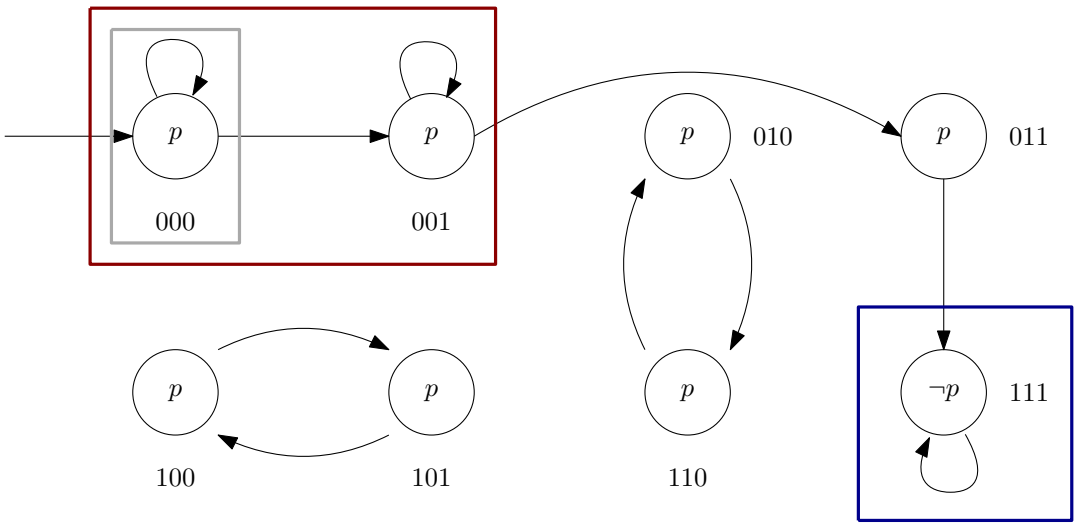
Model checking HW3 - Solution and common mistakes

Step: 1, $k = 1$



$Q = \{000\}$

$A = Q(s_0) \wedge R(s_0, s_1)$

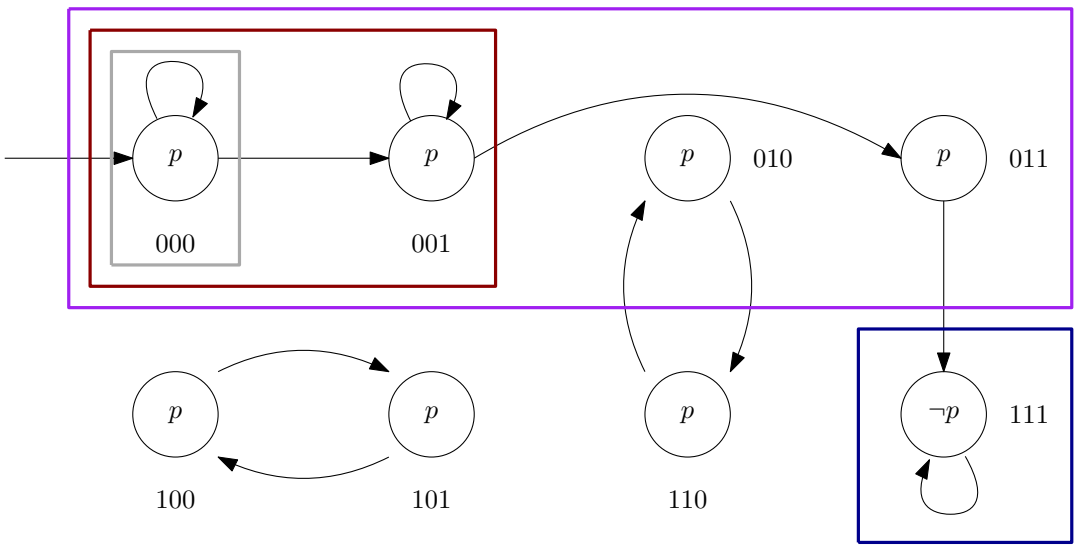
$A = \text{post}(Q) = \{000, 001\}$

$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{111\}$

Model checking HW3 - Solution and common mistakes

Step: 1, $k = 1$



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$B = \{111\}$

$\phi = A \wedge B \rightarrow \text{UNSAT}$

$I = \text{post}(Q) = \neg x_1 \wedge \neg x_2$

- Eliminate x_3? (already not present)
  $\neg x_1 \wedge \neg x_2 \cap B = \emptyset \longrightarrow \checkmark$
- Eliminate x_2?
  $\neg x_1 \cap B = \emptyset \longrightarrow \checkmark$
- Eliminate x_1?
  $\top \cap B \neq \emptyset \longrightarrow \times$

