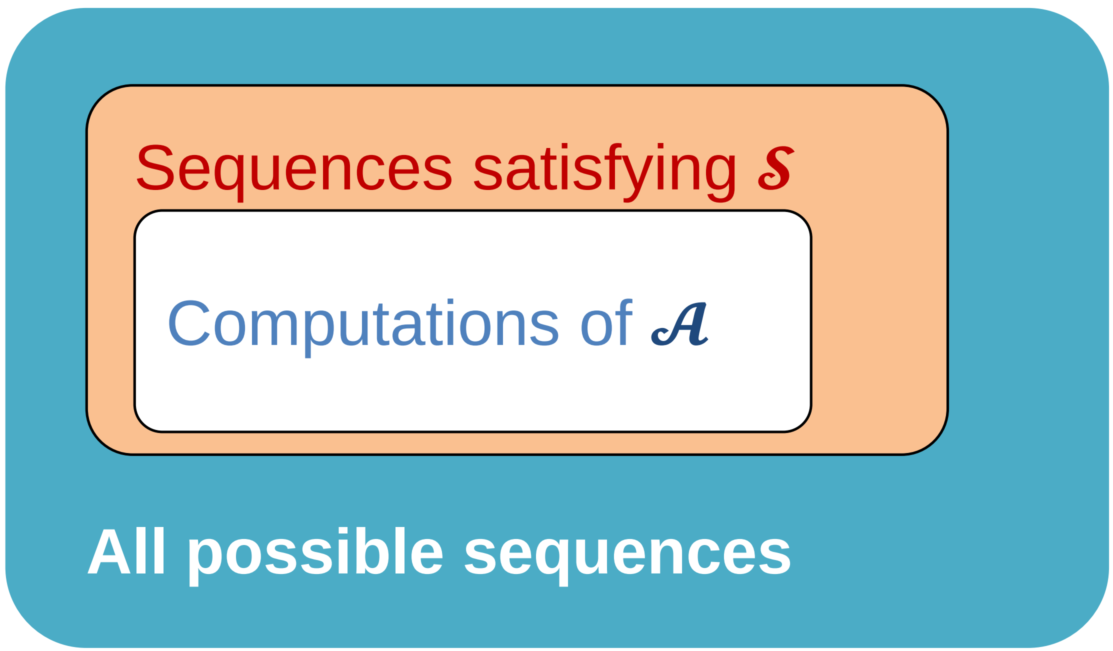
Model Checking when System $\mathcal{A}$ and Spec $\mathcal{S}$ are given as Büchi automata

- $\mathcal{A}$ **satisfies** $\mathcal{S}$ if $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{S})$
  - Is any behavior of $\mathcal{A}$ allowed by $\mathcal{S}$?



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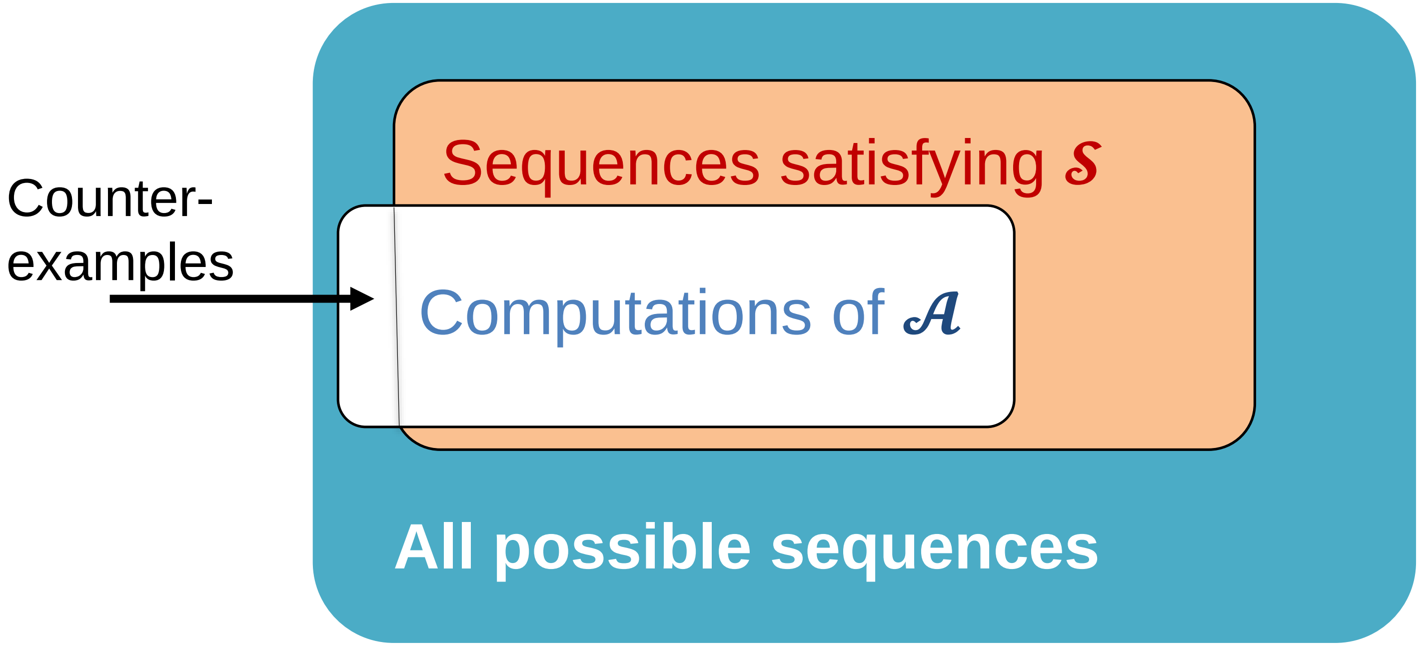
Model Checking when System $\mathcal{A}$ and Spec $\mathcal{S}$ are given as Büchi automata

