Linear Temporal Logic

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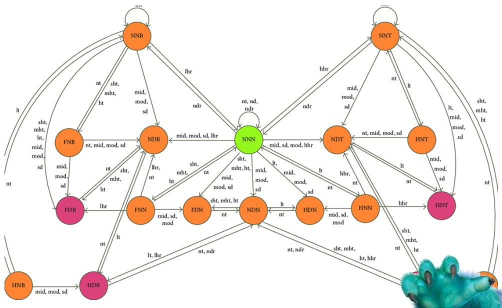
2020-03-31

Formal Methods for System Verification



Logical Formulas

Formal Specification

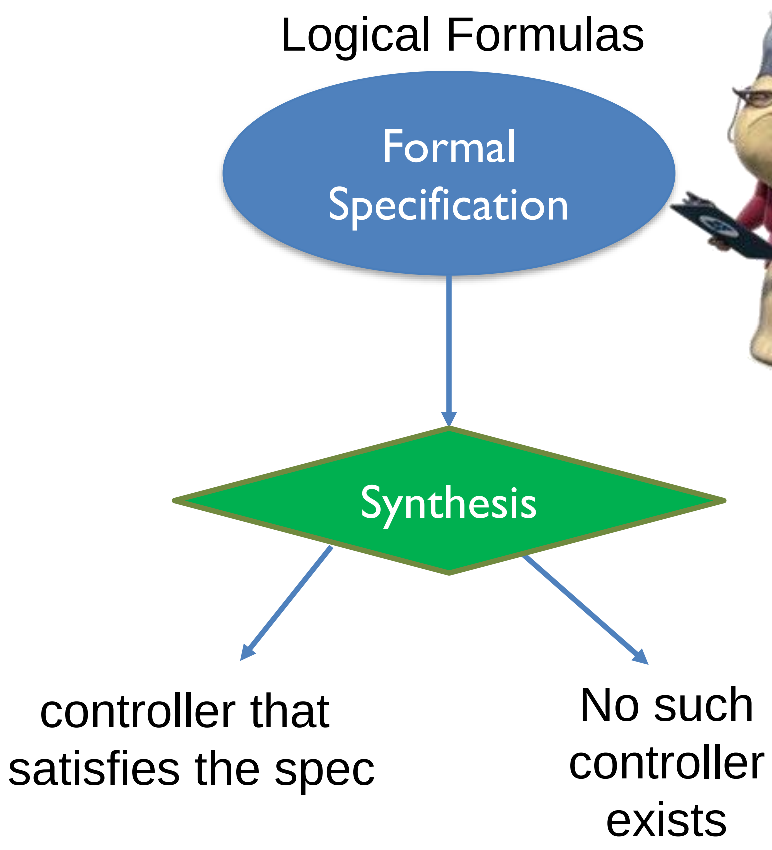
System (Model)

Verification

satisfied
(+ certificate)

violated
(+ counterexample)

Formal Methods for System Verification



Temporal Logic in Verification est. 1977

THE TEMPORAL LOGIC OF PROGRAMS*

Amir Pnueli

Summary:

A unified approach to program verification is suggested, which applies to both sequential and parallel programs. The main proof method suggested is that of temporal reasoning in which the time dependence of events is the basic concept. Two formal systems are presented for providing a basis for temporal reasoning. One forms a formalization of the method of intermittent assertions, while the other is an adaptation of the tense logic system K_b, and is particularly suitable for reasoning about concurrent programs.

Reasoning about Software Systems

- Variables
  - Atomic propositions: p, r, g, i, n
- State
  - An assignment
- Formula
  - Set of states,
  - The set of assignments satisfying it

Reasoning about Software Systems

- Variables
  - Atomic propositions: p, r, g, i, n

- State
  - An assignment

- Formula
  - Set of states,
  - The set of a … ying it

FORMULA?

Linear Temporal Logic (LTL)

- In LTL time is
  - implicit,
  - discrete,
  - has a beginning,
  - runs to infinity.

- The model of an LTL formula is
  - infinite sequence of states: $\pi: s_0, s_1, s_2, \ldots$

LTL: Syntax

- Elements:
  - Atomic propositions: p, r, g, i, n
  - Boolean Operators: $\wedge \vee \neg \rightarrow$
  - Temporal Operators: $\mathbf{G\ F\ X\ U\ R}$

$$\phi \coloneqq (\phi) \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid$$
$$\phi\ \mathbf{U}\ \phi \mid \phi\ \mathbf{R}\ \phi \mid \mathbf{G}\phi \mid \mathbf{F}\phi \mid \mathbf{X}\phi \mid p$$

LTL: Semantic

Formula	Meaning
ϕ	$\phi \to ? \to ? \to \ldots \to ? \to ? \to \ldots$
$\mathbf{X}(\phi)$	$? \to \phi \to ? \to \ldots \to ? \to ? \to \ldots$
$\mathbf{G}(\phi)$	$\phi \to \phi \to \phi \to \ldots \to \phi \to \phi \to \ldots$
$\mathbf{F}(\phi)$	$? \to ? \to ? \to \ldots \to ? \to \phi \to \ldots$
$\phi\ \mathbf{U}\ \psi$	$\phi \to \phi \to \phi \to \ldots \to \psi \to ? \to \ldots$
$\phi\ \mathbf{R}\ \psi$	$\psi \to \psi \to \psi \to \ldots \to \psi,\phi \to ? \to \ldots$
	$\psi \to \psi \to \psi \to \ldots \to \psi \to \psi \to \ldots$

