Computer Organization and Networks

(INB.06000UF, INB.07001UF)

Chapter 3 – State Machines

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What About Storage?

We have so far only considered functions

We also want to store data and define sequences of computation

So far, we have not talked about storage or about time

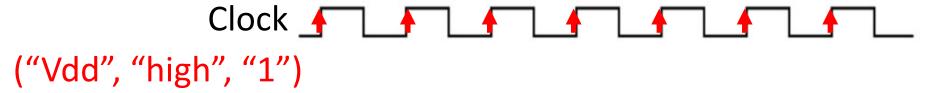
Storage

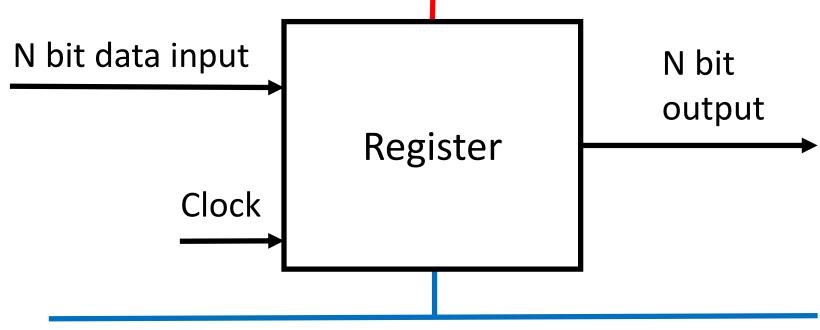
Storage

("Vdd", "high", "1") N bit data input N bit output Register Clock ("GND", "low", "0")

Storage

- The registers sets output = input when the clock switches from low to high;
- In all other cases, the input is ignored; the last "sampled" value is kept at the output





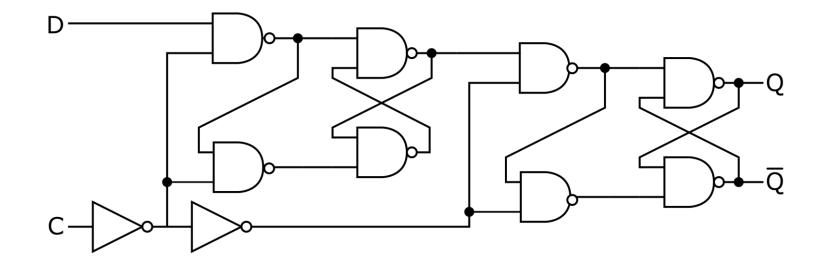
("GND", "low", "0")

Naming Conventions

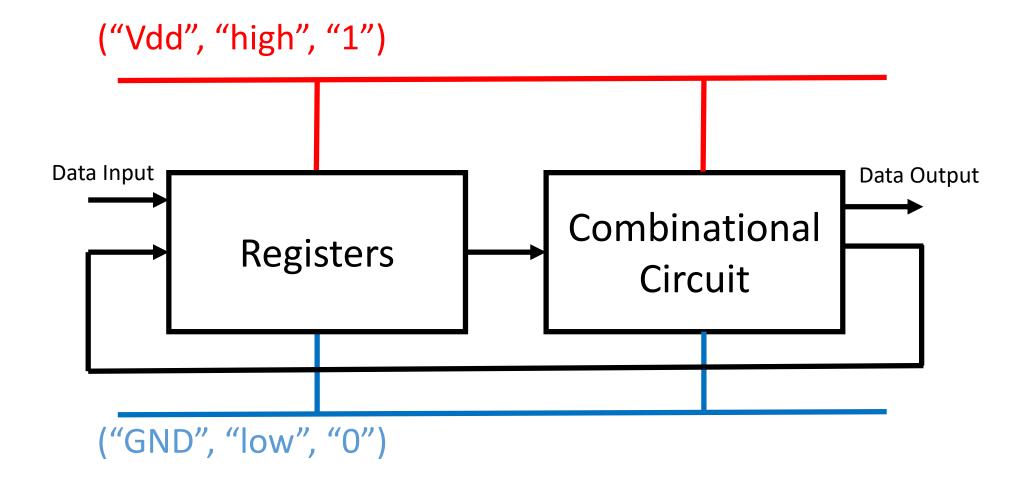
• Register: An n-bit storage sampling data on the rising clock edge

• Flip Flop: A 1-bit storage sampling data on the rising clock edge

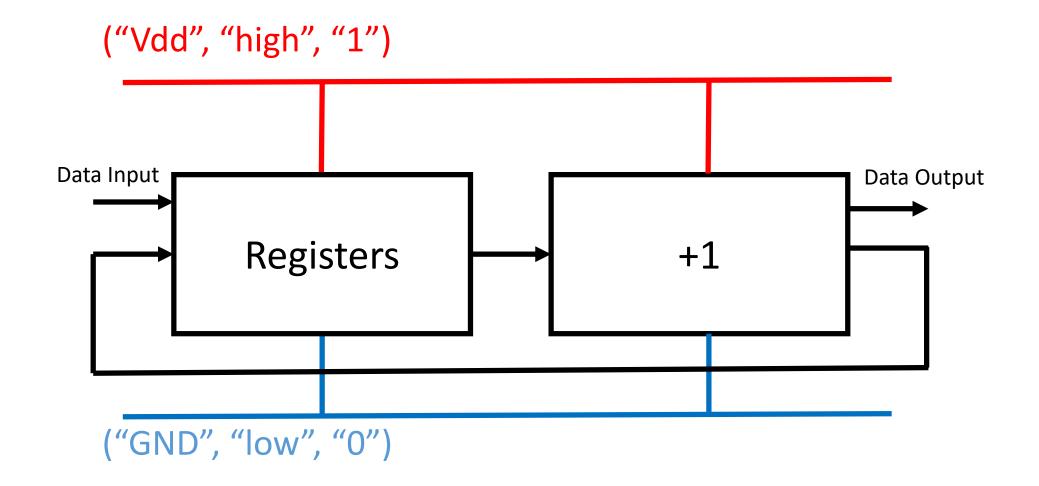
Flip-Flop based on CMOS Gates



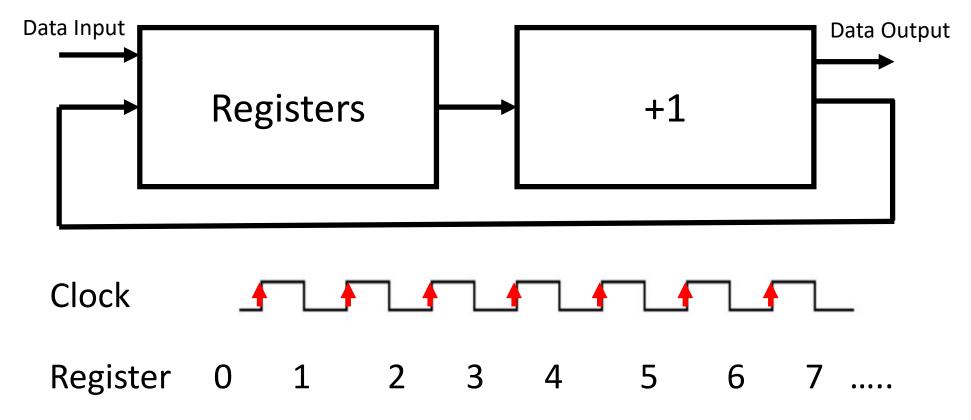
Combining



Example Counter



Example Counter



Let's Build This in Digital

• See examples con03 available at

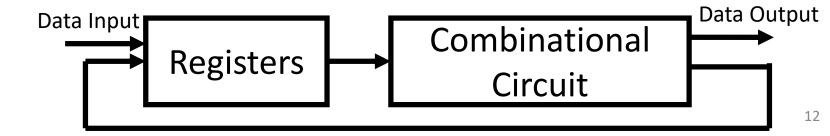
https://extgit.iaik.tugraz.at/con/examples-2021.git

The Clock Frequency

Can we increase the clock frequency arbitrarily?

 The clock frequency is limited by the time the combinational circuit needs to compute its outputs.

• The critical path is the path with the longest propagation delay in the combinational circuit. It defines the maximum clock rate

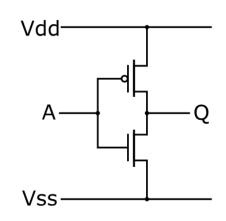


Temperature, Power Consumption

- The higher the temperature, the slower the transistors become and the lower becomes the maximum clock rate
 - → The lower the temperature, the higher clock rates are possible

- Why does a CPU produce heat?
 - Every time a logic gate switches, NMOS and PMOS transistors are open at the same time → there is a short current.
 - Upon a switch, there is also current flowing to charge and discharge parasitics
 - → The more transistors are switching, the more heat is produced





Clock Frequency Too High

What happens, if the clock frequency is too high?

 The circuit stores an intermediate state of the combinational circuit in the registers.

 The intermediate state depends on the physical layout, the temperature, fabrication details, ... → hard to predict; overclocking a processor too much typically leads to a crash

State Machines

Finite State Machines (FSMs)

- FSMs are the "work horse" in digital systems.
- FSMs can be implemented with hardware.
- We look at "synchronous" FSMs only:
 - The "clock signal" controls the action over time
- FSMs can be described with three main "views":
 - The functional view with the "state diagram"
 - The timing view with the "timing diagram"
 - The structural view with the "logic circuit diagram"

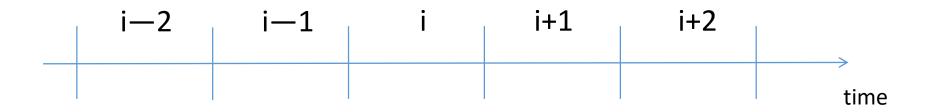
Time flows continuously

time

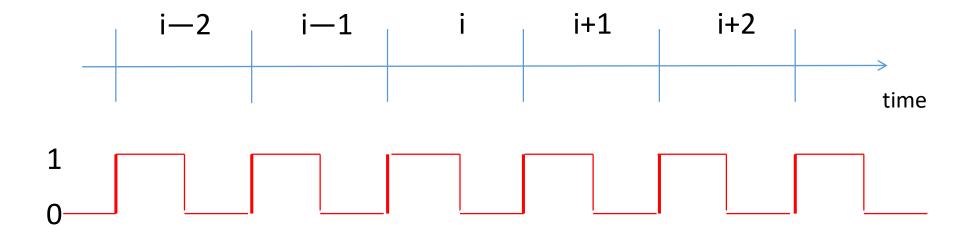
We cut time into slices



Time slices are strictly ordered

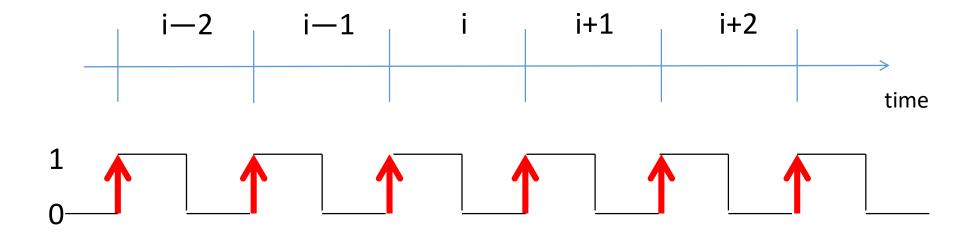


Clock signal



We use the clock signal ("clk") in order to advance time from slice to slice.

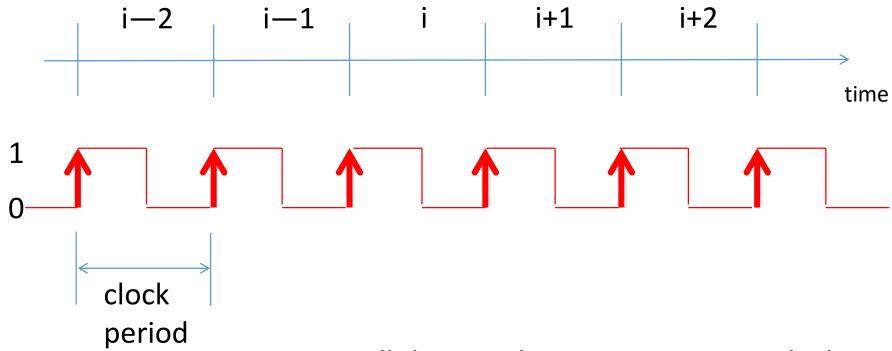
Rising clock edge



With rising clock edges we define the transition between neighboring time slices.

The negative clock edges have no importance.

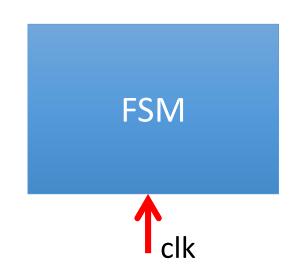
Clock period



We call the time between two rising clock edges also "clock period".

Most often, clock periods have the same length. But this is not necessarily the case.

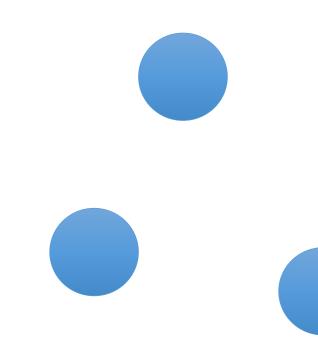
Synchronous finite state machine (= automaton)



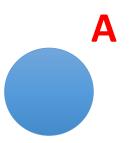
A synchronous FSM is clocked by a clock signal ("clk"). In each clock period, the machine is in a defined (current) state. With each rising edge of the clock signal, the machine advances to a defined next state.

We are interested in "finite state machines" (FSMs). FSMs have only a finite number of states.

The sequence of states can be defined in a state diagram.



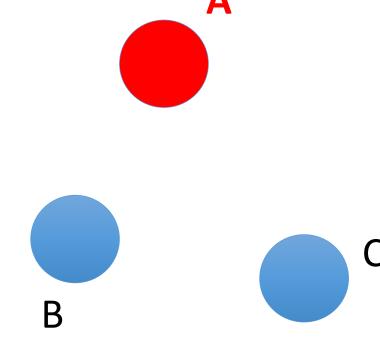
State diagram:



We denote the states with circles and give them symbolic names, e.g. A, B, and C.

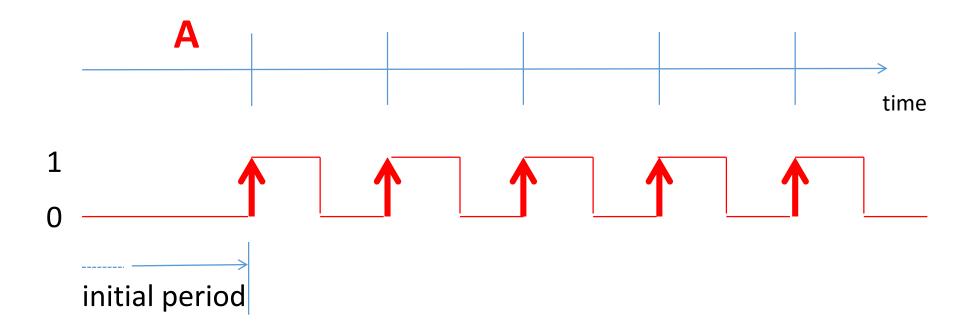


State diagram:



We define one of the states as the initial state.

In the beginning...



Initially, i.e. shortly after switching on the FSM and before the first rising edge of clock, there is the initial period. In this period, the FSM is in the "initial state".