- **$\mathcal{A}$ does not satisfy $\mathcal{S}$** if $\mathcal{L}(\mathcal{A}) \not\subseteq \mathcal{L}(\mathcal{S})$



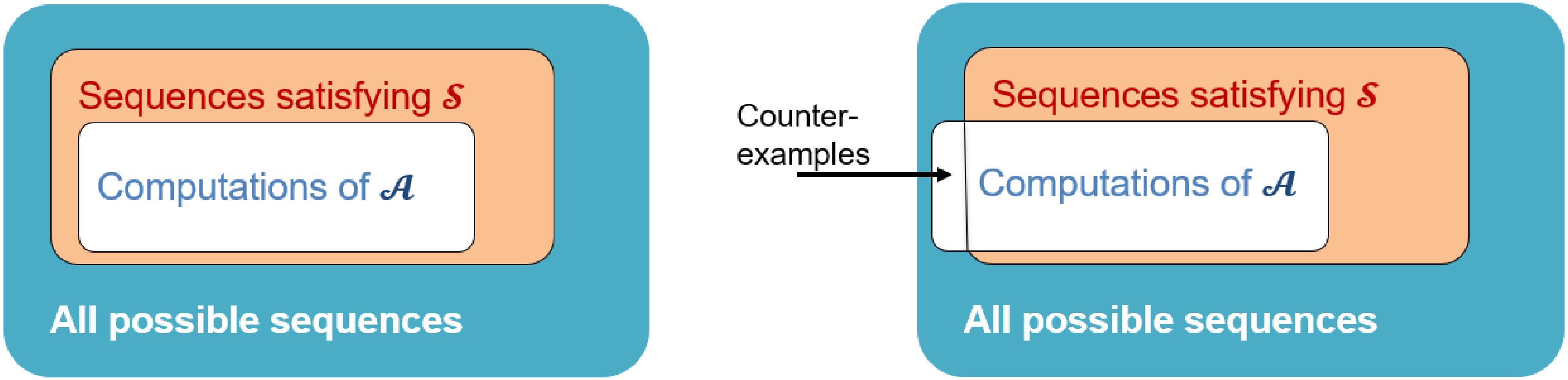
Model Checking when System $\mathcal{A}$ and Spec $\mathcal{S}$ are given as Büchi automata

- Check whether $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{S})$
- Equivalent:

$$\mathcal{L}(\mathcal{A}) \nsubseteq \mathcal{L}(\mathcal{S}) \equiv \boxed{\mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\overline{\mathcal{S}}) \neq \emptyset}$$

$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{S})$

$\mathcal{L}(\mathcal{A}) \nsubseteq \mathcal{L}(\mathcal{S}) \equiv \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\overline{\mathcal{S}}) \neq \emptyset$



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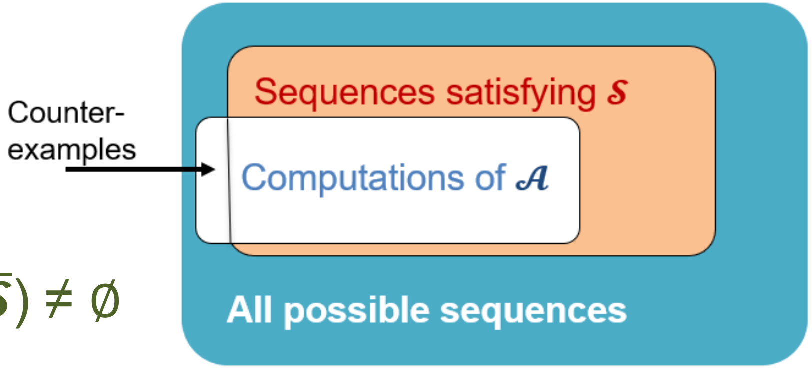
Model Checking – suggested algorithm

when system $\mathcal{A}$ and spec $\mathcal{S}$ are given as Büchi automata

1. Complement $\mathcal{S}$. The resulting Büchi automaton is $\overline{\mathcal{S}}$
2. Construct the automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\overline{\mathcal{S}})$
3. If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies $\mathcal{S}$
4. Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
   - a computation in $\mathcal{A}$ that does not satisfy $\mathcal{S}$



$$\mathcal{L}(\mathcal{A}) \not\subseteq \mathcal{L}(\mathcal{S}) \equiv \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\overline{\mathcal{S}}) \neq \emptyset$$

Model Checking – suggested algorithm

when system $\mathcal{A}$ and spec $\mathcal{S}$ are given as Büchi automata

very hard!

1. Complement $\mathcal{S}$. The resulting Büchi automaton is $\overline{\mathcal{S}}$
2. ✓ Construct the automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{B}) = \mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\overline{\mathcal{S}})$
3. ✓ If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies $\mathcal{S}$
4. ✓ Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
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Model Checking – suggested algorithm

when system $\mathcal{A}$ and spec $\mathcal{S}$ are given as Büchi automata

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4. Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
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How can we avoid building the **complement** of $\mathcal{S}$?