LTL: Semantic

Formula	**Meaning**
ϕ	ϕ holds in begging of time
$\mathbf{X}(\phi)$	ϕ holds in next time step
$\mathbf{G}(\phi)$	ϕ holds in all time steps
$\mathbf{F}(\phi)$	ϕ holds in (at least) one future time step
$\phi\ \mathbf{U}\ \psi$	ϕ holds in all time steps, until eventually ψ holds
$\phi\ \mathbf{R}\ \psi$	ψ holds in all time steps up to and including the step ϕ holds, or ψ holds forever

Example: $\phi = \mathbf{X}a$

step	0	1	2	3	4	5	6	7	8	9	ω
a	0	1	0	0	1	1	1	0	1	1	0
$\mathbf{X}a$	1	0	0	1	1	1	0	1	1	0	0

Q1: $\phi = \mathbf{G}(a \rightarrow b)$

step	0	1	2	3	4	5	6	7	8	9	ω
a	0	1	0	0	1	1	1	0	1	1	0
b	1	1	0	0	1	1	0	0	1	0	0
$a \rightarrow b$											
$\mathbf{G}(a \rightarrow b)$											

Q1: $\phi = \mathbf{G}(a \rightarrow b)$

step	0	1	2	3	4	5	6	7	8	9	ω
a	0	1	0	0	1	1	1	0	1	1	0
b	1	1	0	0	1	1	0	0	1	0	0
$a \rightarrow b$	1	1	1	1	1	1	0	1	1	0	1
$\mathbf{G}(a \rightarrow b)$	0	0	0	0	0	0	0	0	0	0	1

Q2: $\phi = \mathbf{GF}p$

step	0	1	2	3	4	5	6	7	8	ω			
p	0	1	0	0	1	1	1	0	1	0	0	1	0
$\mathbf{F}p$													
$\mathbf{GF}p$													

Q2: $\phi = \mathbf{GF}p$

step	0	1	2	3	4	5	6	7	8	ω			
p	0	1	0	0	1	1	1	0	1	0	0	1	0
$\mathbf{F}p$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\mathbf{GF}p$	1	1	1	1	1	1	1	1	1	1	1	1	1

Q3: $\phi = \mathbf{G}(a \rightarrow \mathbf{X}b \vee \mathbf{F}c)$

step	0	1	2	3	4	5	6	7	8	9	ω
a	0	1	0	0	1	1	1	0	1	1	0
b	1	1	0	0	1	1	0	0	1	0	0
c	1	0	0	1	1	0	0	0	0	0	0
$\mathbf{X}b$											
$\mathbf{F}c$											
$\mathbf{X}b \vee \mathbf{F}c$											
$a \rightarrow \mathbf{X}b \vee \mathbf{F}c$											
ϕ											

Q3: $\phi = \mathbf{G}\big(a \rightarrow \mathbf{X}(b) \vee \mathbf{F}(c)\big)$

step	0	1	2	3	4	5	6	7	8	9	ω
a	0	1	0	0	1	1	1	0	1	1	0
b	1	1	0	0	1	1	0	0	1	0	0
c	1	0	0	1	1	0	0	0	0	0	0
$\mathbf{X}b$	1	0	0	1	1	0	0	1	0	0	0
$\mathbf{F}c$	1	1	1	1	1	0	0	0	0	0	0
$\mathbf{X}b \vee \mathbf{F}c$	1	1	1	1	1	0	0	1	0	0	0
$a \rightarrow \mathbf{X}b \vee \mathbf{F}c$	1	1	1	1	1	0	0	1	0	0	1
ϕ	0	0	0	0	0	0	0	0	0	0	1

Q4: $\phi = a \mathbf{U} b$

step	0	1	2	3	4	5	6	7	8	9	ω
a	1	1	1	0	1	1	1	0	1	1	0
b	1	0	0	1	0	0	0	0	1	0	1
$\phi = a \mathbf{U} b$											

Q4: $\phi = a\ \mathbf{U}\ b$

step	0	1	2	3	4	5	6	7	8	9	ω
a	1	1	1	0	1	1	1	0	1	1	0
b	1	0	0	1	0	0	0	0	1	0	1
$\phi = a\ \mathbf{U}\ b$	1	1	1	1	0	0	0	0	1	1	1

Homework: $\phi = \mathbf{F}(\neg a \land \mathbf{X}(\neg b\ \mathbf{U}\ a))$

step	0	1	2	3	4	5	6	ω
a	0	0	0	0	1	1	1	1
b	0	1	0	0	0	1	1	1
$\neg a$								
$\neg b$								
$\neg b\ \mathbf{U}\ a$								
$\mathbf{X}(\neg b\ \mathbf{U}\ a)$								
$\neg a \land \mathbf{X}(\neg b\ \mathbf{U}\ a)$								
ϕ								