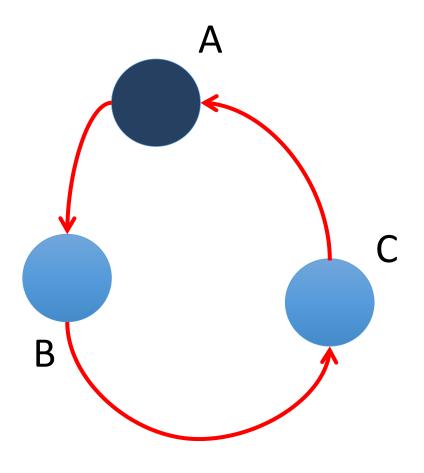
State diagram:

B

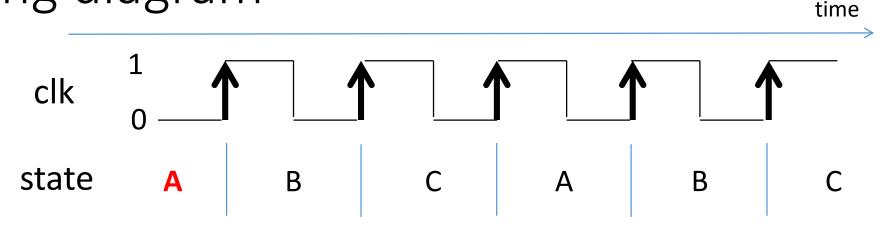
With arrows we define the sequence of states.

The sequence of states can also be defined in a state transition table.

present	next
state	state
Α	В
В	С
С	Α



Timing diagram

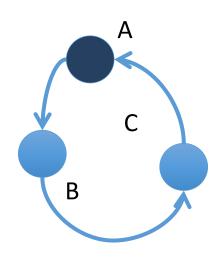


Initially, the FSM is in the initial state A.

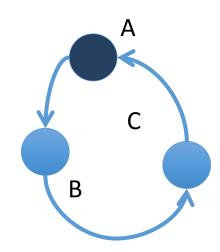
With every positive clock edge, the next state becomes the current state.

An FSM has always a next state.

In order to technically realize ("implement") a state diagram, we start by giving each state a unique number.



Popular state-encoding schemes

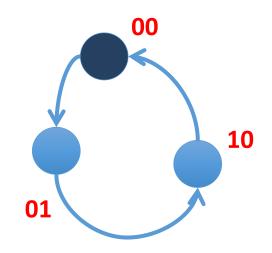


- Binary encoding
 - needs minimum amount of flip-flops.

- One-hot encoding
 - Tends to have a simpler next-state logic.

Binary encoding

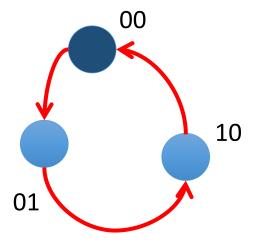
state	number
Α	00
В	01
С	10
	33



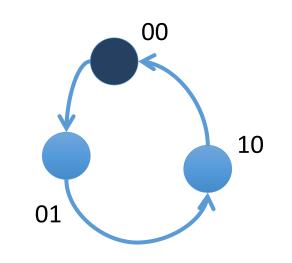
Binary encoding:

The state transition table has also only binary numbers.

present	next
state	state
00	01
01	10
10	00

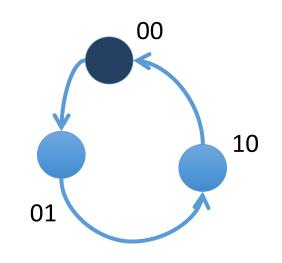


We also enter the unused combination "11". This state does not exist. "x" stands for "Don't care".



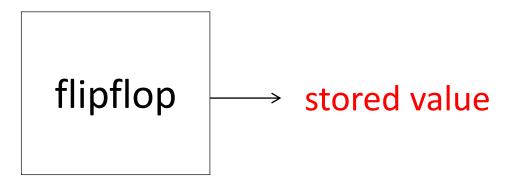
present	next
state	state
00	01
01	10
10	00
11	XX

We define names for the two state bits, e.g. s1, s0.

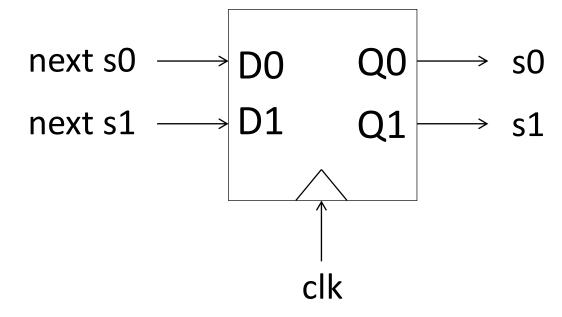


present		next	
s1	s0	s1 s	0
0	0	0 1	1
0	1	1 ()
1	0	0 ()
1	1	XX	(

Each state bit is stored in a flipflop

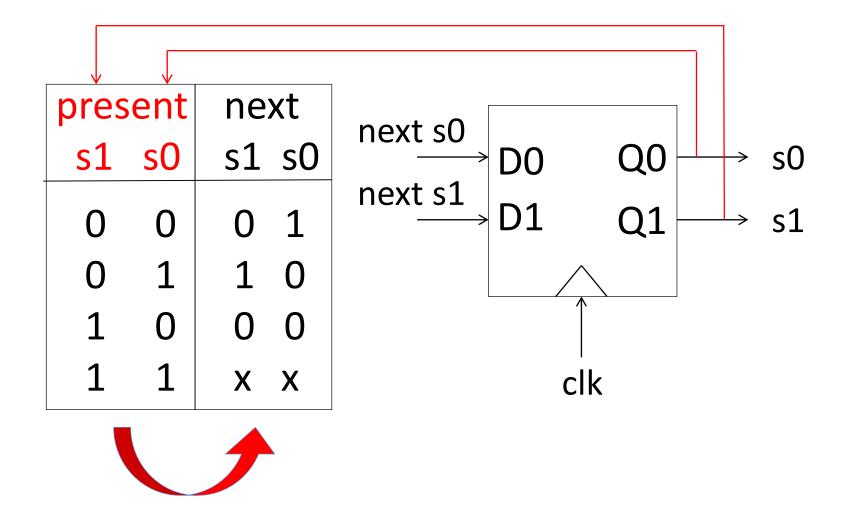


In our example, we need 2 flip-flops for storing the state bits.

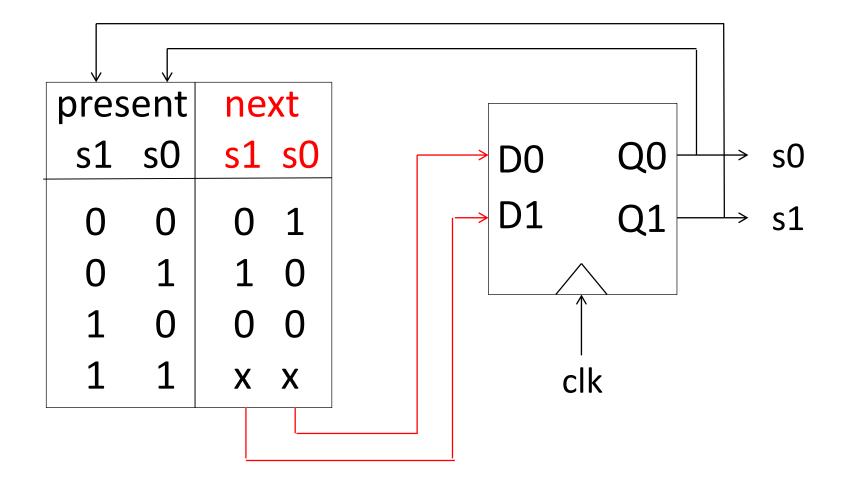


Flipflops which can store several bits are also called "registers".

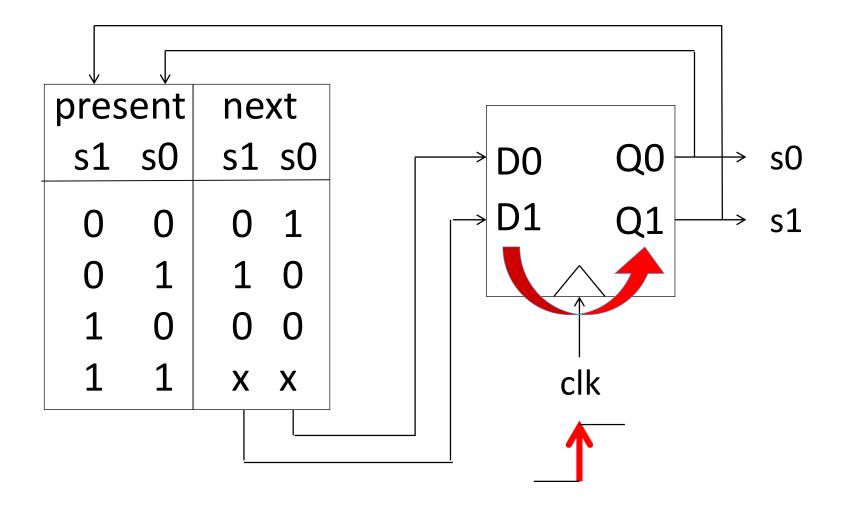
We use the state transition table as a "lookup table"...



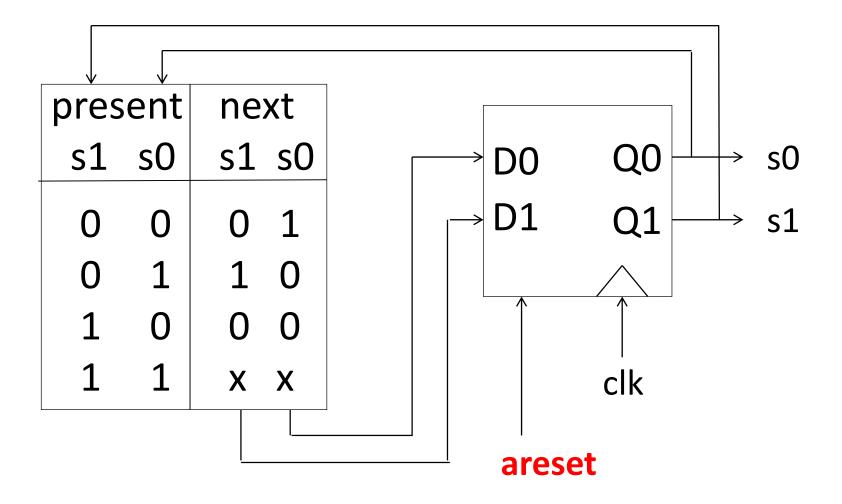
...and thus find out the next state



At each positive edge of clk, the next state gets stored as the current state.

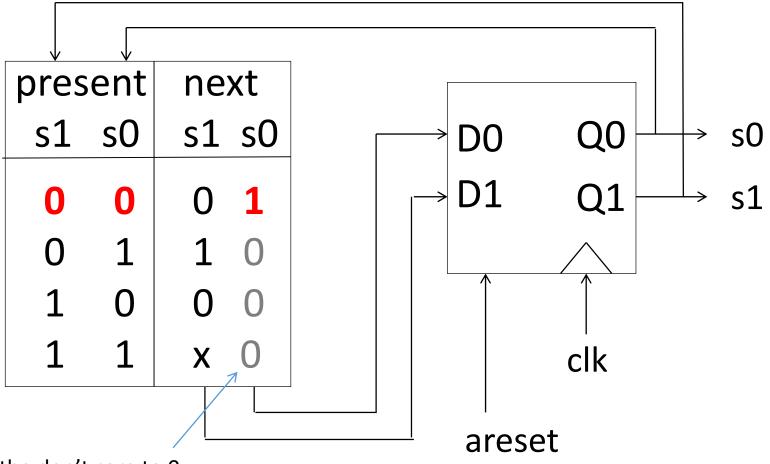


In order to get the initial state "00", we use flipflops with an "asynchronous reset input". Shortly after switching on the circuit, we apply "areset".



The state transition function in this example can be derived from the truth table:

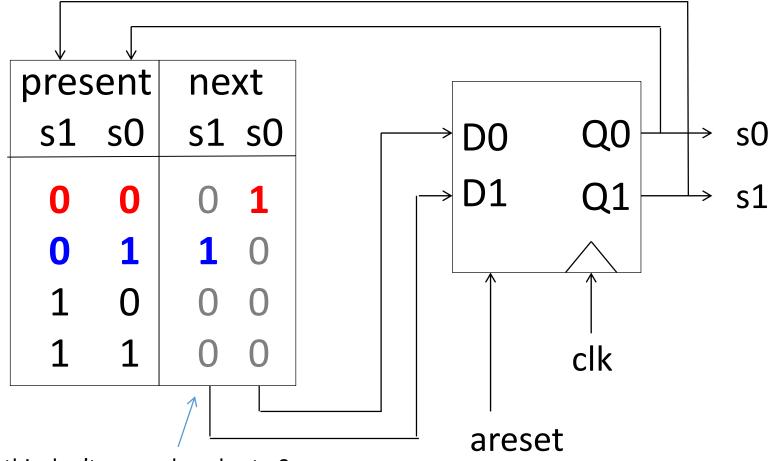
$$next s0 = (\sim s1) & (\sim s0)$$



We set the don't care to 0.

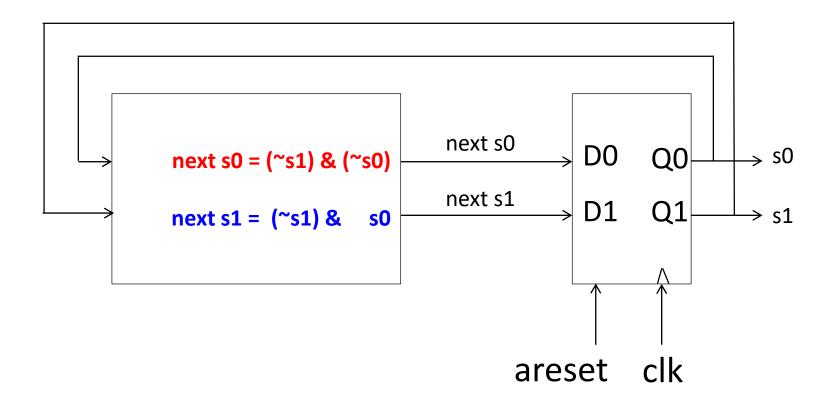
The state transition function:

next s0 = (
$$^{\sim}$$
s1) & ($^{\sim}$ s0)
next s1 = ($^{\sim}$ s1) & s0



We set this don't care value also to 0.

Structural diagram of the FSM: State-transition function, storage elements, and feedback of state.



Essence so far

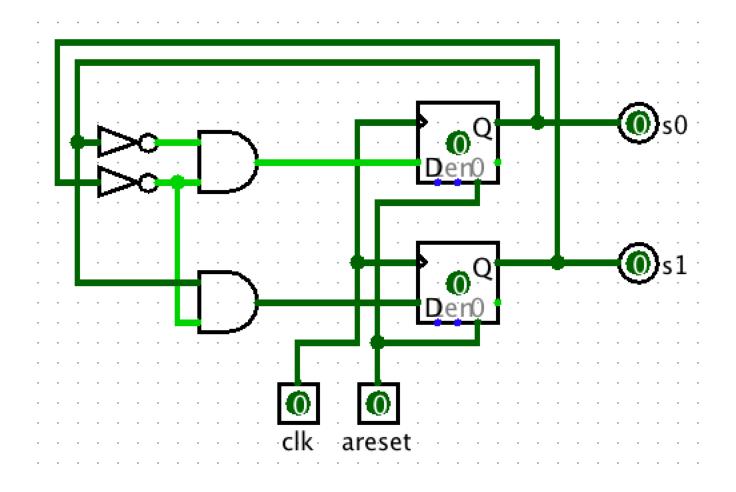
State diagram:

"next state" is a function of (present) "state"

State transitions:

the "next state" becomes (the present) "state" on the rising edge of the clock

The FSM modeled and simulated with Digital



For an example of this FSM see

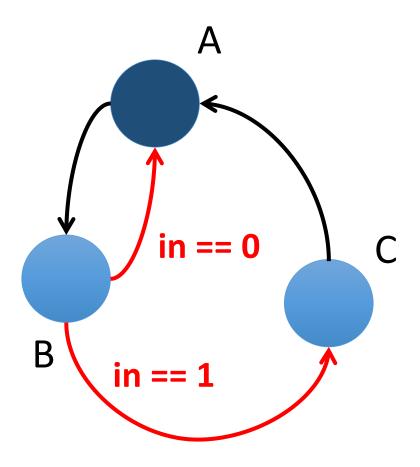
fsm_moore_no_input

Inputs

FSMs can also have **inputs** influencing the transition to the next state

next state = f(state, input)

In this example we see that the one-bit input "in" influences the choice of the state after B.