Model Checking of LTL

given **an LTL property φ** and a Kripke structure M
check whether $M \models \varphi$

Model Checking of LTL

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check whether M $\models \varphi$

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5. If $\mathcal{L}(\mathcal{B}) = \emptyset \Rightarrow \mathcal{A}$ satisfies φ
6. Otherwise, a word $v \cdot w^{\omega} \in \mathcal{L}(\mathcal{B})$ is a counterexample
   - a computation in M that does not satisfy φ

Model Checking of LTL

given **an LTL property φ** and a Kripke structure M
check whether $M \models \varphi$

next topic

1. Construct $\neg\varphi$
2. Construct a Büchi automaton $\mathcal{S}_{\neg\varphi}$
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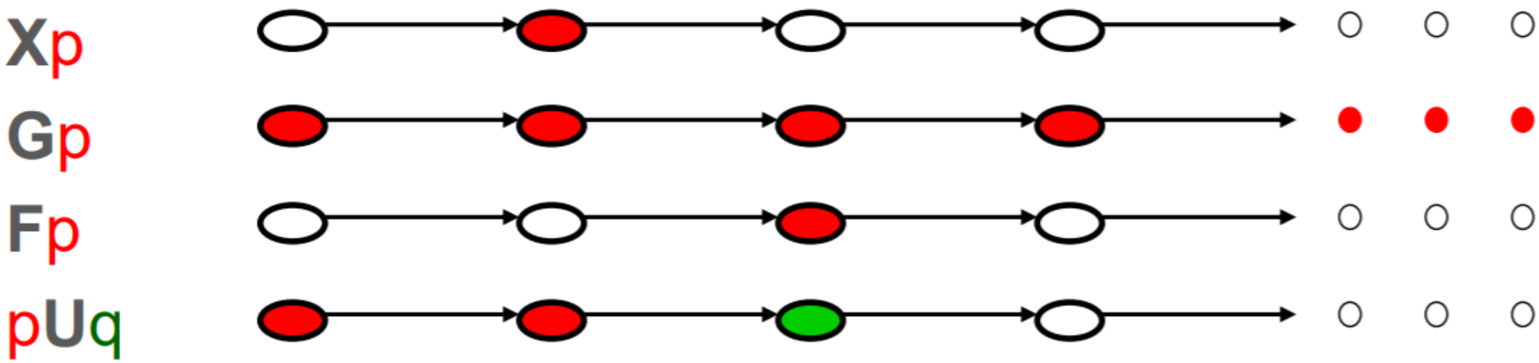
Translation of LTL to Büchi automata

1. Translate φ into **generalized Büchi Automaton**
   - $\mathcal{A}_\varphi$ accepts exactly all the traces that satisfy φ

2. Translate **generalized Büchi** to **Büchi** automaton ✓

Recall LTL Semantics



Rewriting

- Algorithm only handles
  - $\neg, \wedge, \vee, \boldsymbol{X}, \boldsymbol{U}$
  - Use rewriting Rules
    - $\neg G\varphi = F\neg\varphi$
    - $F\varphi = true\ U\varphi$
    - $G\varphi = \neg F\neg\varphi$

From LTL formula φ to GBA $\mathcal{A}_\varphi$

- Step 1: Define the state space $\mathcal{A}_\varphi$ based on φ

The set of all **good sets** of **cl(φ)** defines the state space of $\mathcal{A}_\varphi$

- cl(φ) are subformulas of φ and their negation
- Formally:
  - $\varphi \in cl(\varphi)$.
  - **If** $\boldsymbol{\varphi_1 \in cl(\varphi), then \neg\varphi_1 \in cl(\varphi)}$.
  - **If** $\boldsymbol{\neg\varphi_1 \in cl(\varphi), then\ \varphi_1 \in cl(\varphi)}$.
  - If $\varphi_1 \vee \varphi_2 \in cl(\varphi), then\ \varphi_1 \in cl(\varphi)\ and\ \varphi_2 \in cl(\varphi)$.
  - If $X\ \varphi_1 \in cl(\varphi), then\ \varphi_1 \in cl(\varphi)$.
  - If $\varphi_1\ U\ \varphi_2 \in cl(\varphi), then\ \varphi_1 \in cl(\varphi)\ and\ \varphi_2 \in cl(\varphi)$.

From LTL formula φ to GBA $\mathcal{A}_\varphi$

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The set of all **good sets** of **cl(φ)** defines the state space of $\mathcal{A}_\varphi$

- cl(φ) are subformulas of φ and their negation
- Example cl(φ) for $\varphi \coloneqq (\neg p \; U \; ((Xq) \vee r))$

$$\mathrm{cl}(\varphi) = \{\varphi, \neg\varphi, \neg p, p, q, \neg q, r, \neg r\ , (Xq), \neg(Xq), ((Xq) \vee r), \neg((Xq) \vee r), \}$$