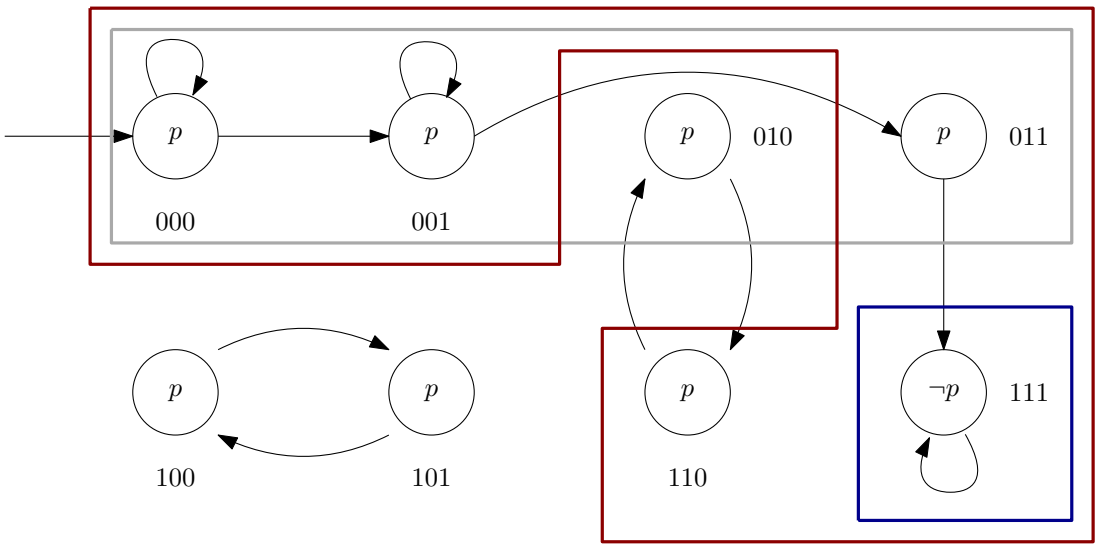
$I = \neg x_1$

$I(s_0) \subseteq Q? \longrightarrow \times$

$Q := Q \vee I$

Model checking HW3 - Solution and common mistakes

Step: 2, $k = 1$



$Q = \{000, 001, 010, 011\}$

$A = Q(s_0) \wedge R(s_0, s_1)$

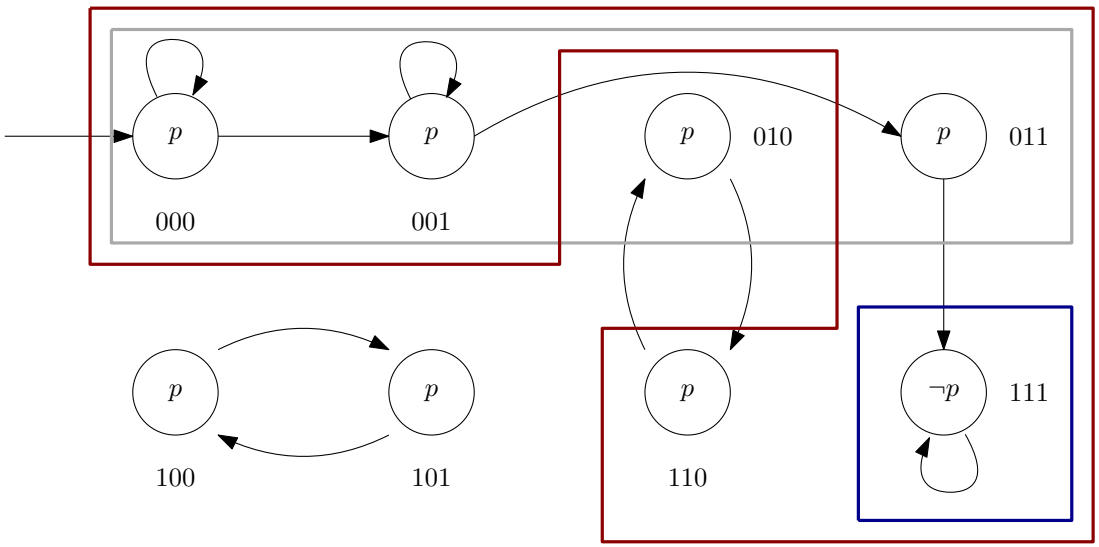
$A = \text{post}(Q) = \{000, 001, 011, 110, 111\}$