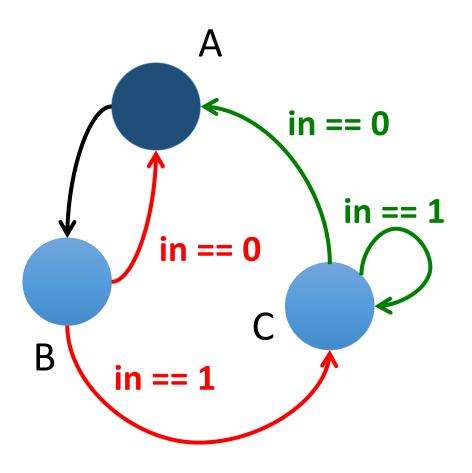


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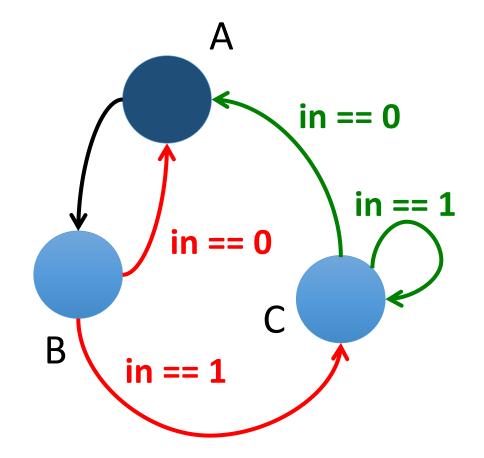
In this example we see that the one-bit input "in" influences the choice of the state after B.

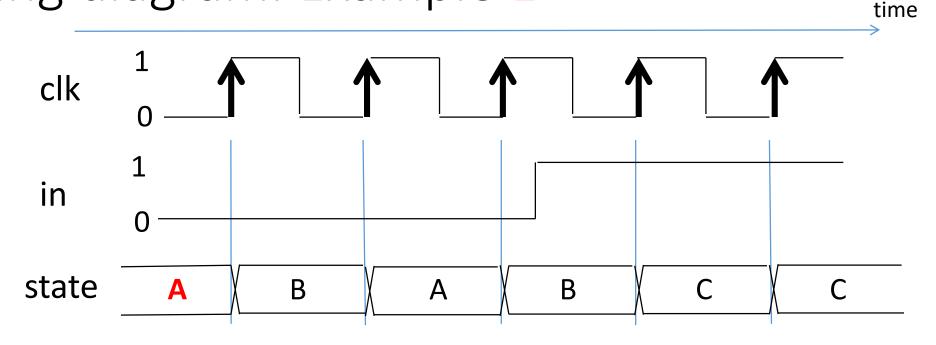
The following state can also be the same as the current state.

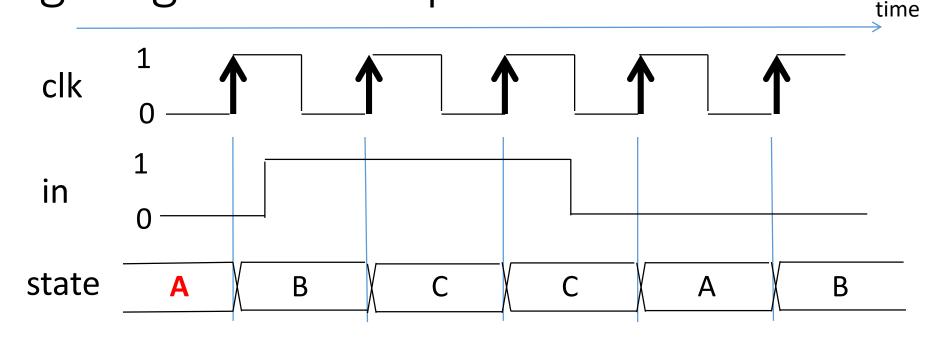


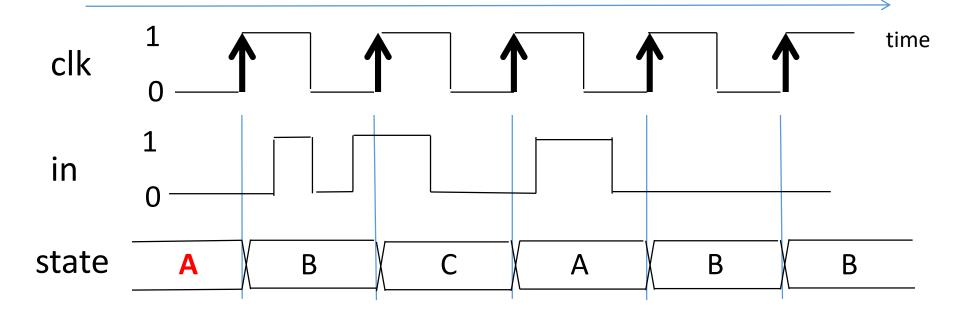
The state transition table

present in state		next state
A	0	B
A	1	B
B	0	A
B	1	C
C	0	A



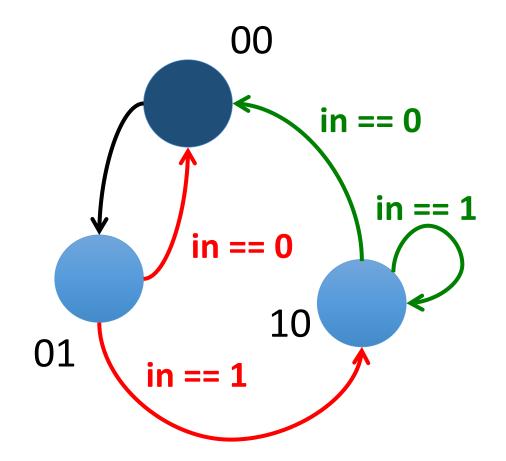






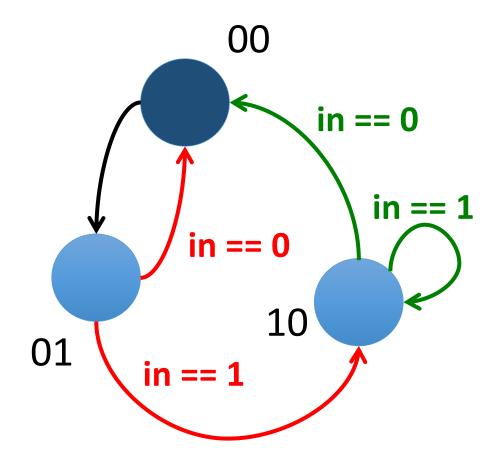
Binary state encoding: Instead of symbolic state names we use numbers

'	present in state		-		nex stat	
0 0 0 0 1 1	0 0 1 1 0	0 1 0 1 0	0 0 0 1 0	1 0 0 0		



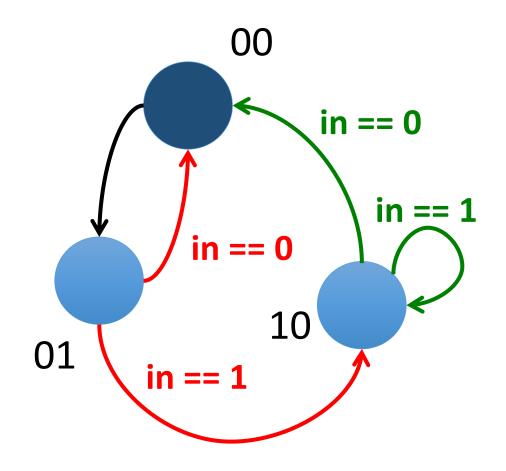
"11" does not exist: We use "Don't Care" as the following state

	present in state		nex stat	
0	0	0	0	1
0	0	1	0	1
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	0
1	1	0	X	X
1	1	1	X	X



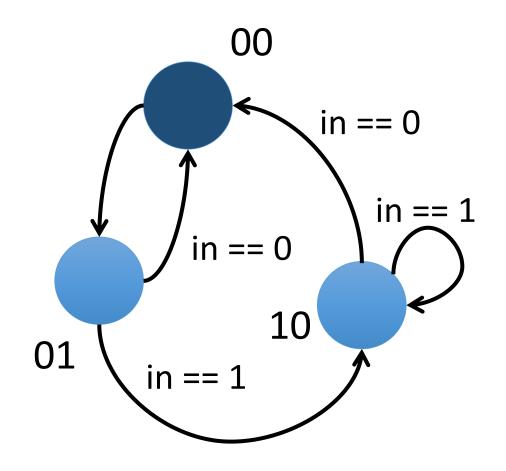
I have ordered the lines in the state transition table from 0 to 7. This makes it easier to "read" the table.

	'		nex stat		
0	0	0	0	0	1
1	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	1	0
4	1	0	0	0	0
5	1	0	1	1	0
6	1	1	0	X	X
7	1	1	1	X	X
1					



We call the state bits "s1" and "s0"

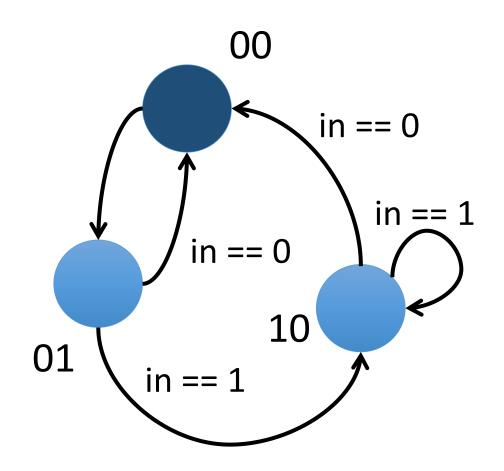
	present in		ne	xt	
	s1	s0		s1	s0
0	0	0	0	0	1
1	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	1	0
4	1	0	0	0	0
5	1	0	1	1	0
6	1	1	0	Х	Х
7	1	1	1	Х	X



next s0 =
$$((^s1) & (^s0) & (^in))$$

| $((^s1) & (^s0) & in)$

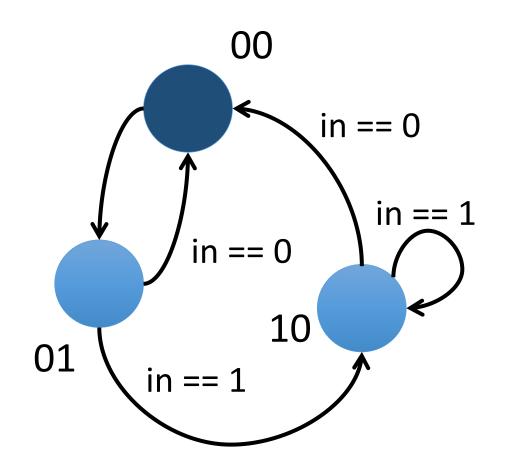
	present		in	ne	xt
	s1	s0		s1	s0
0	0	0	0	0	1
1	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	1	0
4	1	0	0	0	0
5	1	0	1	1	0
6	1	1	0	Х	0
7	1	1	1	Х	0



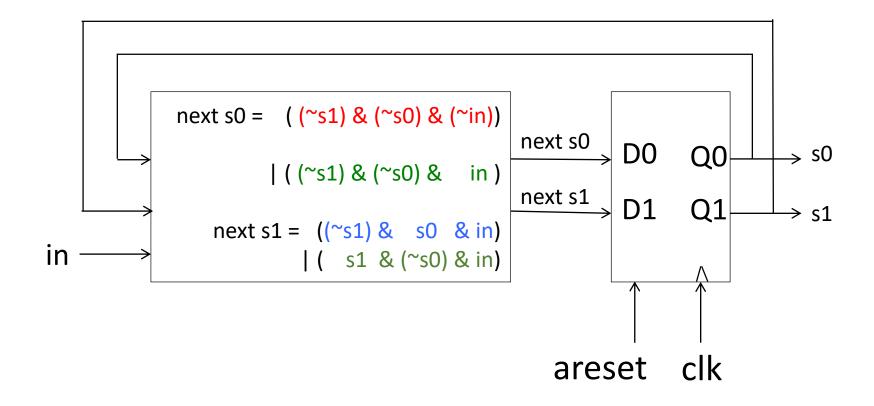
next s1 =
$$((^s1) & s0 & in)$$

| $(s1 & (^s0) & in)$

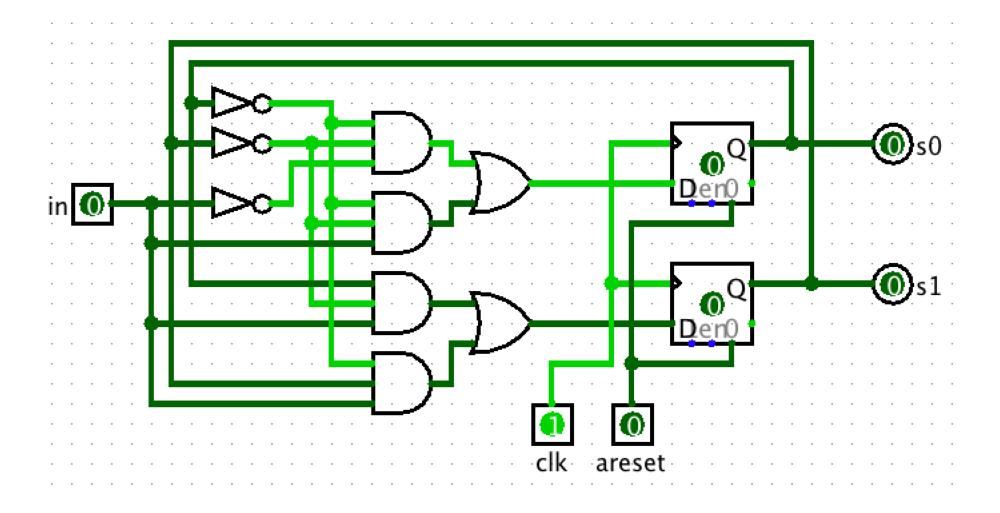
	present		in		ext
	s1	s0		s1	s0
0	0	0	0	0	1
1	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	1	0
4	1	0	0	0	0
5	1	0	1	1	0
6	1	1	0	0	0
7	1	1	1	0	0



Structural diagram of the FSM



Implementation with Digital



For an example of this FSM see

fsm_moore_no_output_function

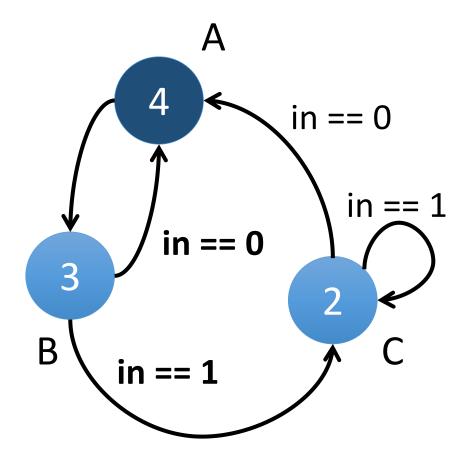
Outputs

FSMs typically also have outputs

In this example we see that the outputs are a function of the state. We write the output values into the circles.