From LTL formula φ to GBA $\mathcal{A}_\varphi$

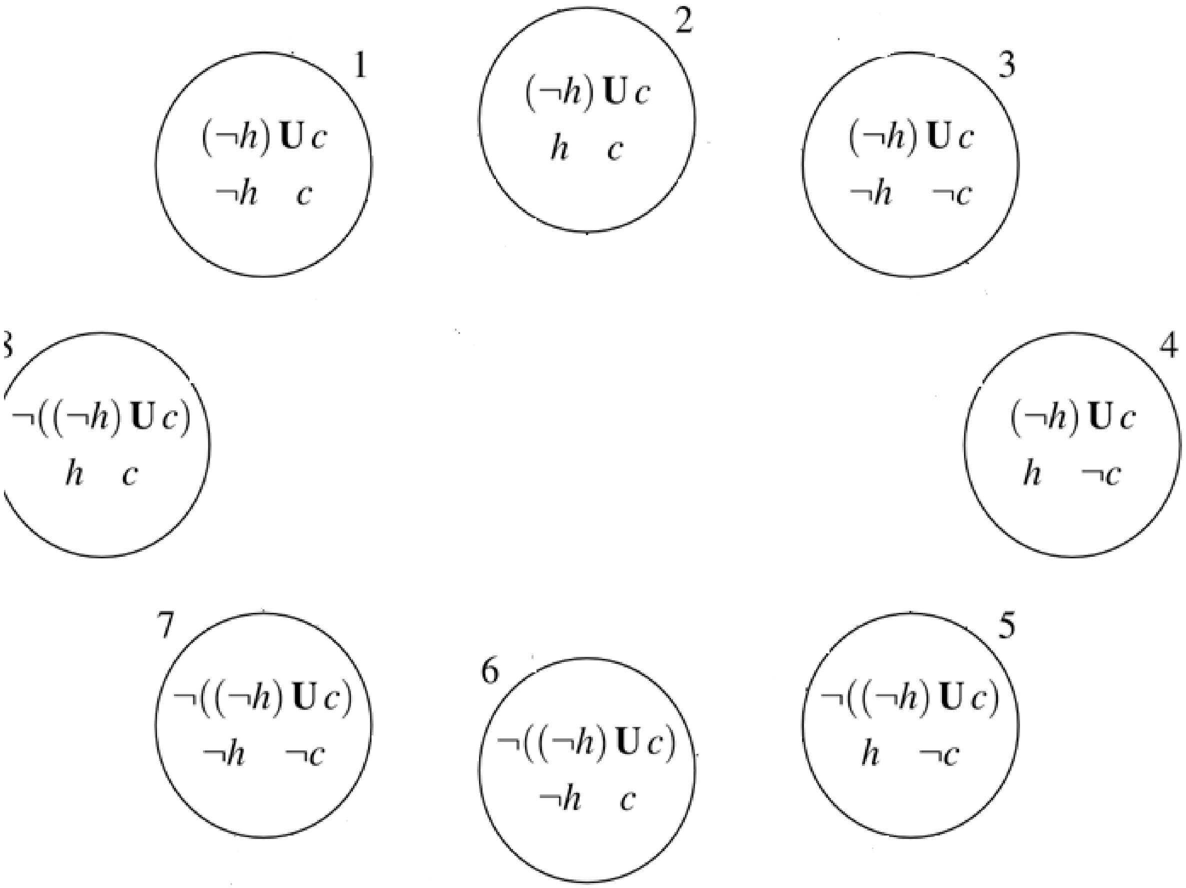
- Step 1: Define the state space $\mathcal{A}_\varphi$ based on φ

The set of all **good sets** of **cl(φ)** defines the state space of $\mathcal{A}_\varphi$

- cl(φ) are subformulas of φ and their negation
- ***Good sets:***
  S $\subseteq$ *cl*(φ) is ***good*** *in cl(φ)* if *S* is a maximal set of formulas in *cl(φ)* that is **consistent**
  - For all $\varphi_1 \in cl(\varphi)$: $\varphi_1 \in S \Leftrightarrow \neg \varphi_1 \notin S$

Give the state space of $\mathcal{A}_\varphi$ for the formula $\varphi = (\neg h \ \mathbf{U}\ c)$

$cl(\varphi) = \{\neg h, h, \neg c, c, (\neg h \ \mathbf{U}\ c), \neg(\neg h \ \mathbf{U}\ c)\}$



From LTL formula φ to GBA $\mathcal{A}_\varphi$

$\mathcal{A}_\varphi = (\mathcal{P}(AP), \mathbf{Q}, \mathbf{\Delta}, \mathbf{Q}^0, \mathbf{F})$

- $\mathbf{Q} \subseteq \mathcal{P}(cl(\varphi))$ is the set of all the good sets in $cl(\varphi)$.
- Next: $\mathbf{\Delta}$

Each state of $\mathcal{A}_\varphi$ is **labelled** with **a set of properties** that should be satisfied **on all paths starting at that state**

From LTL formula φ to GBA $\mathcal{A}_\varphi$

$\mathcal{A}_\varphi = (\mathcal{P}(AP), \mathbf{Q}, \mathbf{\Delta}, \mathbf{Q}^0, \mathbf{F})$

- For q, q' ∈Q and $\sigma \subseteq AP$, $(q,\sigma,q') \in \mathbf{\Delta}$ if:

1. $\sigma = q' \cap AP$ (push labels backwards)
2. $\boldsymbol{X}\varphi_1 \in q \Leftrightarrow \varphi_1 \in q'$
3. $\neg(\boldsymbol{X}\varphi_1) \in q \Leftrightarrow \neg\varphi_1 \in q'$