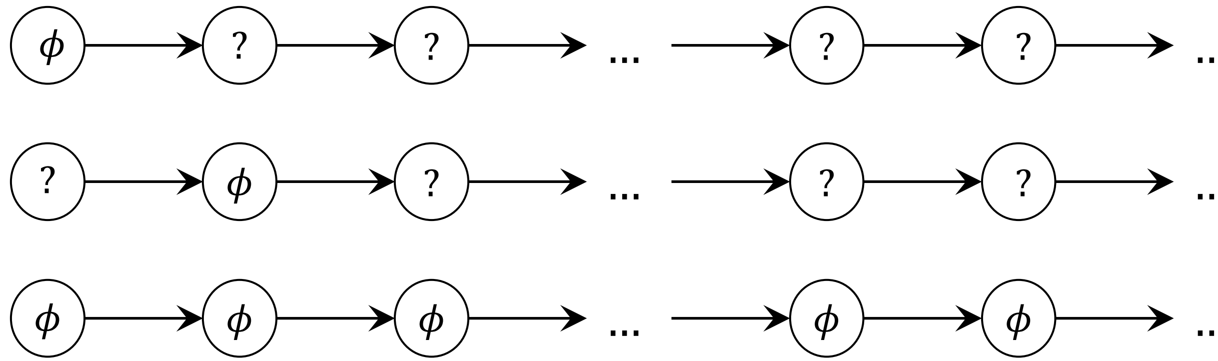
Temporal Expansion: Globally

Formula

ϕ

$\mathbf{X}(\phi)$

Semantic



Temporal Expansion: Globally

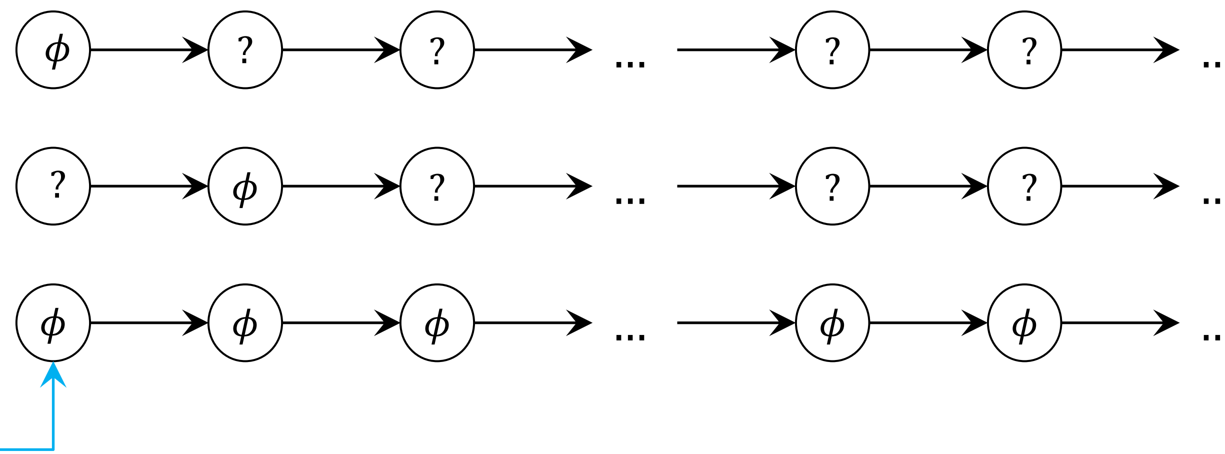
Formula

ϕ

$\mathbf{X}(\phi)$

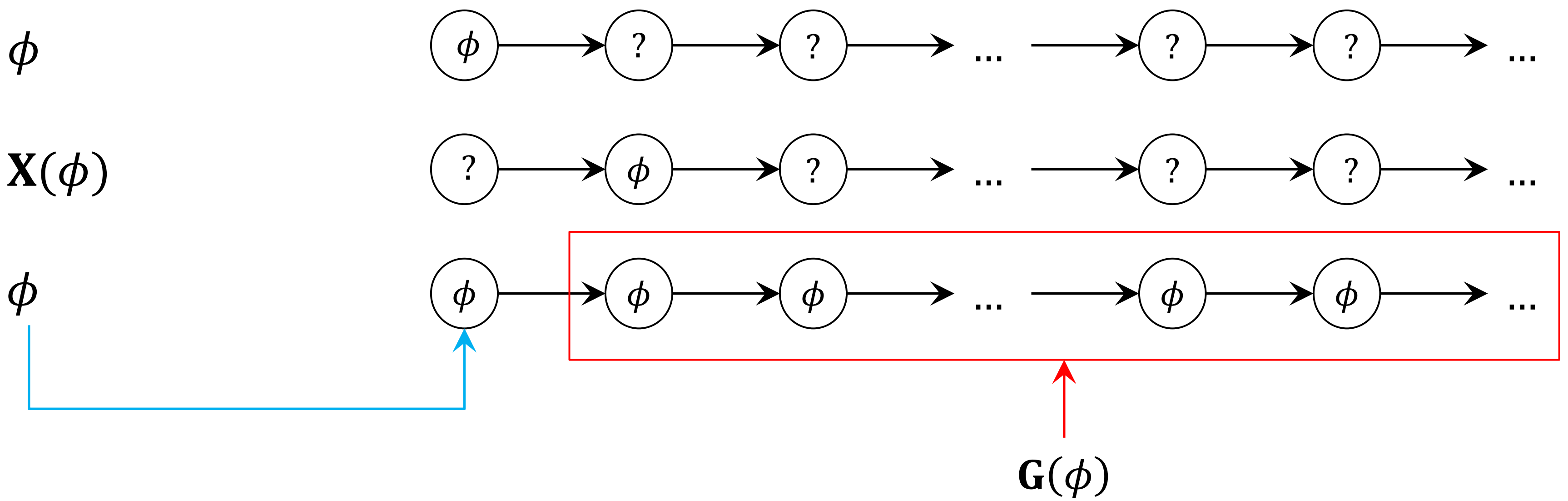