We call such machines also "Moore machines":

output = f(state)

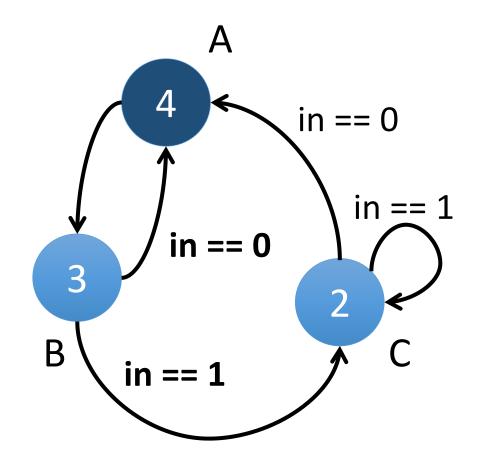


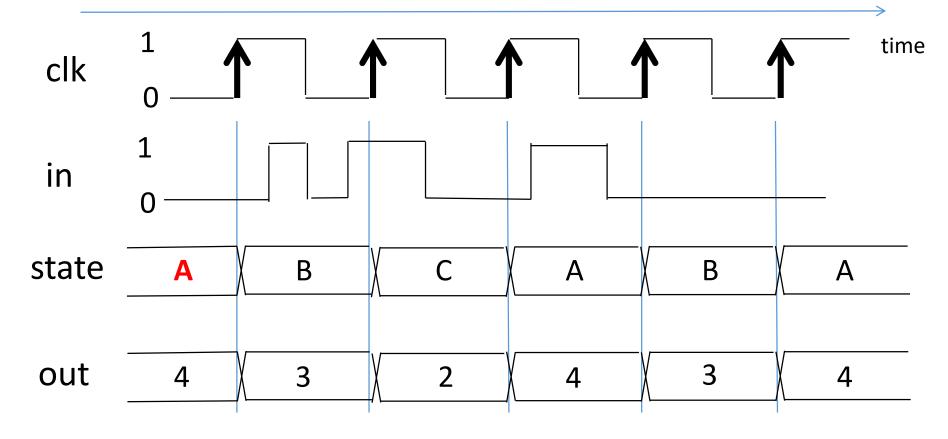
We define the outputs with the "output function"

Moore machines:

output = f(state)

state	output
Α	4
В	3
С	2



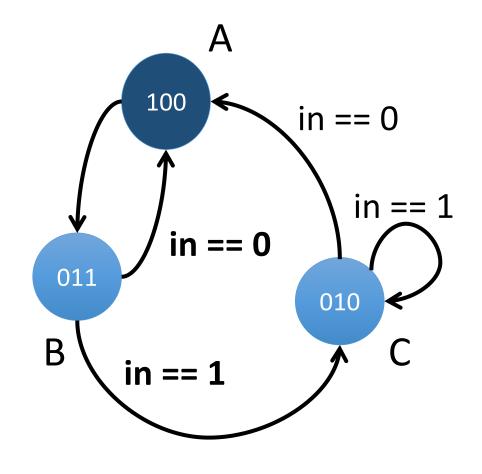


We define the outputs with binary values

Moore machines:

output = f(state)

state	output	
Α	100	(=4)
В	011	(=3)
С	010	(=2)

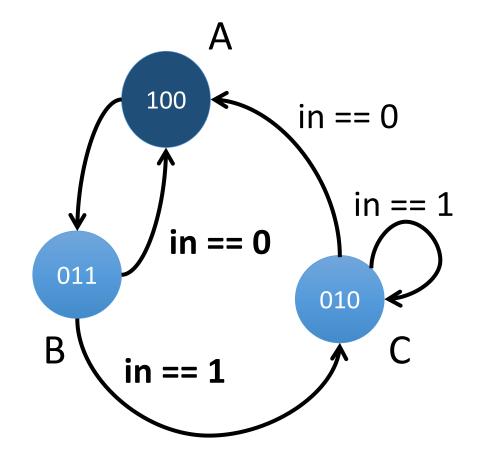


We define the outputs with binary values

Moore machines:

output = f(state)

state	o2	o1	00
Α	1	0	0
В	0	1	1
С	0	1	0

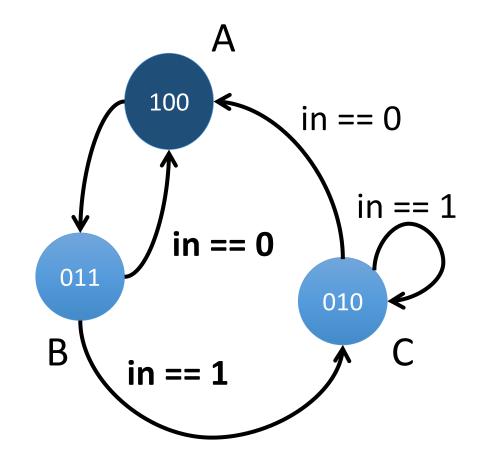


For an example of this FSM see

fsm_moore_with_output_function

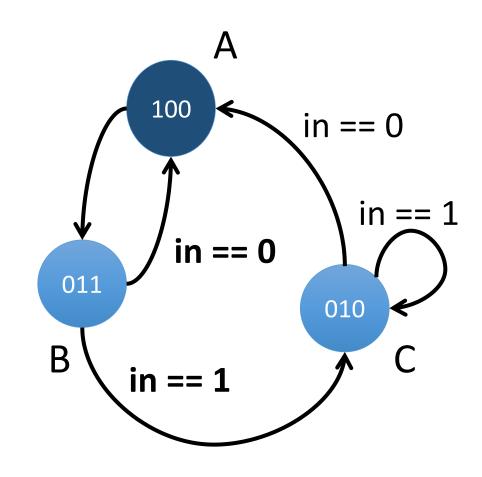
Binary state encoding: We define the outputs with binary values

state		02 01 00			
0	0	1	0	0	
0	1	0	1	1	
1	0	0	1	0	



We define the outputs with binary values

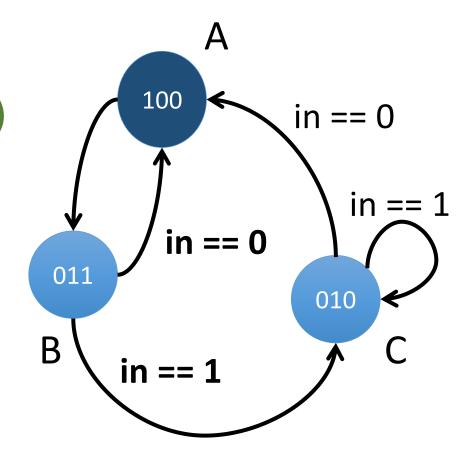
s1	s0	02	o1	00
0	0	1	0	0
0	1	0	1	1
1	0	0	1	0



We define the outputs with binary values

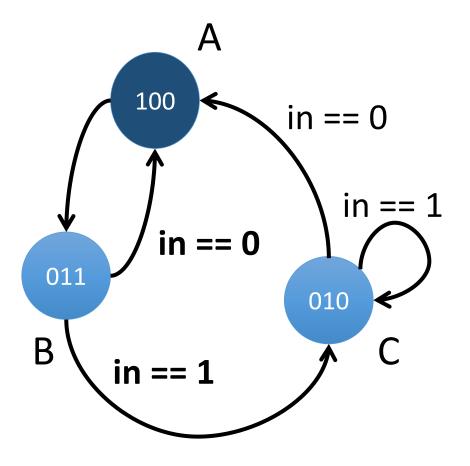
$$o1 = (\sim s1 \& s0) | (s1 \& \sim s0)$$

s1	s0	о2	o1	о0
0	0	1	0	0
0	1	0	1	1
1	0	0	1	0

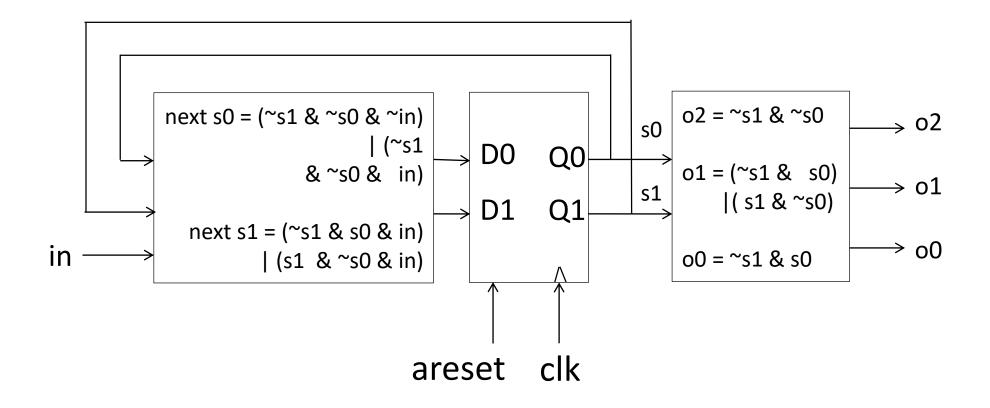


We define the outputs with binary values

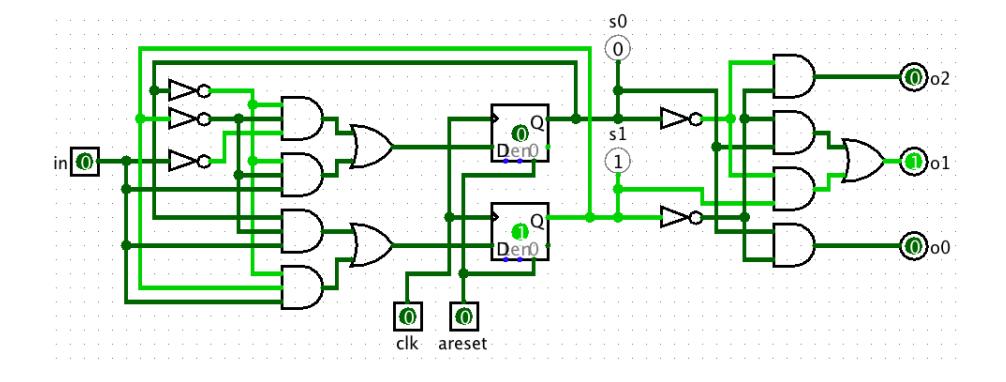
s1	s0	о2	o1	00
0	0	1	0	0
0	1	0	1	1
1	0	0	1	0