4. φ_1 U $\varphi_2 \in$ q ⇔ either $\varphi_2 \in$ q **or both** $\varphi_1 \in$ q **and** φ_1 U $\varphi_2 \in$ q'

5. $\neg(\varphi_1$ U $\varphi_2) \in$ q ⇔ either $\neg\varphi_2 \in$ q and **either** $\neg\varphi_1 \in$ q **or** $\neg(\varphi_1$ U $\varphi_2) \in$ q'

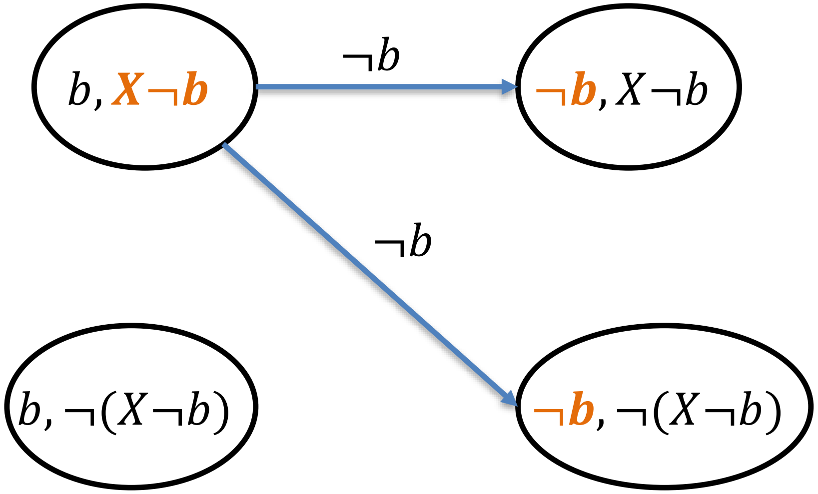
$\varphi = X\neg b$

ToDo

1. $\sigma = q' \cap AP$ (push labels backwards)
2. $\boldsymbol{X}\varphi_1 \in q \Leftrightarrow \varphi_1 \in q'$
3. $\neg(\boldsymbol{X}\varphi_1) \in q \Leftrightarrow \neg\varphi_1 \in q'$

Draw the state space and the transitions.

$cl(\varphi) = \{b, \neg b, X\neg b, \neg(X\neg b)\}$



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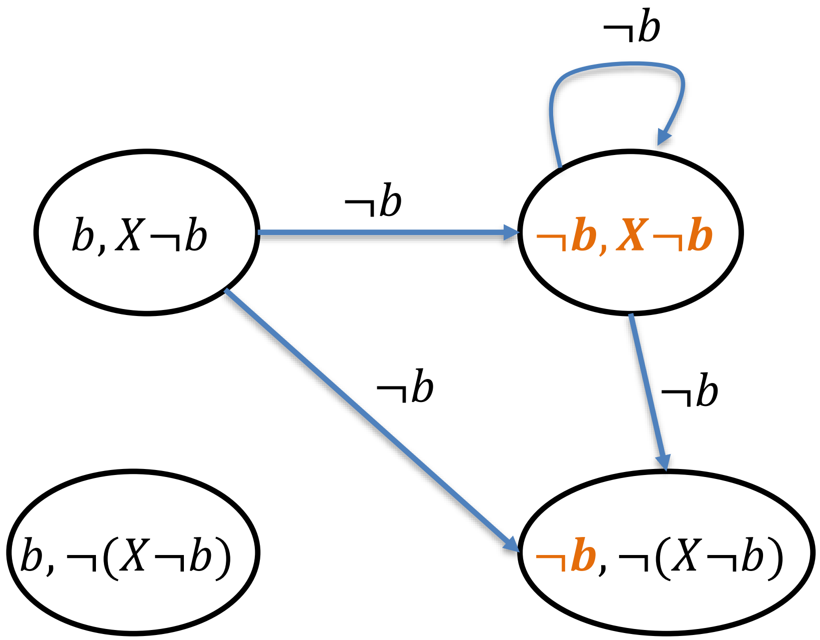

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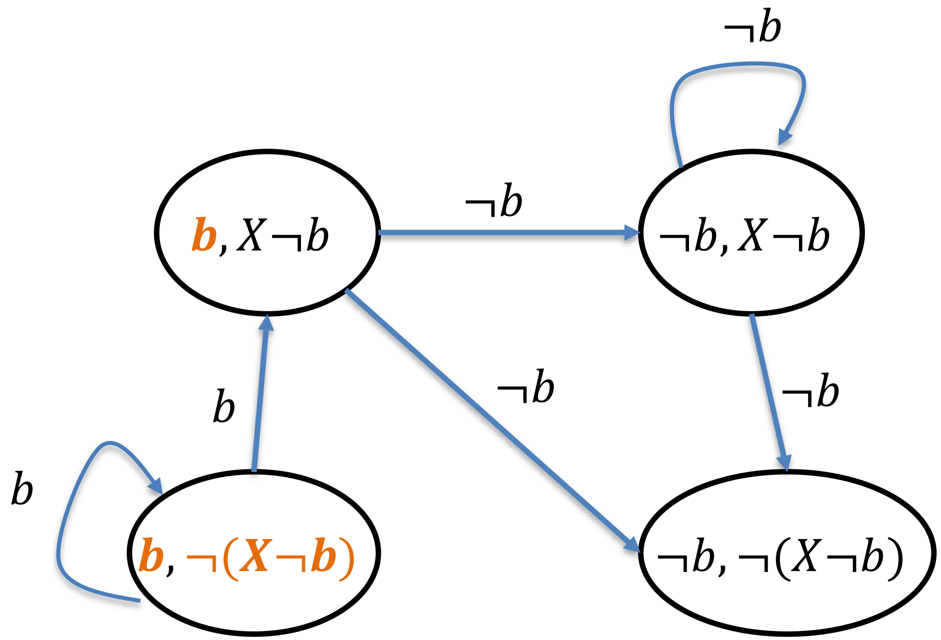
3. $\neg(\boldsymbol{X}\varphi_1) \in q \Leftrightarrow \neg\varphi_1 \in q'$