$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{111\}$

Model checking HW3 - Solution and common mistakes

Step: 2, $k = 1$



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$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{111\}$

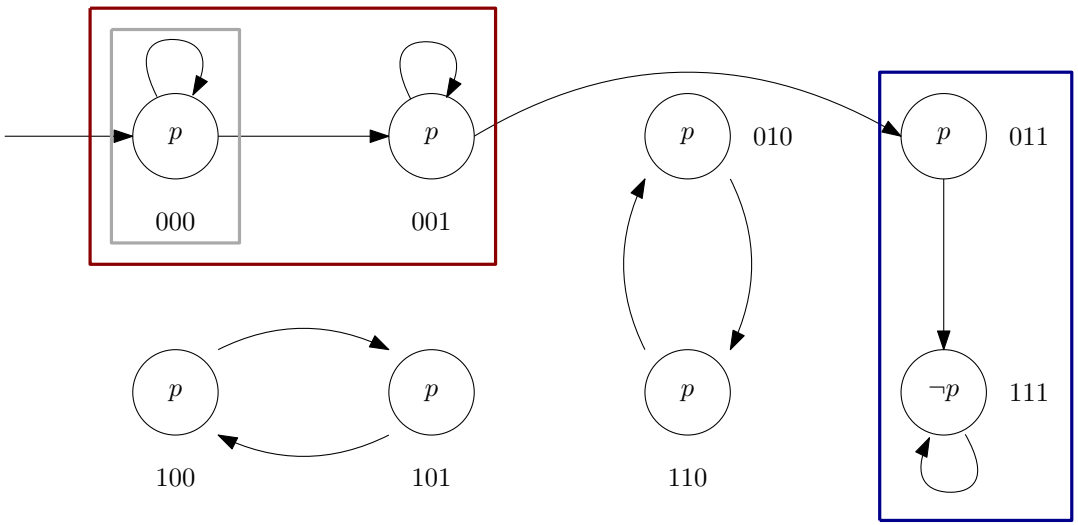
$\phi = A \wedge B \rightarrow \text{SAT}$