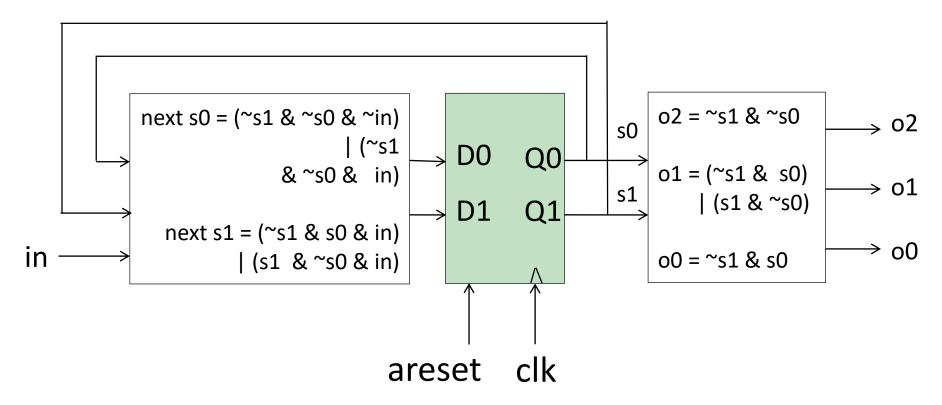
Structural diagram of the FSM



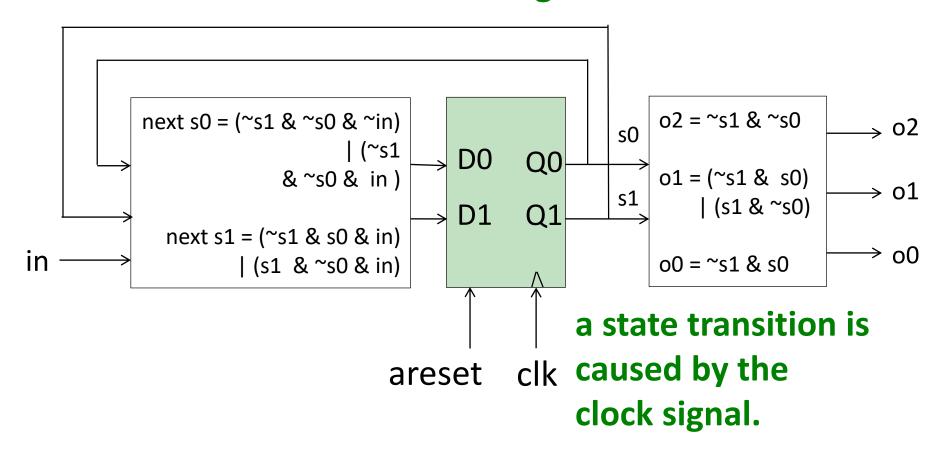
Implementation with Digital



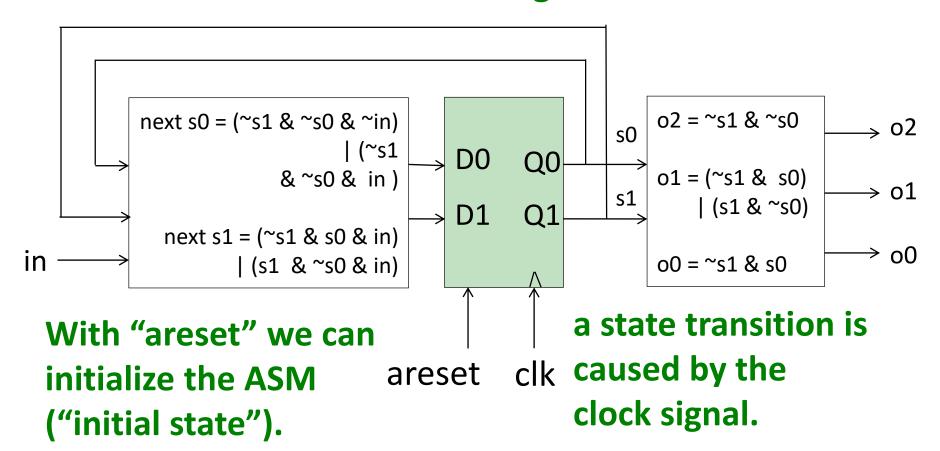
the state is stored in a register



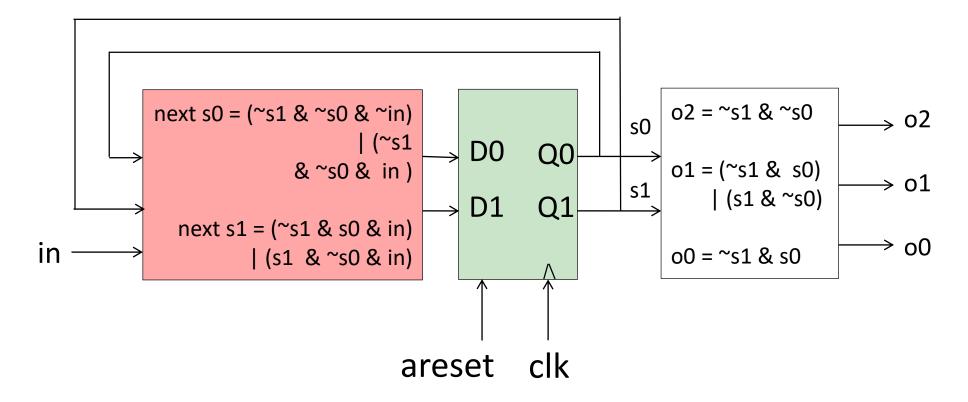
the state is stored in a register



the state is stored in a register

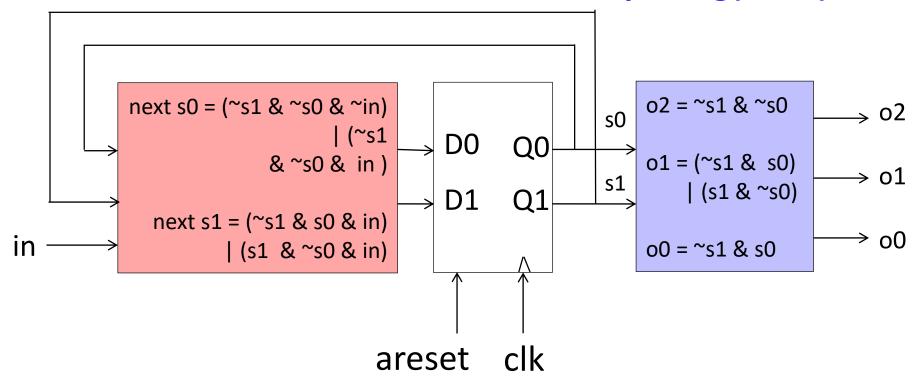


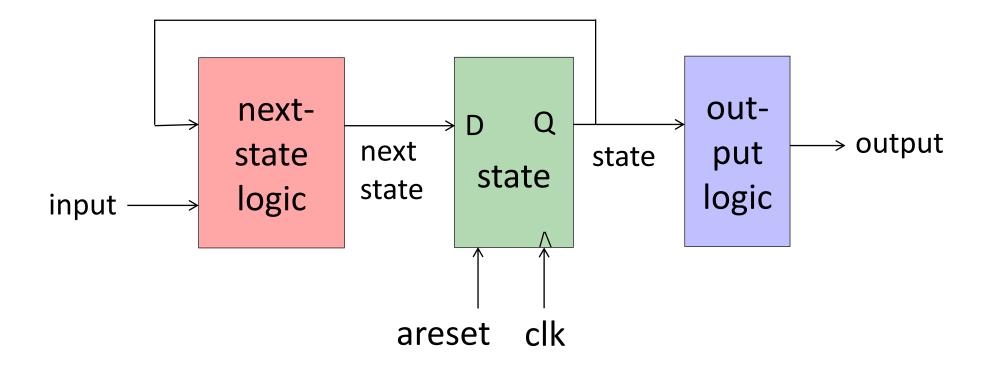
With the next-state function f we compute the next state: next state = f(state, input)



With the output function we compute the output values:

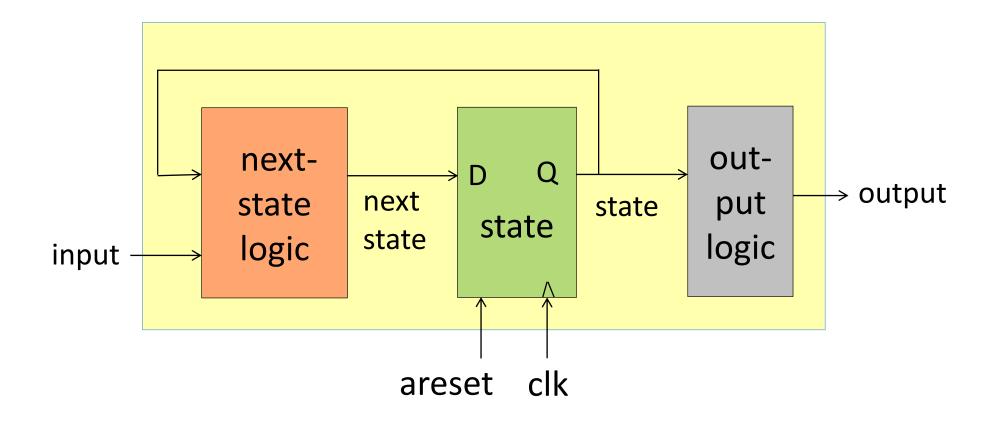
output = g(state)





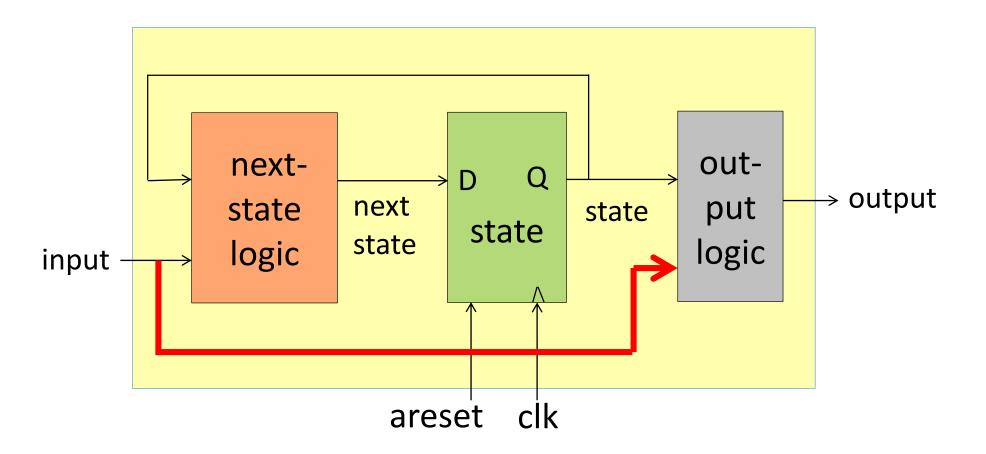
There exist 2 types of machines: check out the LITTLE but IMPORTANT difference

- Moore Machines
 - next state = function of present state and input
 - output = function of present state
- Mealy Machines
 - next state = function of present state and input
 - output = function of present state and input



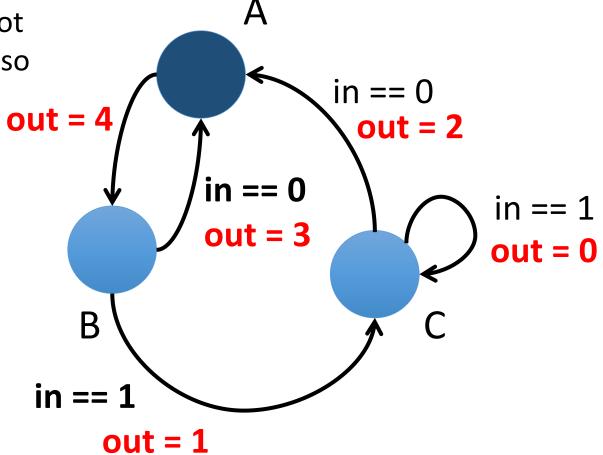
Essence of Mealy Machines

output = g(state, input)



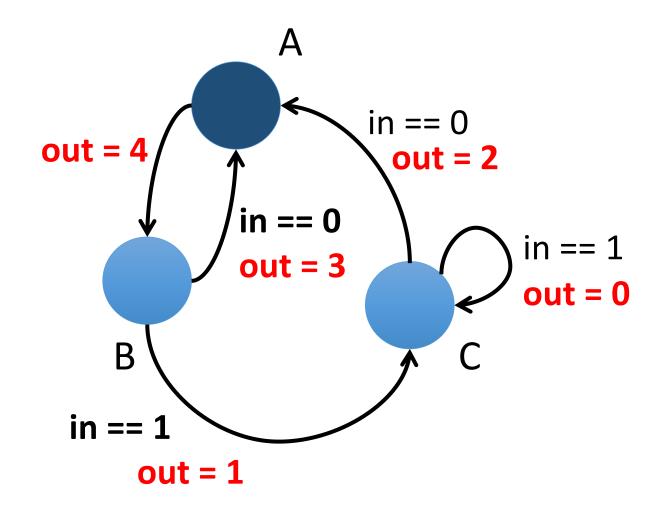
An example for a Mealy Machine

We write the **output values** next to the transition arrows, since the output depends not only on the state, but can also depend on the input.



The state transitions are the same as in the previous example with the Moore Machine

present state	in	next state
A	0	B
A	1	A
B	0	C
B	1	A
C	0	C

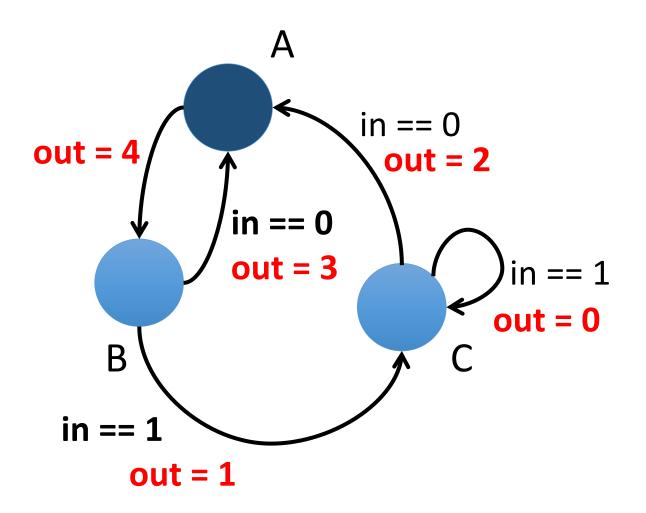


The output function

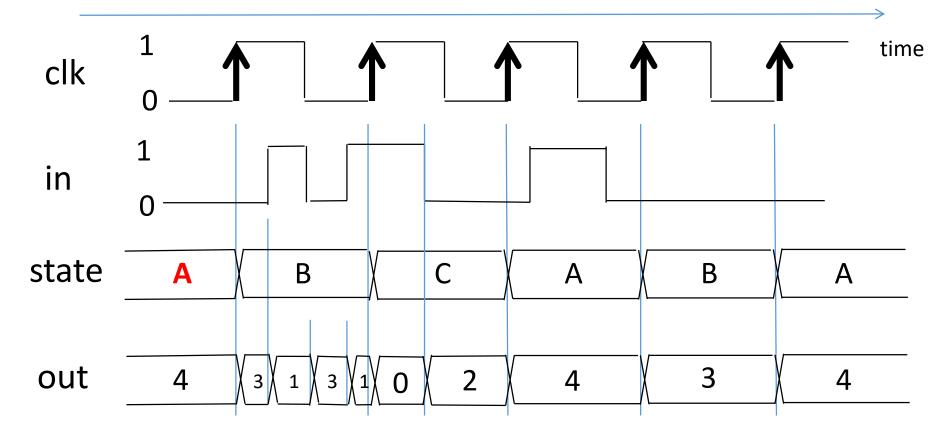
The output function can be derived from the state diagram.

output = g(state, input)

state	in	output
Α	0	4
Α	1	4
В	0	3
В	1	1
С	0	2
С	1	0



Timing diagram

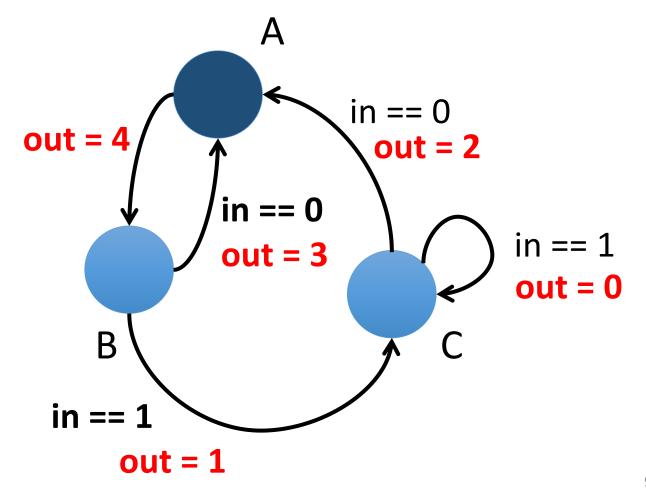


For a timing diagram, we need to choose values for the input signal "in". Only after some choice for "in" we can derive the sequence of states from the state diagram. We see here also, how the value of "in" immediately influences the value of "out".

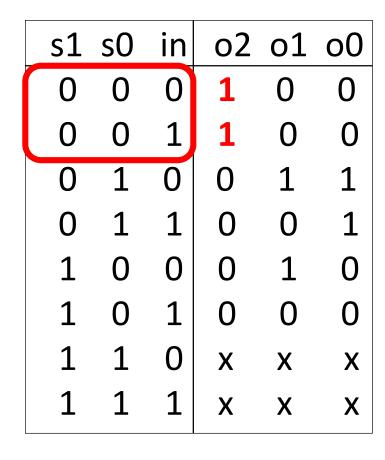
Binary encoding: Re-writing the output table with binary values.

And completing the table with the unused bit combinations.

s1	s0	in	o2	o1	00
0	0	0	1	0	0
0	0	1	1	0	0
0	1	0	0	1	1
0	1	1	0	0	1
1	0	0	0	1	0
1	0	1	0	0	0
1	1	0	X	X	X
1	1	1	X	X	X



We can derive the logic functions for o2, o1, and o0



We can derive the logic functions for o2, o1, and o0

S	1	s0	in	02	o1	00
()	0	0	1	0	0
()	0	1	1	0	0
)	1	0	0	1	1
()	1	1	0	0	1
1	L	0	0	0	1	0
	L	0	1	0	0	0
	L	1	0	X	X	X
	L	1	1	X	X	X

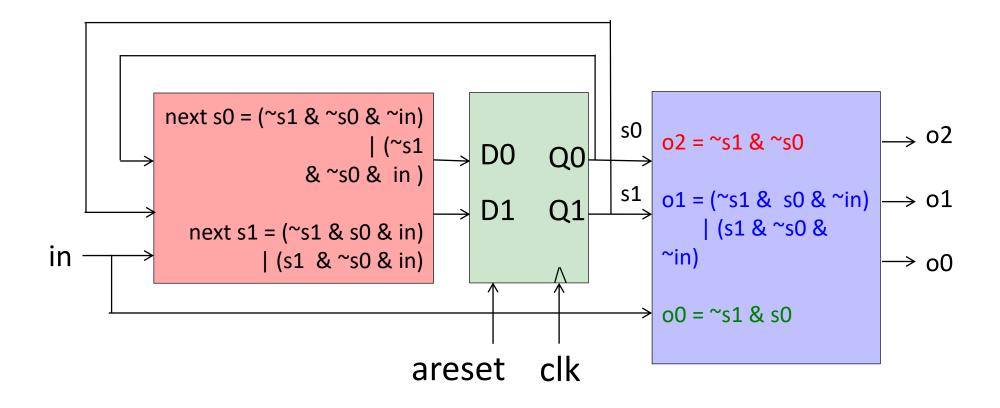
```
o2 = ~s1 & ~s0
o1 = (~s1 & s0 & ~in)
    | (s1 & ~s0 & ~in)
```

We can derive the logic functions for o2, o1, and o0

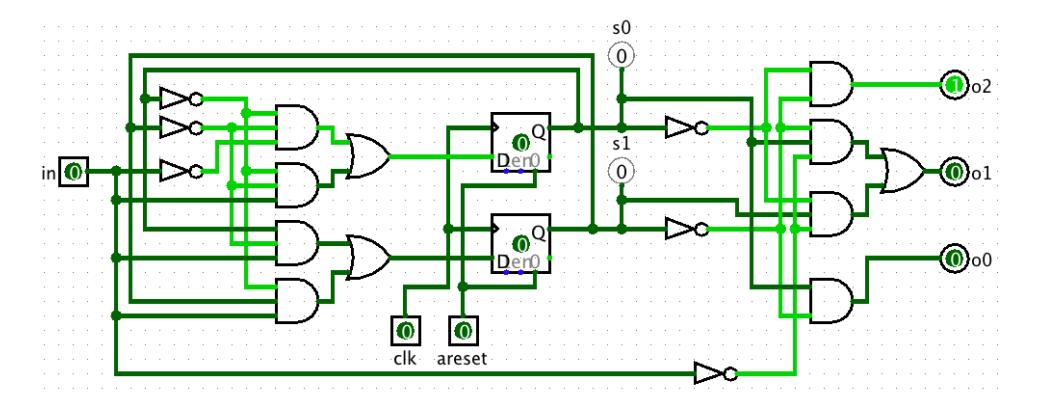
	s1	s0	in	o2	o1	00
	0	0	0	1	0	0
	0	0	1	1	0	0
	0	1	0	0	1	1
l	0	1	1	0	0	1
	1	0	0	0	1	0
	1	0	1	0	0	0
	1	1	0	X	X	X
	1	1	1	X	X	X

```
o2 = ~s1 & ~s0
o1 = (~s1 & s0 & ~in)
    (s1 & ~s0 & ~in)
00 = ~s1 \& s0
```

The result



Modeling with Digital



For a SystemVerilog example of this FSM see

con03_mealy_fsm

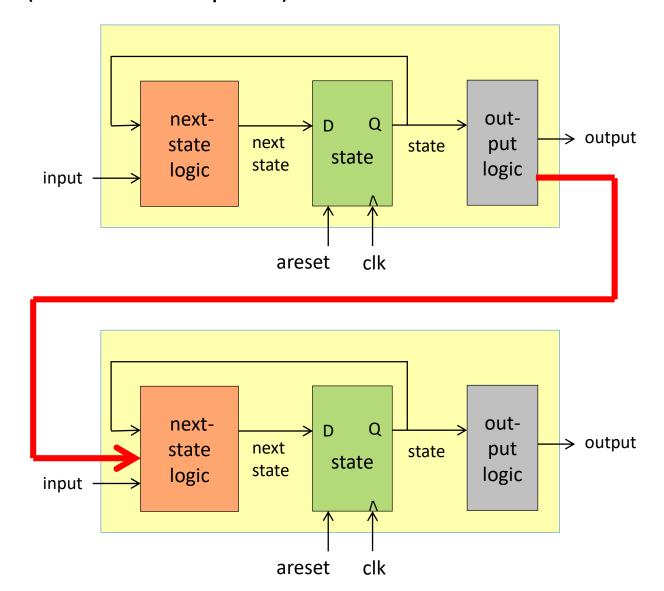
We can combine machines

 Combining Moore Machines causes no problem. We get another Moore Machine.