ϕ

Semantic

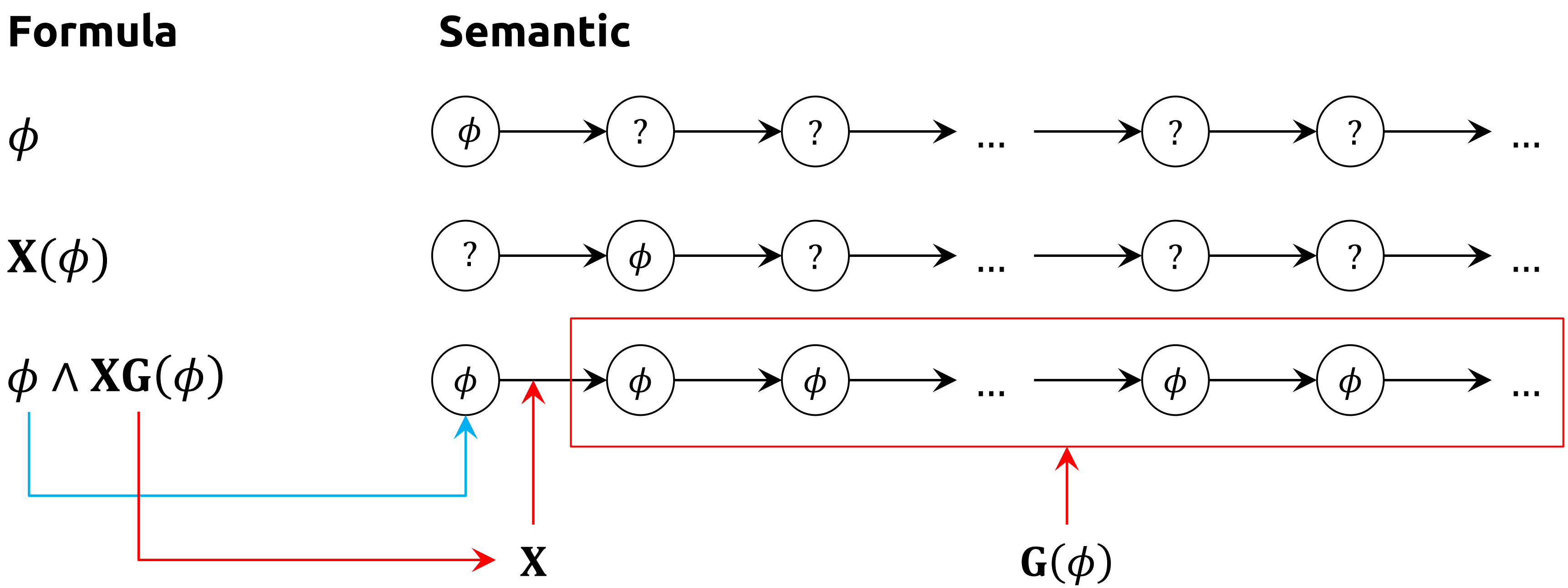


Temporal Expansion: Globally

Formula **Semantic**



Temporal Expansion: Globally



Temporal Expansion: Future

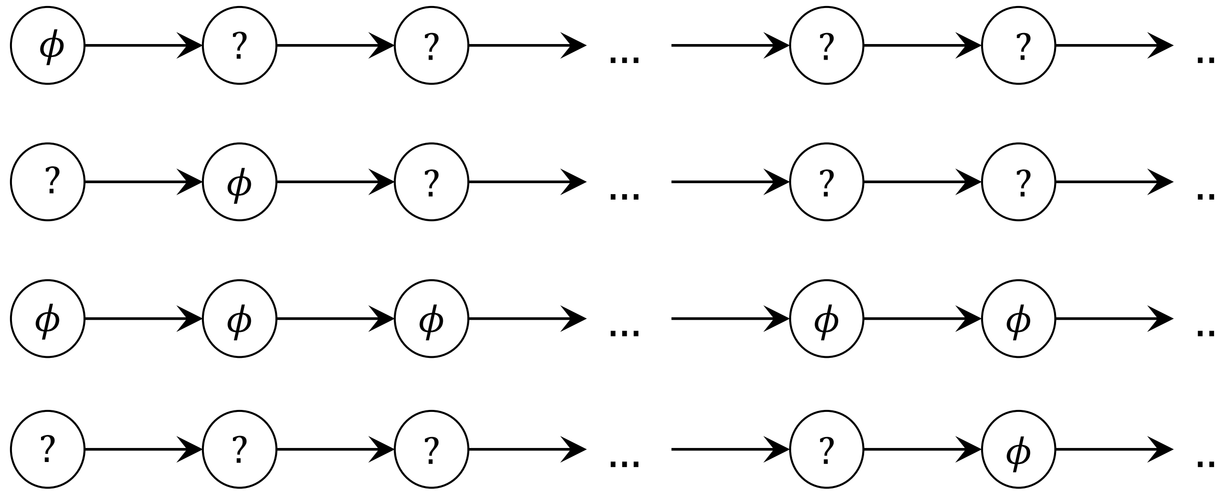
Formula

ϕ

$\mathbf{X}(\phi)$