Draw the state space and the transitions.

$cl(\varphi) = \{b, \neg b, X\neg b, \neg(X\neg b)\}$



$\varphi = X\neg b$

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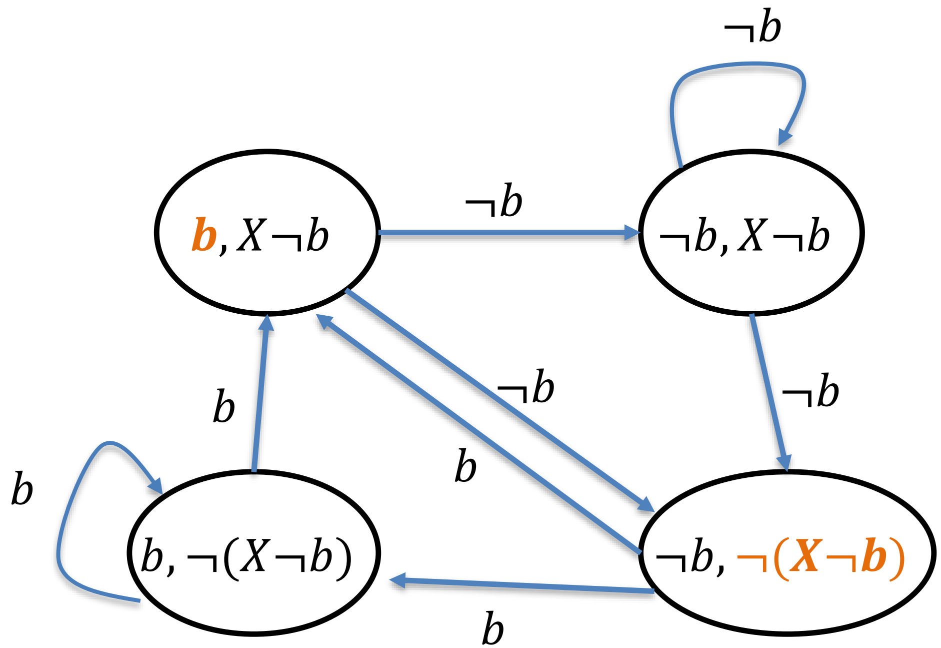
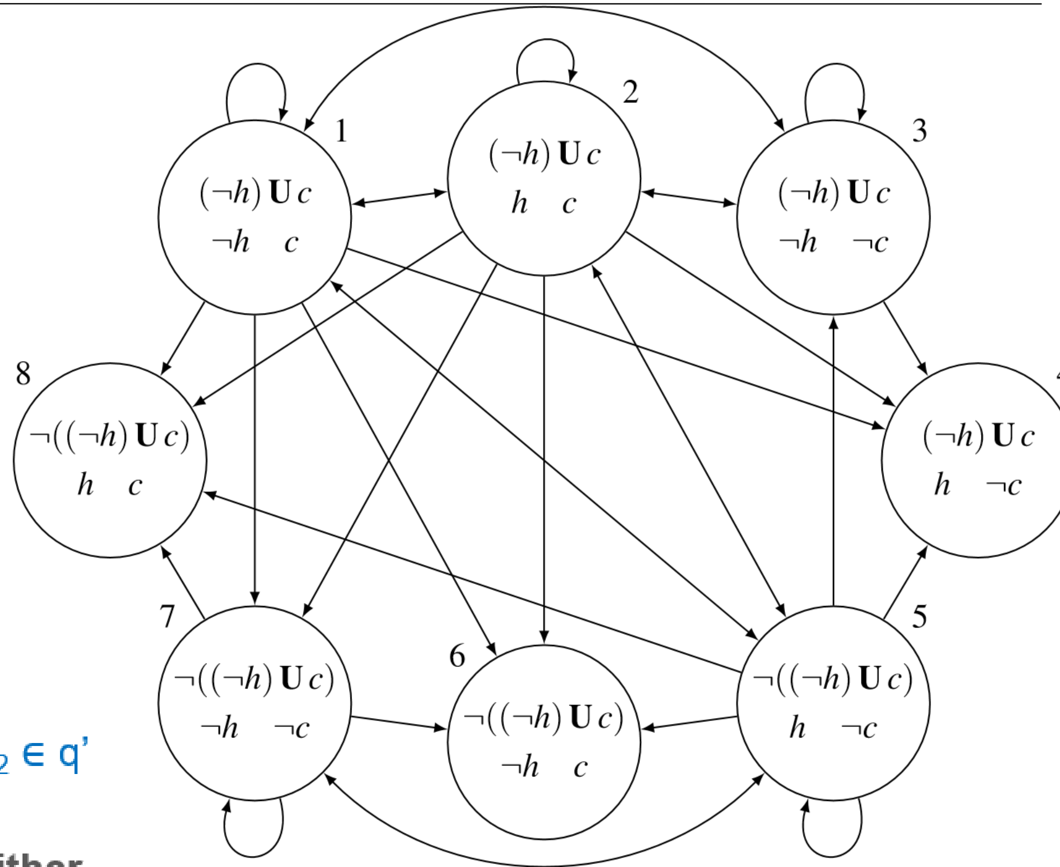
ToDo

Draw the state space and the transitions.