$Q = S_0? \longrightarrow$ ✗

$k := k + 1$

Model checking HW3 - Solution and common mistakes

Step: 3, $k = 2$



$Q = \{000\}$

$A = Q(s_0) \wedge R(s_0, s_1)$

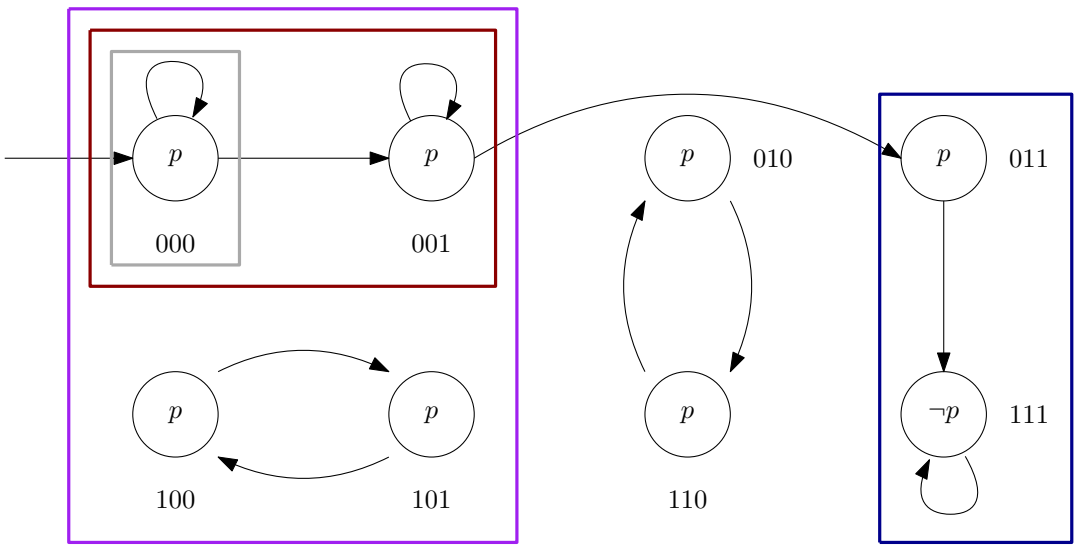
$A = \text{post}(Q) = \{000, 001\}$

$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{011, 111\}$

Model checking HW3 - Solution and common mistakes

Step: 3, $k = 2$



$Q = \{000\}$

$A = Q(s_0) \wedge R(s_0, s_1)$

$A = \text{post}(Q) = \{000, 001\}$

$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{011, 111\}$

$\phi = A \wedge B \rightarrow \text{UNSAT}$

$I = \text{post}(Q) = \neg x_1 \wedge \neg x_2$

- Eliminate x_3? (already not present)
  $\neg x_1 \wedge \neg x_2 \cap B = \emptyset \longrightarrow$ ✓
- Eliminate x_2?
  $\neg x_1 \cap B \neq \emptyset \longrightarrow$ ✗
- Eliminate x_1?
  $\neg x_2 \cap B = \emptyset \longrightarrow$ ✓

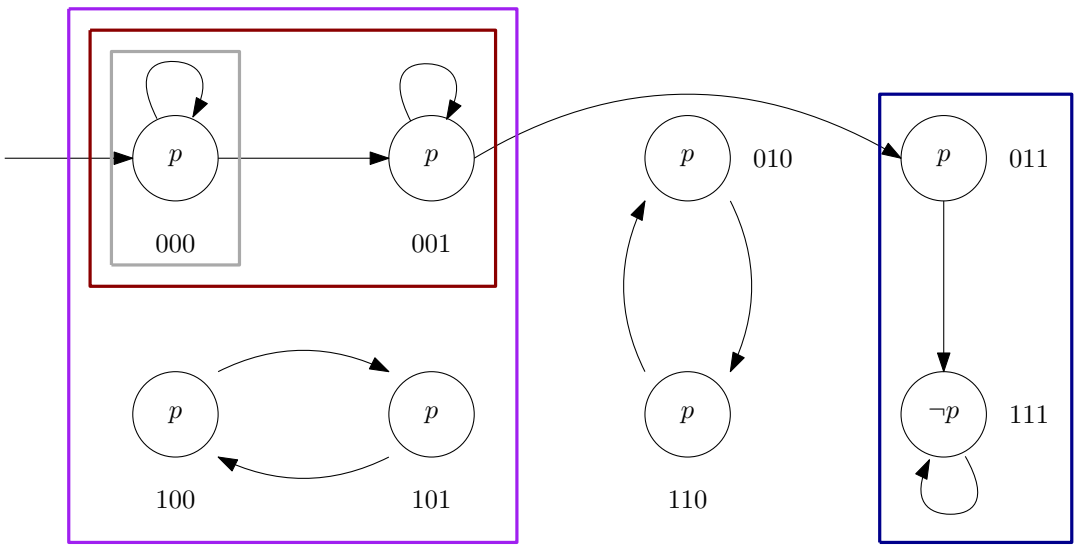
$I = \neg x_2$

$I(s_0) \subseteq Q? \longrightarrow$ ✗

$Q := Q \vee I$

Model checking HW3 - Solution and common mistakes

Step: 3, $k = 2$



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$A = Q(s_0) \wedge R(s_0, s_1)$

$A = \text{post}(Q) = \{000, 001\}$

$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{011, 111\}$

$\phi = A \wedge B \rightarrow$ UNSAT

$I = \text{post}(Q) = \neg x_1 \wedge \neg x_2$

- Eliminate x_3? (already not present)
  $\neg x_1 \wedge \neg x_2 \cap B = \emptyset \longrightarrow$ ✓
- Eliminate x_2?
  $\neg x_1 \cap B \neq \emptyset \longrightarrow$ ✗
- Eliminate x_1?
  $\neg x_2 \cap B = \emptyset \longrightarrow$ ✓