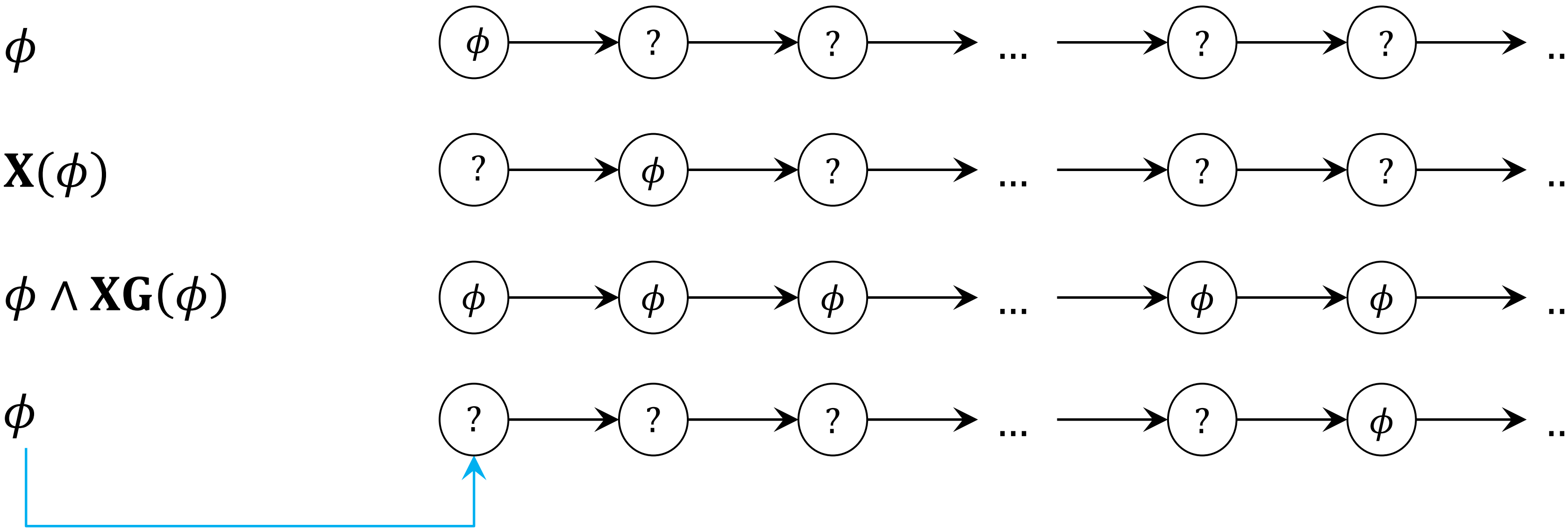
$\phi \land \mathbf{XG}(\phi)$

Semantic



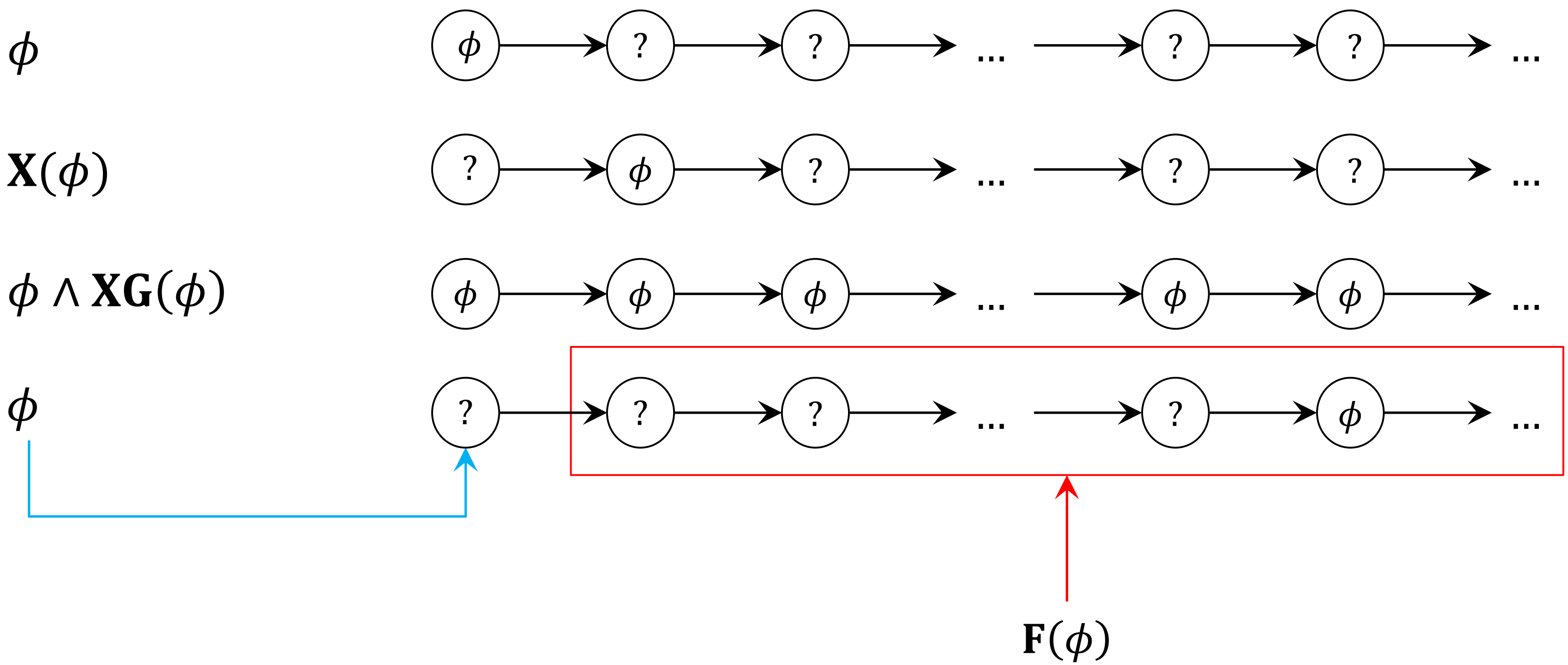
Temporal Expansion: Future

Formula	Semantic
ϕ	
$\mathbf{X}(\phi)$	
$\phi \wedge \mathbf{XG}(\phi)$	
ϕ	



Temporal Expansion: Future