$cl(\varphi) = \{b, \neg b, X\neg b, \neg(X\neg b)\}$



φ= (¬*h* U *c*)



1. $\sigma = q' \cap AP$

3. φ_1 U $\varphi_2 \in$ q ⇔ either $\varphi_2 \in$ q **or both** $\varphi_1 \in$ q **and** φ_1 U $\varphi_2 \in$ q'

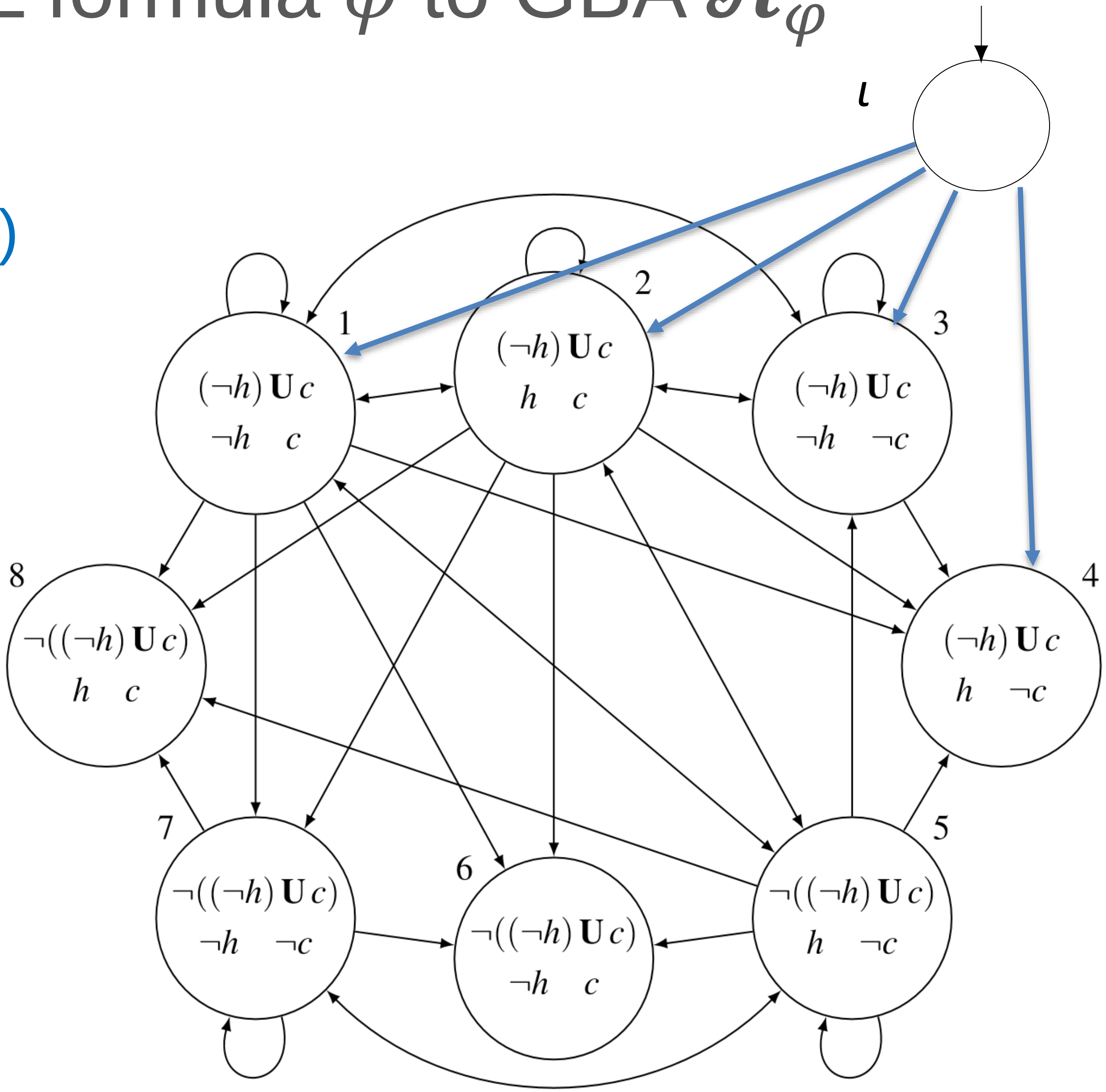
4. ¬(φ_1 U φ_2) ∈ q ⇔ either ¬$\varphi_2 \in$ q and **either** ¬ $\varphi_1 \in$ q **or** ¬(φ_1 U φ_2) ∈ q'

From LTL formula φ to GBA $\mathcal{A}_\varphi$

$\mathcal{A}_\varphi = (\mathcal{P}(AP), \mathbf{Q}, \mathbf{\Delta}, \{\iota\}, \mathbf{F})$

- $\mathbf{Q} \subseteq \mathcal{P}(cl(\varphi)) \cup \{\iota\}$ is the set of all the good sets in $cl(\varphi) \cup \{\iota\}$.