$I = \neg x_2$

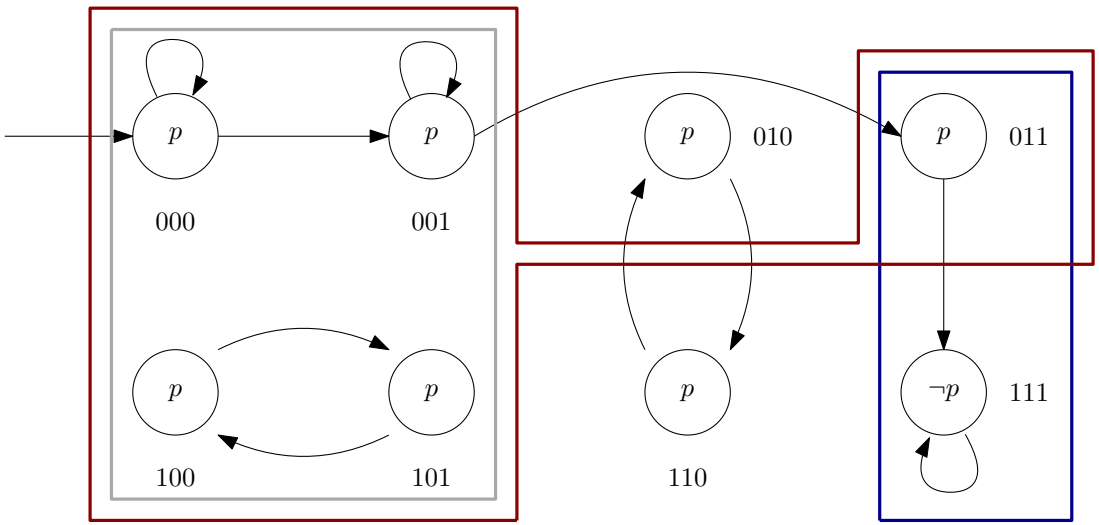
$I(s_0) \subseteq Q? \longrightarrow$ ✗

$Q := Q \vee I$

Note: Some students stop here, don't check whether x_1 can be eliminated or not. In that case, the interpolant you get is $I = \neg x_1 \wedge \neg x_2$. This is also an interpolant, albeit smaller than it could have been.

Model checking HW3 - Solution and common mistakes

Step: 4, $k = 2$



$Q = \{000, 001, 100, 101\}$

$A = Q(s_0) \wedge R(s_0, s_1)$

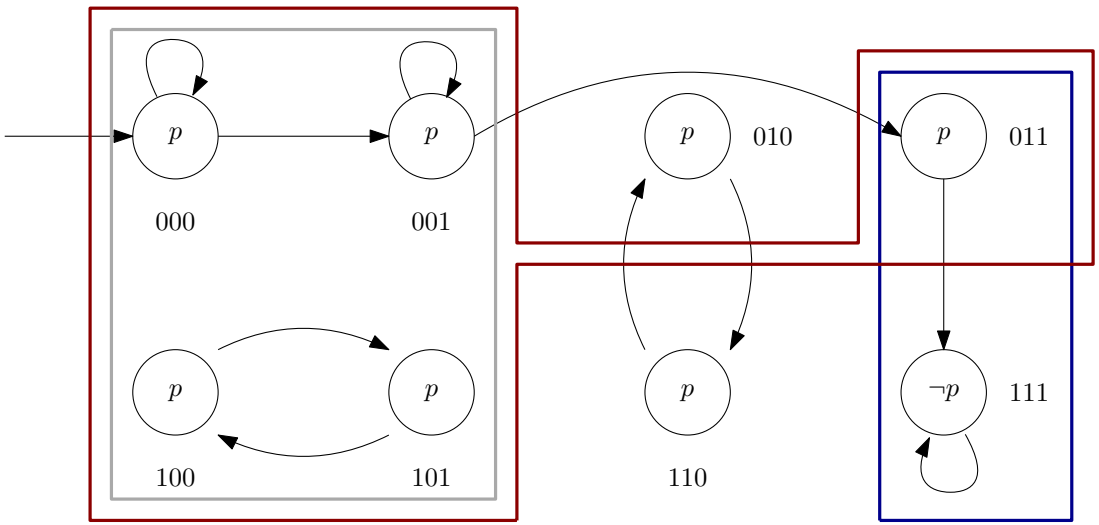
$A = \text{post}(Q) = \{000, 001, 100, 101, 011\}$