Formula **Semantic**



Temporal Expansion: Future

Formula

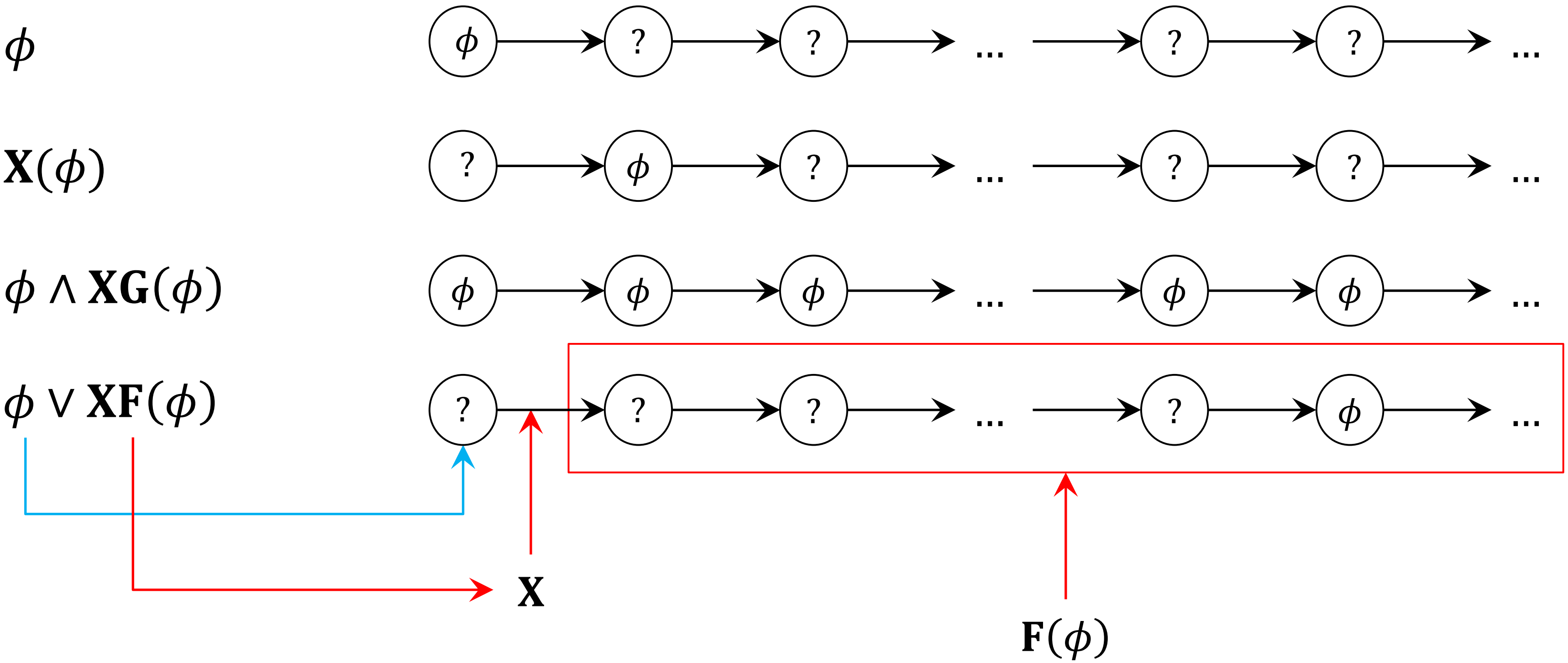
ϕ

$\mathbf{X}(\phi)$

$\phi \wedge \mathbf{XG}(\phi)$

$\phi \vee \mathbf{XF}(\phi)$

Semantic



Temporal Expansion: Until

Formula