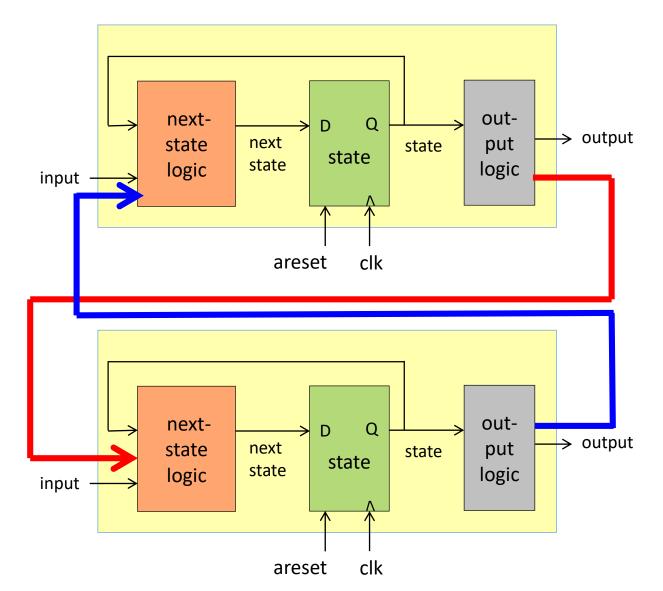
• Combining a Moore Machine with a Mealy Machine causes also no problem. We get a Moore Machine or a Mealy Machine.

 Combining two Mealy Machines can cause troubles: One needs to avoid combinational loops!

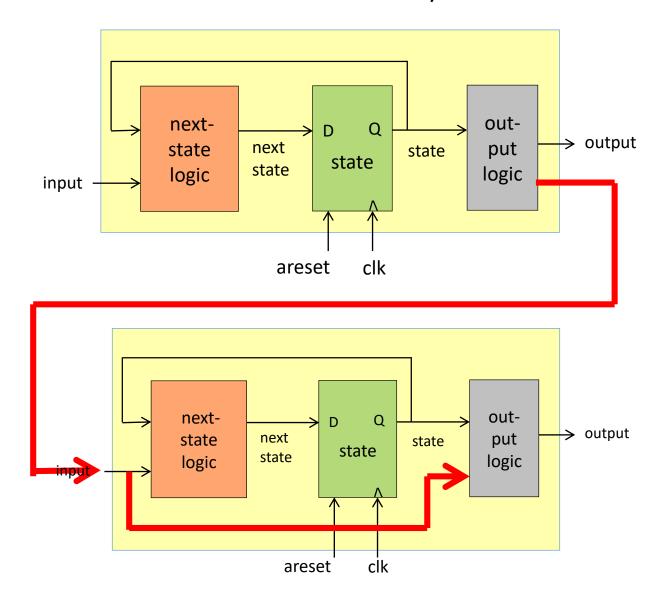
The combination of two Moore Machines creates again a (more complex) Moore Machine



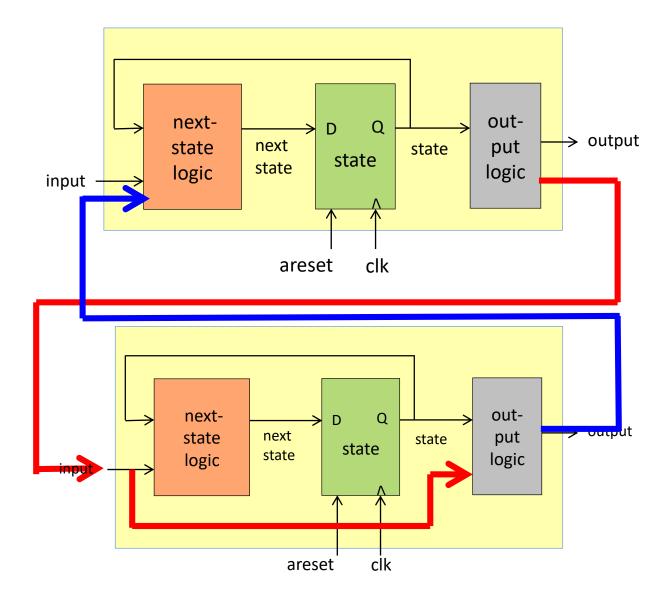
We can even connect More Machines in a loop-like fashion



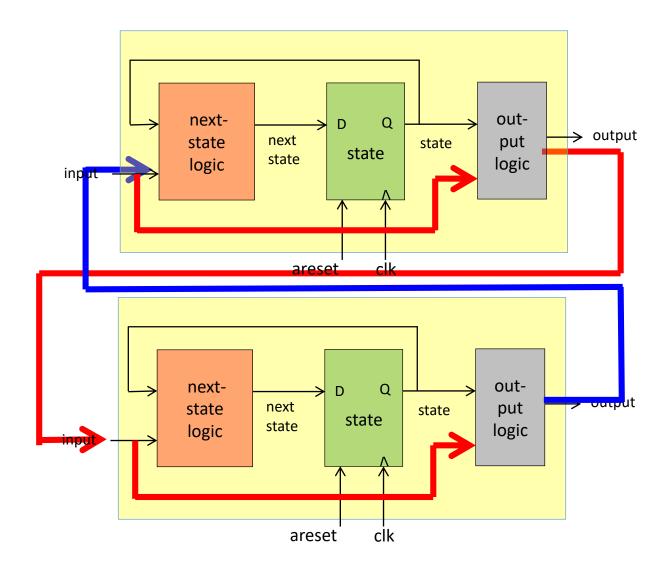
The combination of a Moore Machine with a Mealy Machine creates a Moore Machine or a Mealy Machine



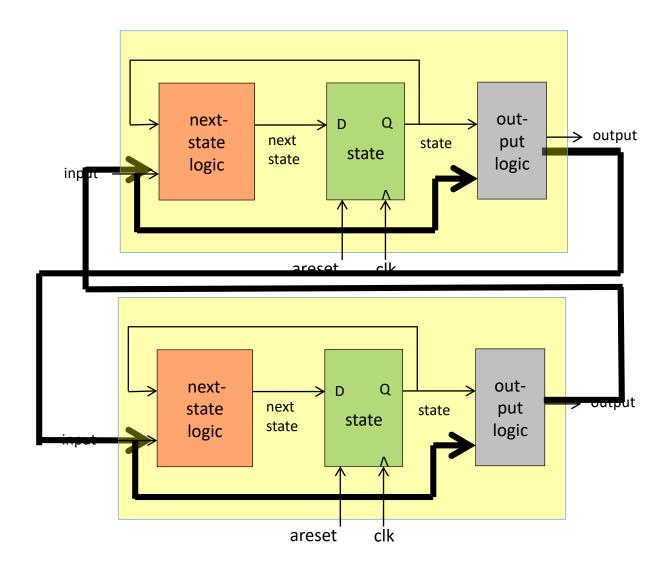
The combination of a Moore Machine with a Mealy Machine creates a Moore Machine or a Mealy Machine



The combination of two Mealy Machines is "dangerous": You need to avoid "combinational loops"



The combination of two Mealy Machines is "dangerous": You need to avoid "combinational loops"



Summary

 All digital logic can in principle be built with Moore Machines and Mealy Machines.

You always start by defining the function with a state diagram.

• If you choose values for the input signal(s), then you can derive the timing diagram by using the state diagram.

• From a state diagram, you can always derive a circuit diagram.

Algorithmic State Machines

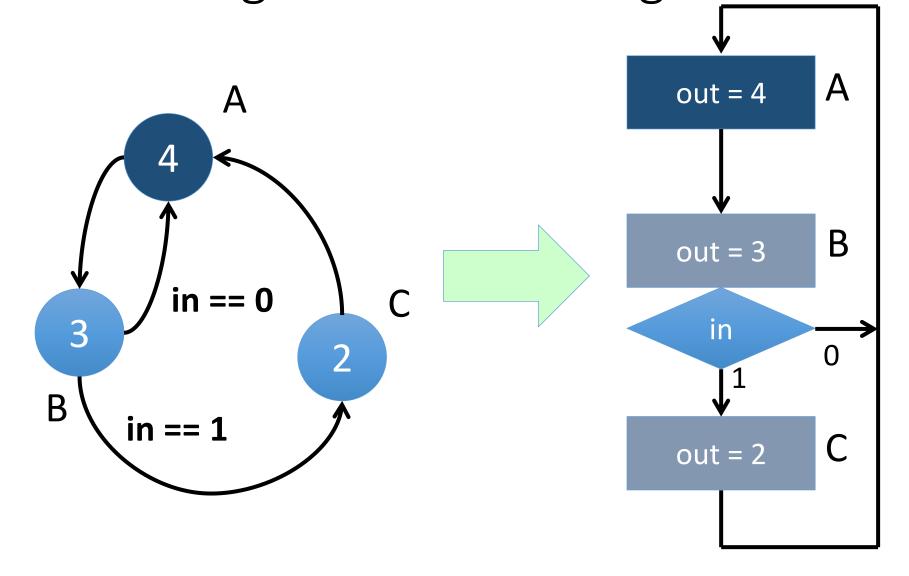
Algorithmic State Machines (ASMs)

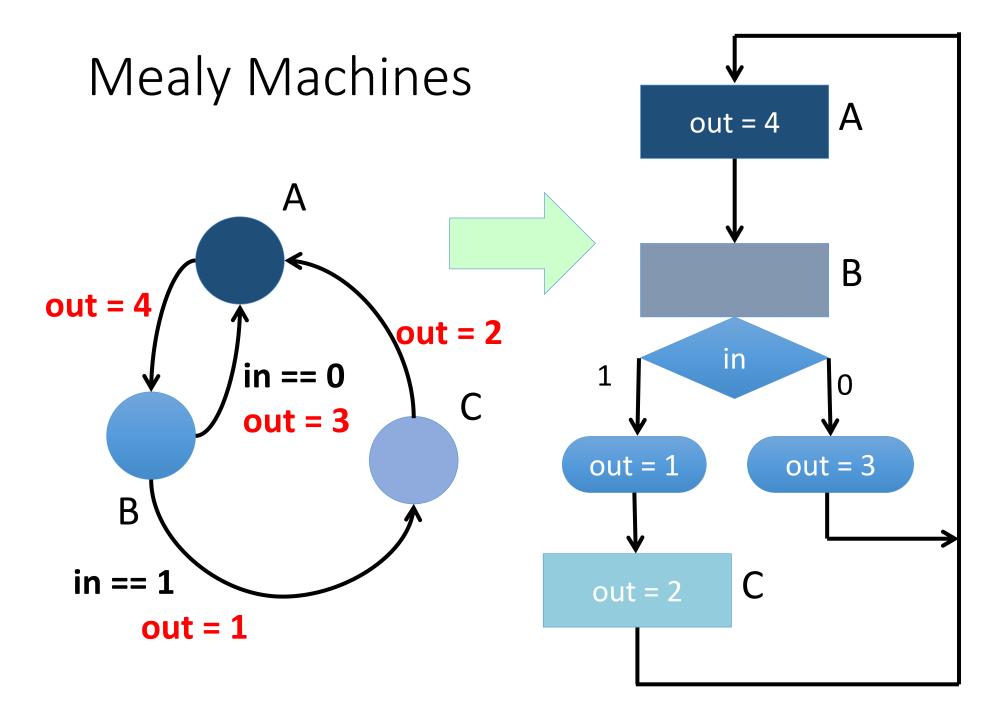
ASMs are a useful extension to finite state machines

 ASMs allow to specify a system consisting of a data path together with its control logic

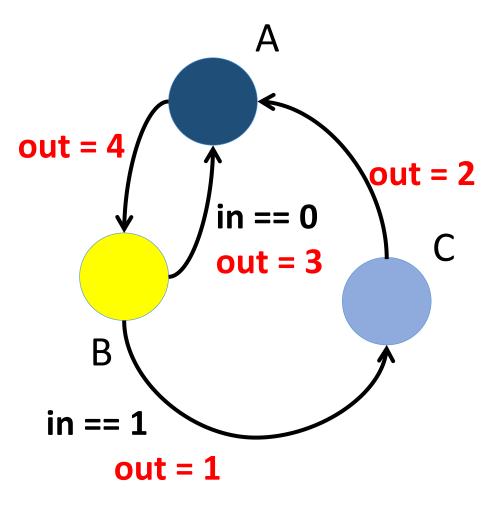
All FSM state diagrams have an equivalent ASM diagram