$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{011, 111\}$

Model checking HW3 - Solution and common mistakes

Step: 4, $k = 2$



$Q = \{000, 001, 100, 101\}$

$A = Q(s_0) \wedge R(s_0, s_1)$

$A = \text{post}(Q) = \{000, 001, 100, 101, 011\}$

$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{011, 111\}$

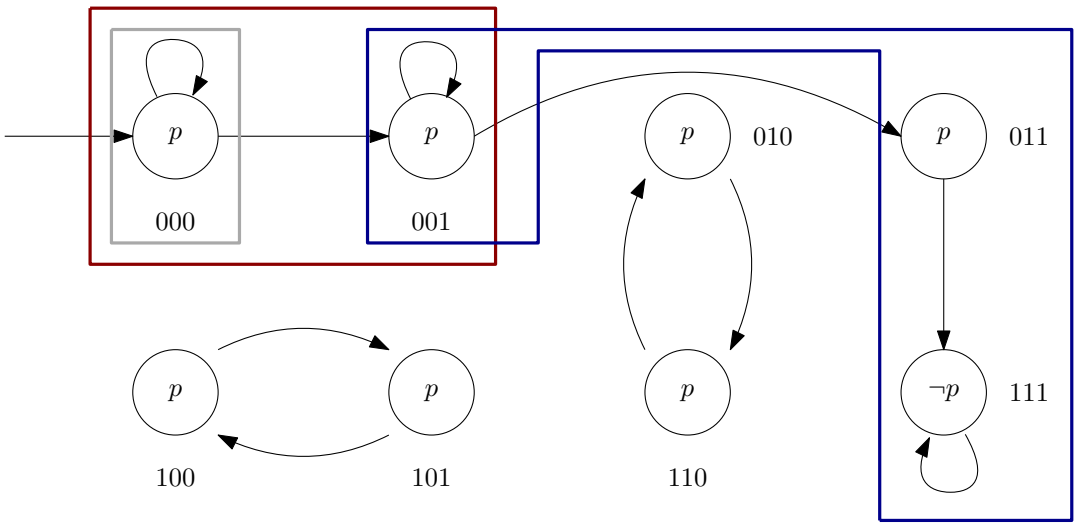
$\phi = A \wedge B \rightarrow \text{SAT}$

$Q = S_0? \longrightarrow$ ✗

$k := k + 1$

Model checking HW3 - Solution and common mistakes

Step: 5, $k = 3$



$Q = \{000\}$

$A = Q(s_0) \wedge R(s_0, s_1)$

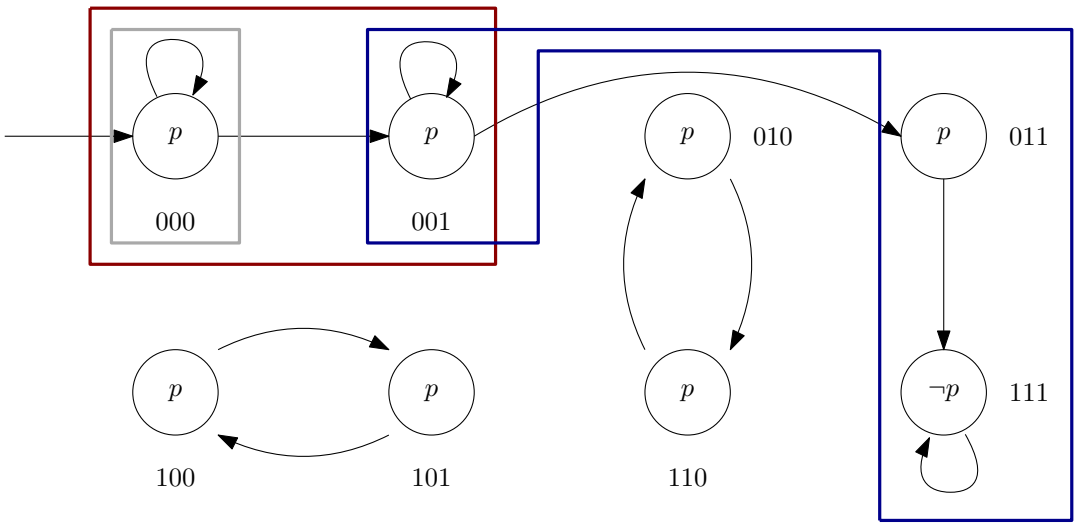
$A = \text{post}(Q) = \{000, 001\}$

$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{001, 011, 111\}$

Model checking HW3 - Solution and common mistakes

Step: 5, $k = 3$



$Q = \{000\}$

$A = Q(s_0) \wedge R(s_0, s_1)$

$A = \text{post}(Q) = \{000, 001\}$

$B = \bigwedge_{i=1}^{k-1} R(s_i, s_{i+1}) \wedge \bigwedge_{i=1}^{k} \neg p(s_i)$

$B = \{001, 011, 111\}$

$\phi = A \wedge B \rightarrow \text{SAT}$

$Q = S_0? \longrightarrow \checkmark$

`return` $M \not\models AG\, p$

Note: The algorithm finishes here. There is no need to continue to $k = 4$.