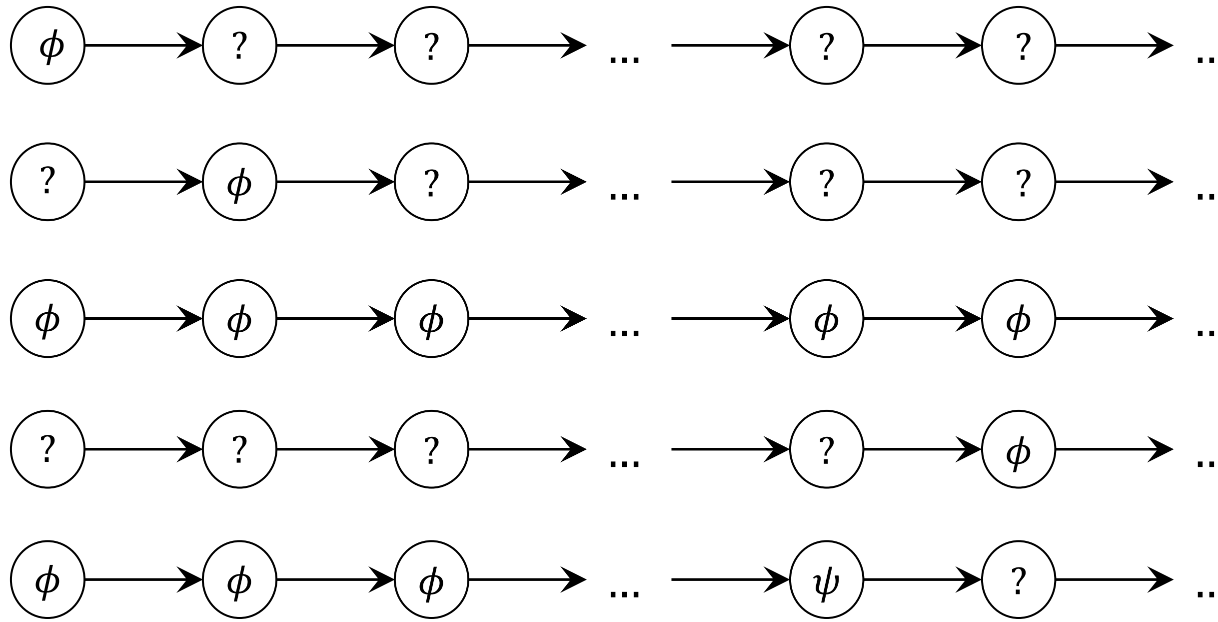
ϕ

$\mathbf{X}(\phi)$

$\phi \wedge \mathbf{XG}(\phi)$

$\phi \vee \mathbf{XF}(\varphi)$

Semantic



$(\phi \mathbf{U} \psi)$

Temporal Expansion: Until

Formula | **Semantic**

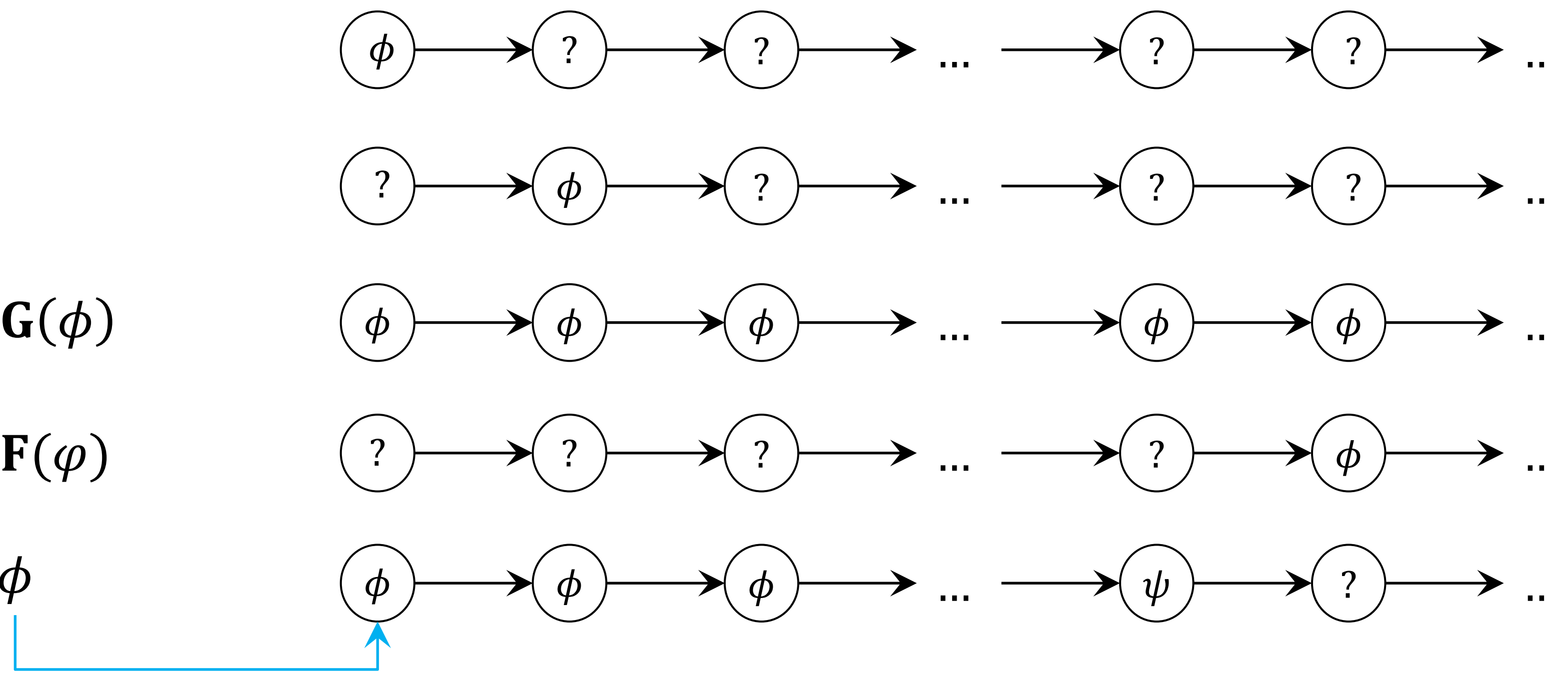
ϕ