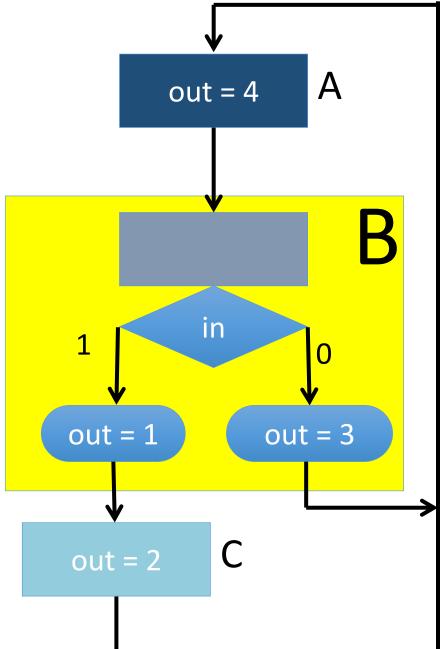
FSM state diagram -> ASM diagrams





Mealy Machines

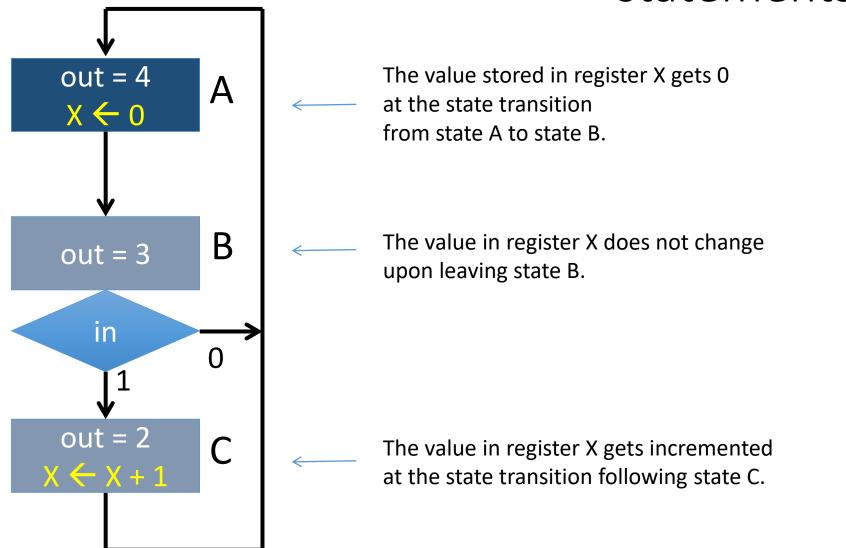




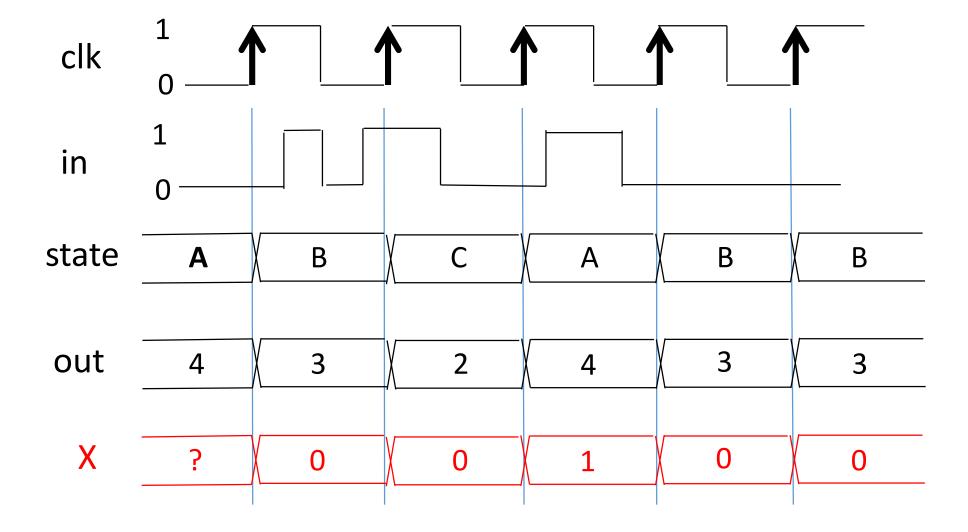
Register-Transfer Statements

- Register-transfer statements define the change of a value stored in a register.
- Values in registers can only change at the active (= rising) edge of clock.
- We denote "register-transfer statements" with a "left arrow" ("←")
- Example: "a \leftarrow x" means that the value in the register "a" gets the value of "x" at the "next" active (= rising) edge of clock.
- We can specify register-transfer statement in an ASM diagram.

ASM diagram with two register-transfer statements



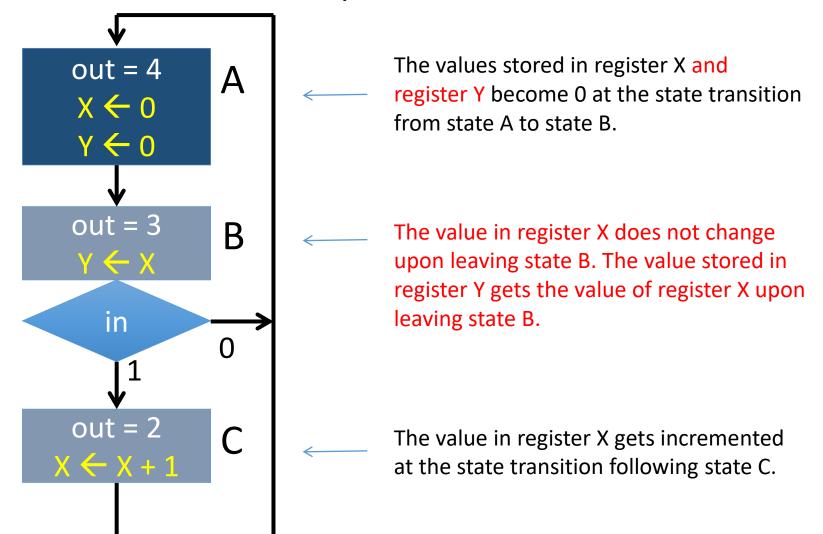
Timing diagram



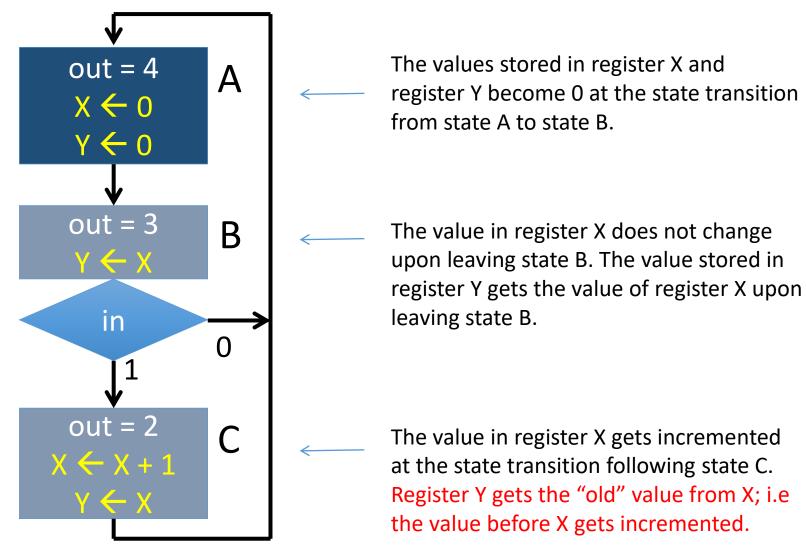
• With the equal sign ("=") we denote that the output of the FSM has a certain value during a particular state.

• With the left-arrow ("←") we denote a register-transfer statement: The register value left of the arrow changes to whatever is defined right of the arrow upon the next active (= rising) edge of clock.

Several register-transfer statements can be specified within one state

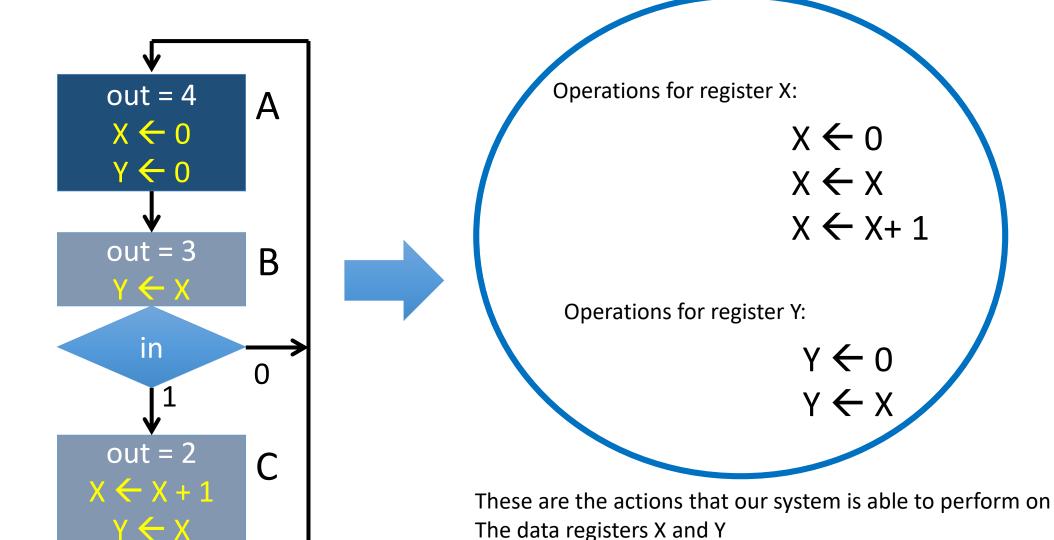


Several register-transfer statements can be specified within one state



Separating Control and Data Path

Register-Transfer Statements Define the Data Path



Control Unit

 State machine generating control signals for the data path



Data Path

- Contains all functional units and registers related to data processing
- Receives control signals to perform operations on the data.
- Provides status signals to the controlrelated data to the control unit

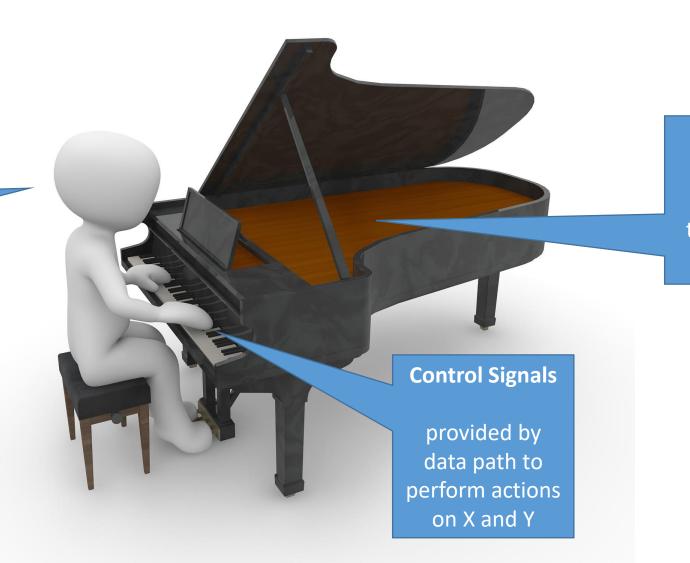


"Piano"

Music

Control

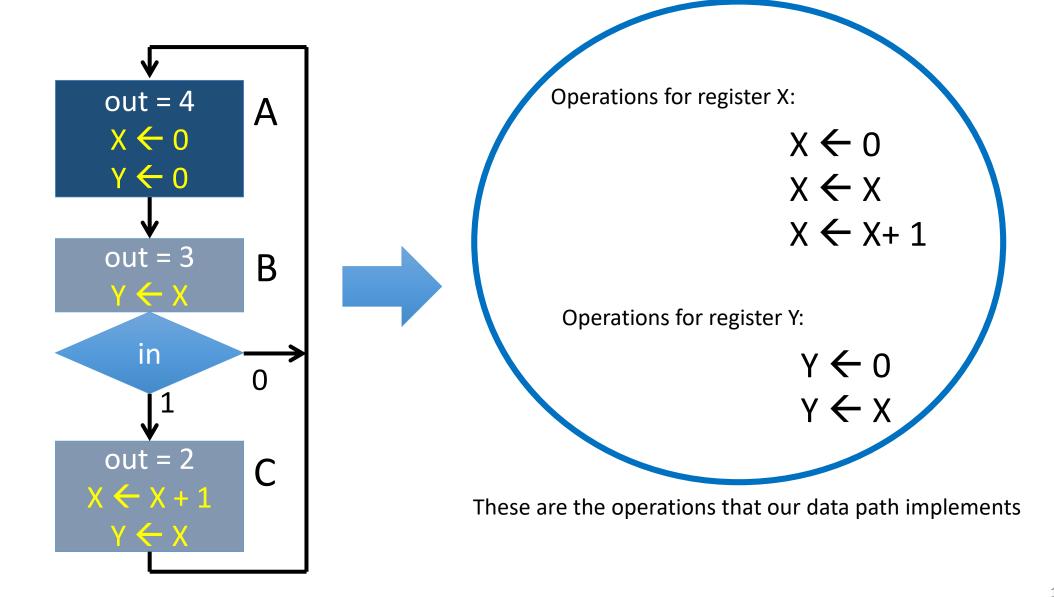
defines when what action should be performed



Data Path

implementing the actions on X, Y

Register-Transfer Statements Define the Data Path



Operations for register X

Case 0: $X \leftarrow X$

Case 1: $X \leftarrow X+1$

Case 2: $X \leftarrow 0$

→ A one bit control signal is not enough. We need two control signals.

We need to distinguish

between 3 cases.

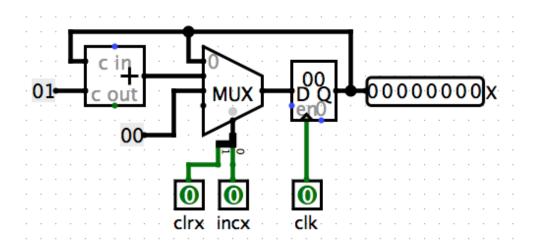
The neighborhood of register X

clrx	incx	action
0	0	$X \leftarrow X$
0	1	$X \leftarrow X+1$
1	0	$X \leftarrow 0$

We use binary notation and name the two binary select variables.

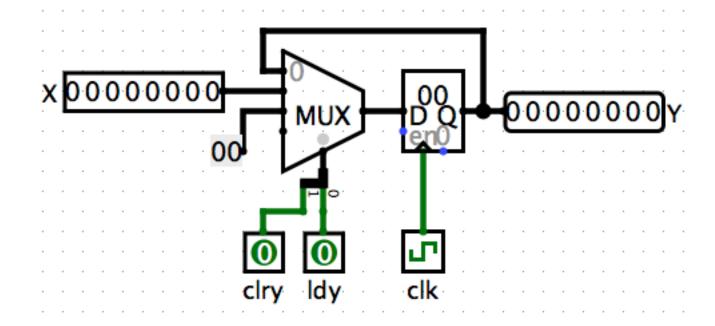
The neighborhood of register X

clrx	incx	action	
0	0	$X \leftarrow X$	
0	1	$X \leftarrow X+1$	With Logisim we can model the neighborhood of
1	0	$X \leftarrow 0$	Register and also simulate.

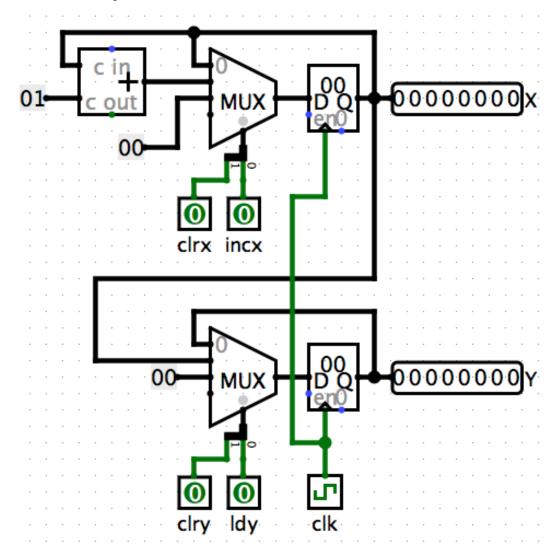


The neighborhood of register Y

clry	ldy	action	
0	0	$Y \leftarrow Y$	In a similar way wa san
0	1	$Y \leftarrow X$	In a similar way, we can model the neighborhood
1	0	$Y \leftarrow 0$	of Y.