- $(\iota, \alpha, q) \in \mathbf{\Delta} \Leftrightarrow$ $\varphi \in q$ and $\sigma = q \cap AP$



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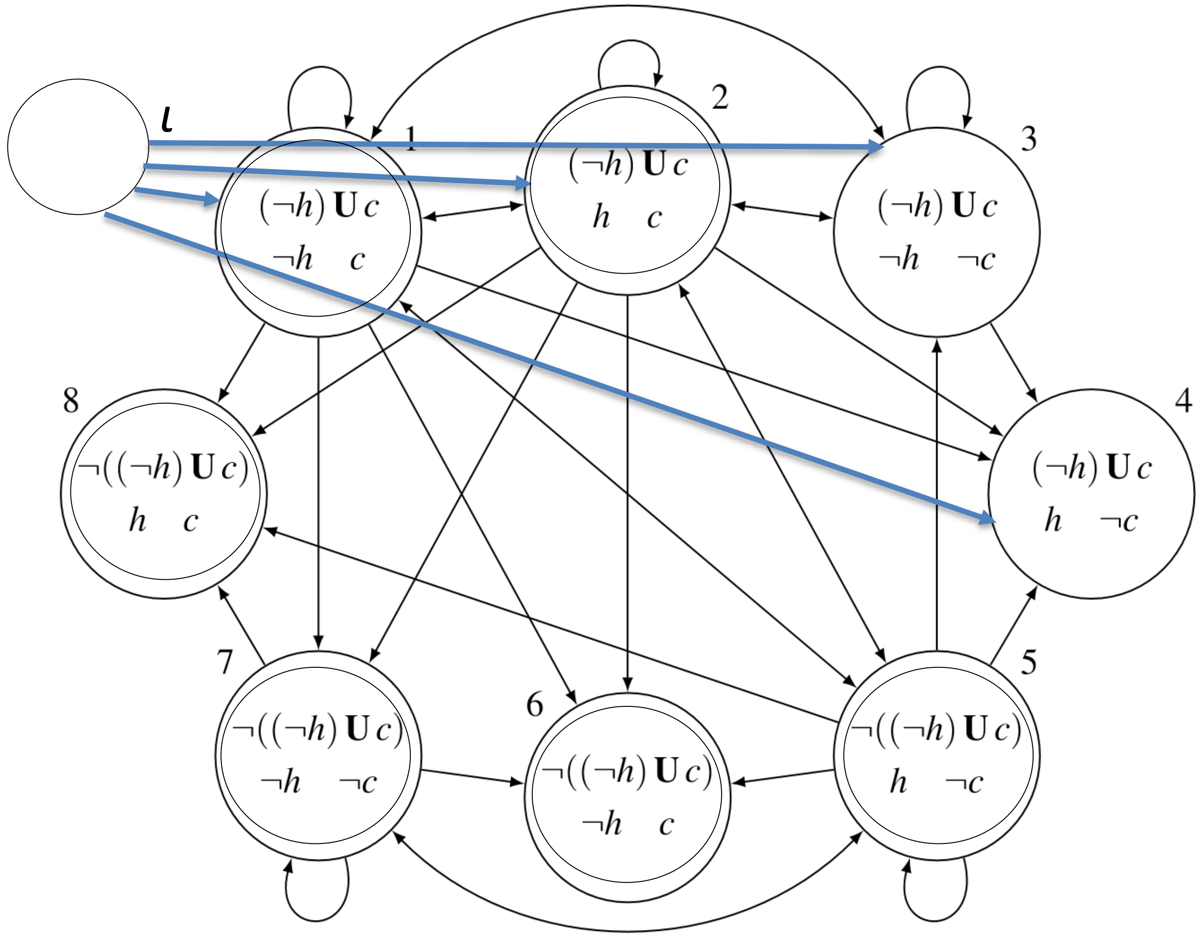
  - $(\iota, \alpha, q) \in \mathbf{\Delta} \Leftrightarrow \varphi \in q$ and $\sigma = q \cap AP$
- For every $\varphi_1 \mathbin{U} \varphi_2 \in cl(\varphi)$, $\mathbf{F}$ includes the set
  - $F_{\varphi_1 \mathbin{U} \varphi_2} = \{q \in \mathbf{Q} \mid \varphi_2 \in q \text{ or } \neg(\varphi_1 \mathbin{U} \varphi_2) \in q\}$.

$\varphi = (\neg h \text{ U } c)$



- F = {{1, 2, 5, 6, 7, 8}}

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- What is the complexity?

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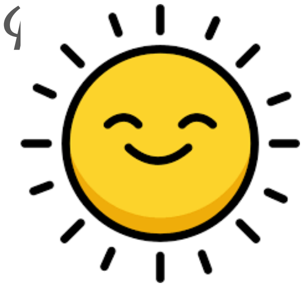

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- What is the complexity?
  - $\mathcal{A}_\varphi$ is **always exponential** in the size of φ.


THANK
YOU!