The datapath



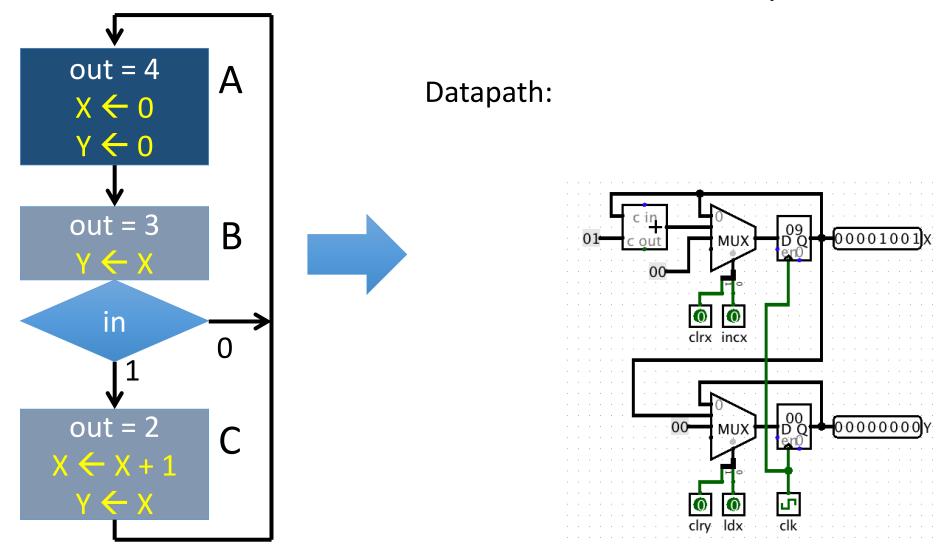
We combine the two neighborhoods.

Note that both neighborhoods are Moore machines.

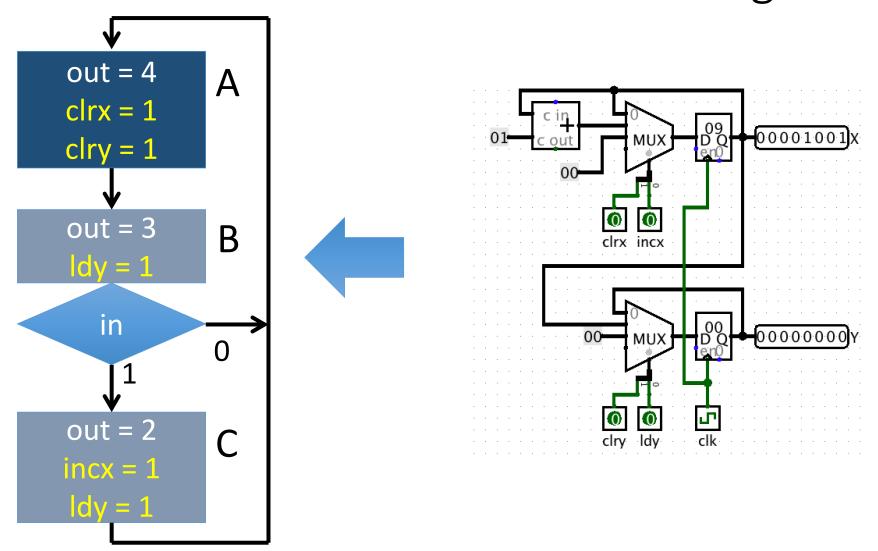
The Moore machine for X has 2 inputs: clrx and incx. Since we have chosen an 8-bit register for X, we have 256 possible states. The output function is the identity function.

The connection of the two is again a Moore machine. Thus, The datapath is a Moore machine.

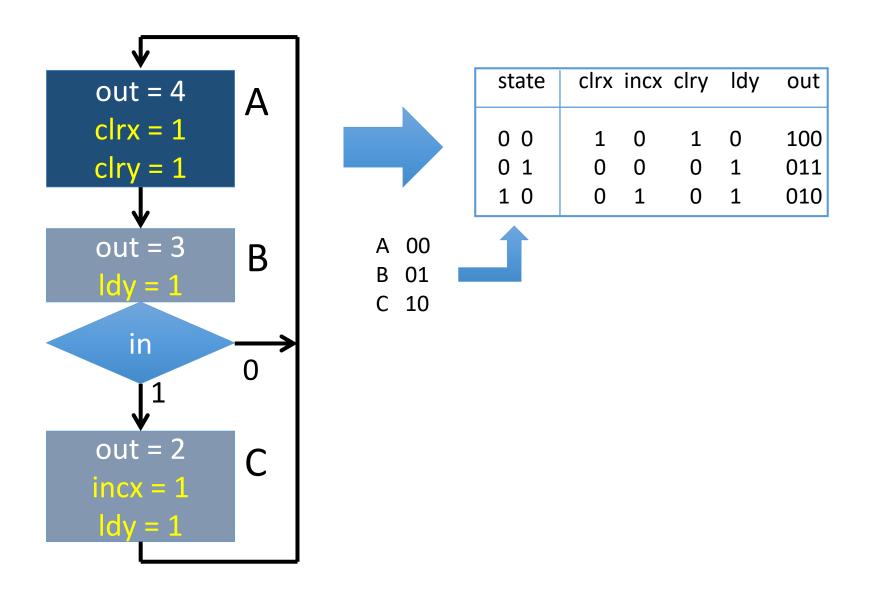
Register-transfer statements define the data path



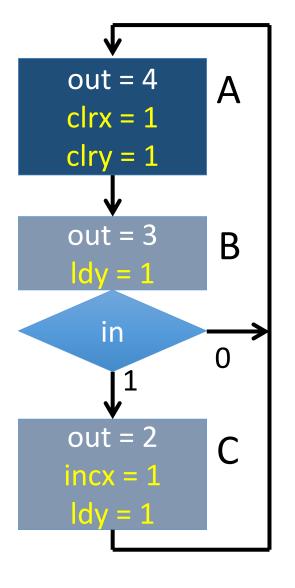
The control logic needs to provide the control signals



Truth table of output logic of controller



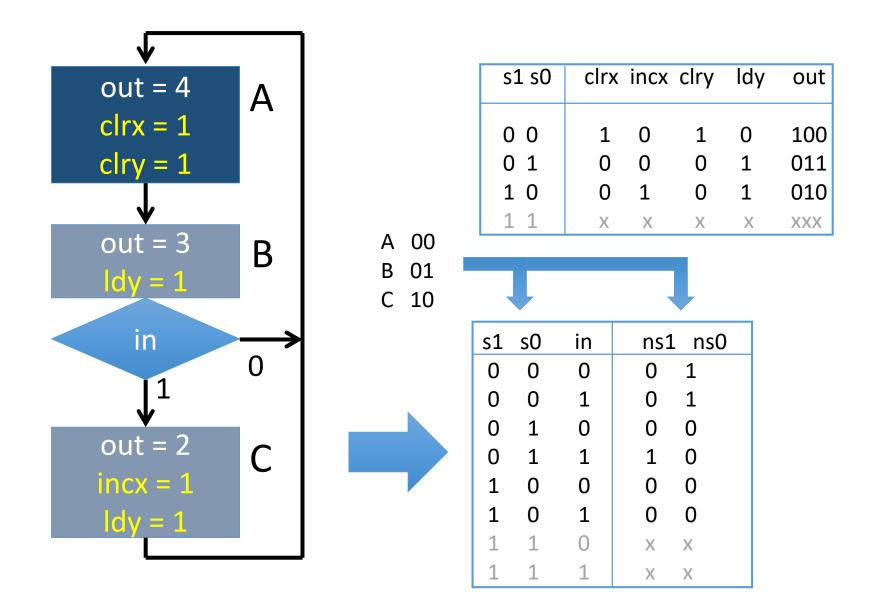
Truth table of next-state logic of controller



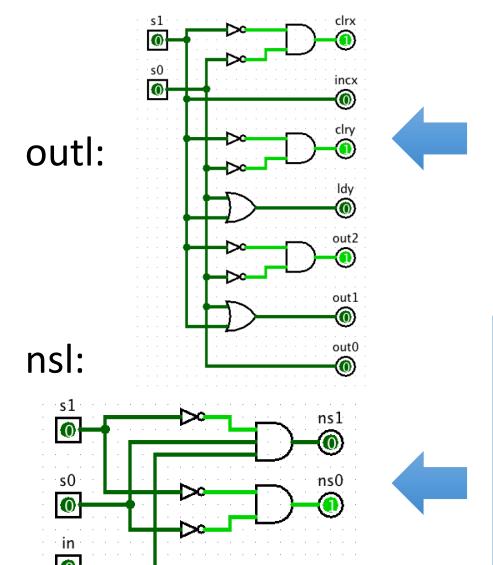
state	clrx	incx	clry	ldy	out
0 0	1	0	1	0	100
0 1	0	0	0	1	011
1 0	0	1	0	1	010

state	in	next_state
Α	0	В
Α	1	В
В	0	А
В	1	С
С	0	А
С	1	Α

Truth table of next-state logic of controller



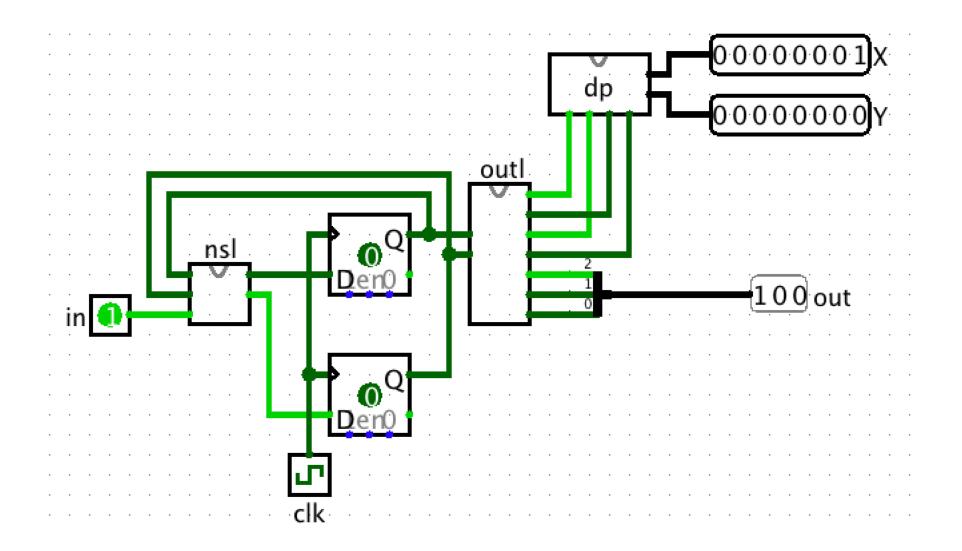
From truth table to implementation



s1 s0	clrx	incx	clry	ldy	out
0 0	1	0	1	0	100
0 1	0	0	0	1	011
1 0	0	1	0	1	010
1 1	Χ	X	X	Χ	XXX

s1	s0	in	ns1 ns0
0	0	0	0 1
0	0	1	0 1
0	1	0	0 0
0	1	1	1 0
1	0	0	0 0
1	0	1	0 0
1	1	0	хх
1	1	1	ХХ

The controller and the data path



SystemVerilog

(Watch the tutorials for the practicals for details)

System Verilog coding style

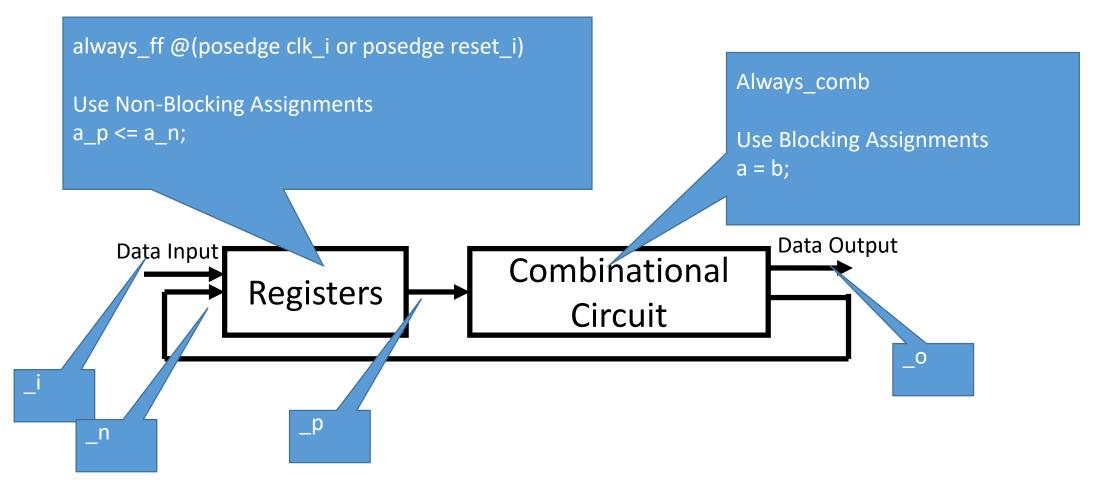
- Suffix _o for module outputs, _i for module inputs
- Register variables with suffix _p for previous and _n for next value
- Array range with [MSB:LSB], like e.g. [31:0]
- Clocked processes use non-blocking (<=) others use blocking assignments (=)
- Clocked processes only update registers, everything else has to be done in combinational blocks
- Filename corresponds to module name: module MyDesign in file mydesign.sv
- Module instantiation always with named assignments (.A(C))

With significant implications beyond style:

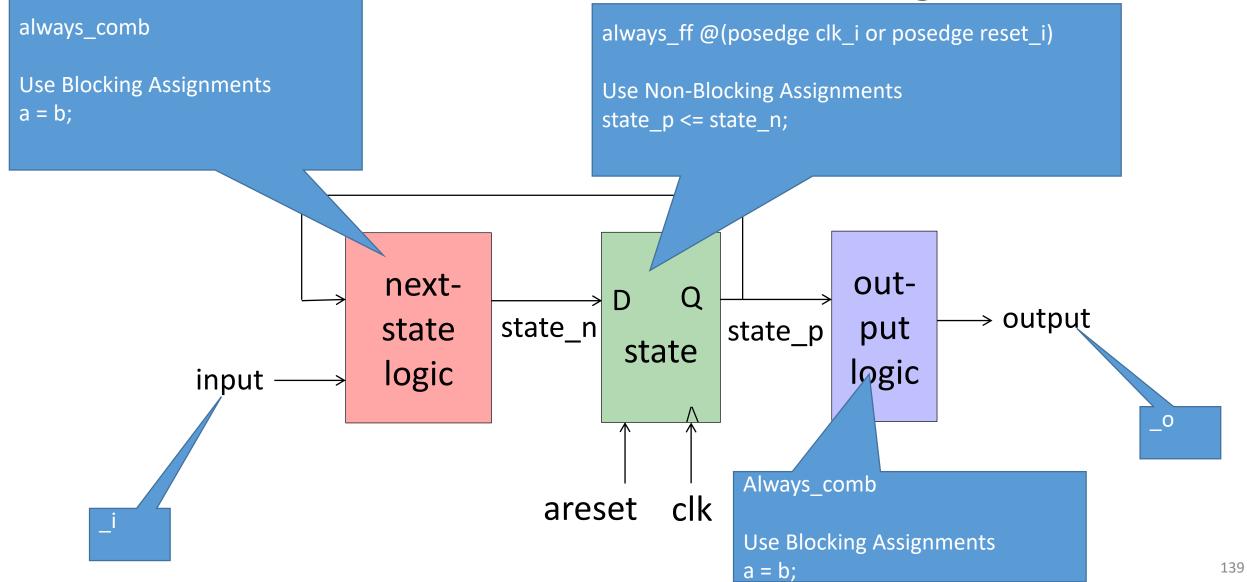
- Always use default assignments (e.g. state_n = state_p)
- Always use default branches (default:) in case statements

→ If you do not assign the output of a combinational block for all input conditions, latches are created for data storage!

Overview of Coding Guidelines



Essence of Moore Machines (Verilog)



System Verilog Examples

All FSMs of this lecture are available in SystemVerilog as well