$\mathbf{X}(\phi)$

$\phi \wedge \mathbf{XG}(\phi)$

$\phi \vee \mathbf{XF}(\varphi)$

ϕ



Temporal Expansion: Until

Formula

ϕ

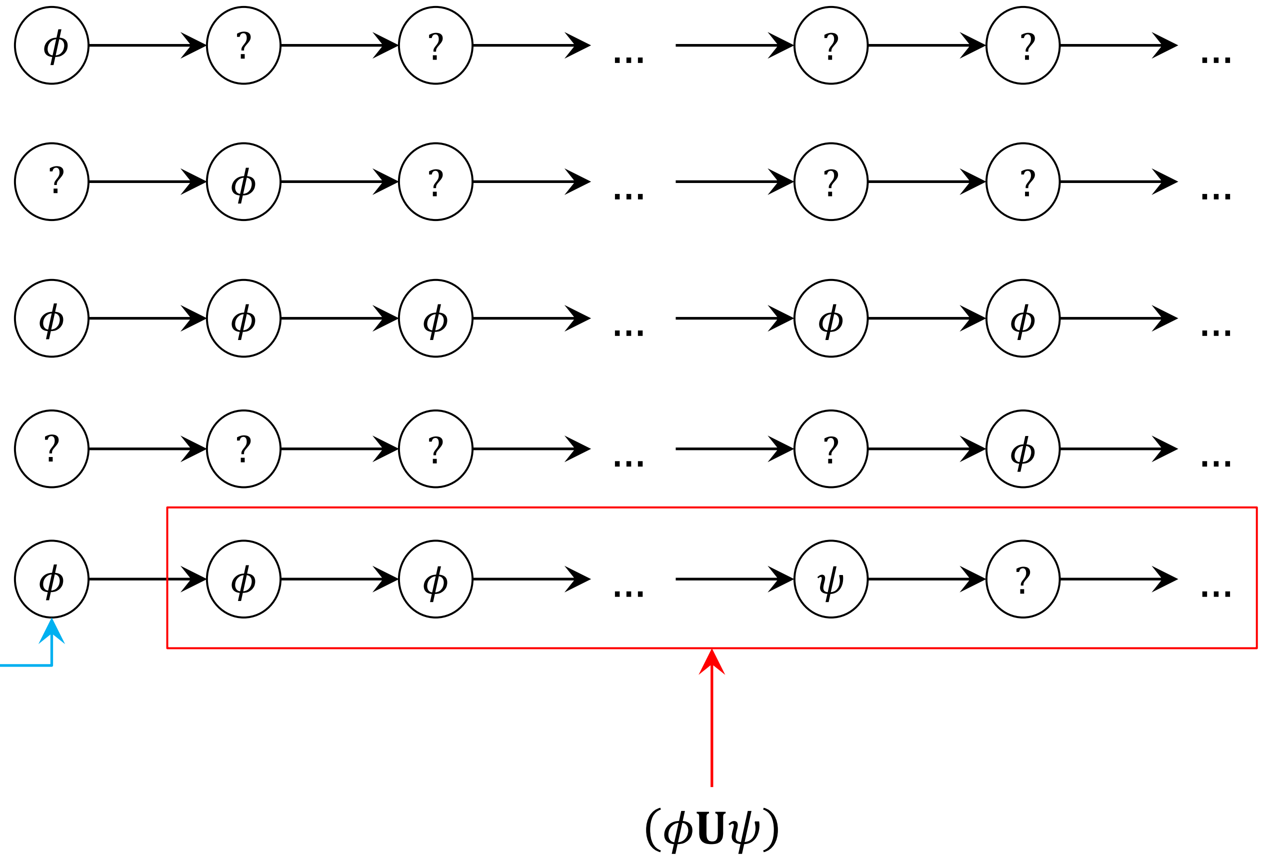
$\mathbf{X}(\phi)$

$\phi \wedge \mathbf{XG}(\phi)$

$\phi \vee \mathbf{XF}(\varphi)$

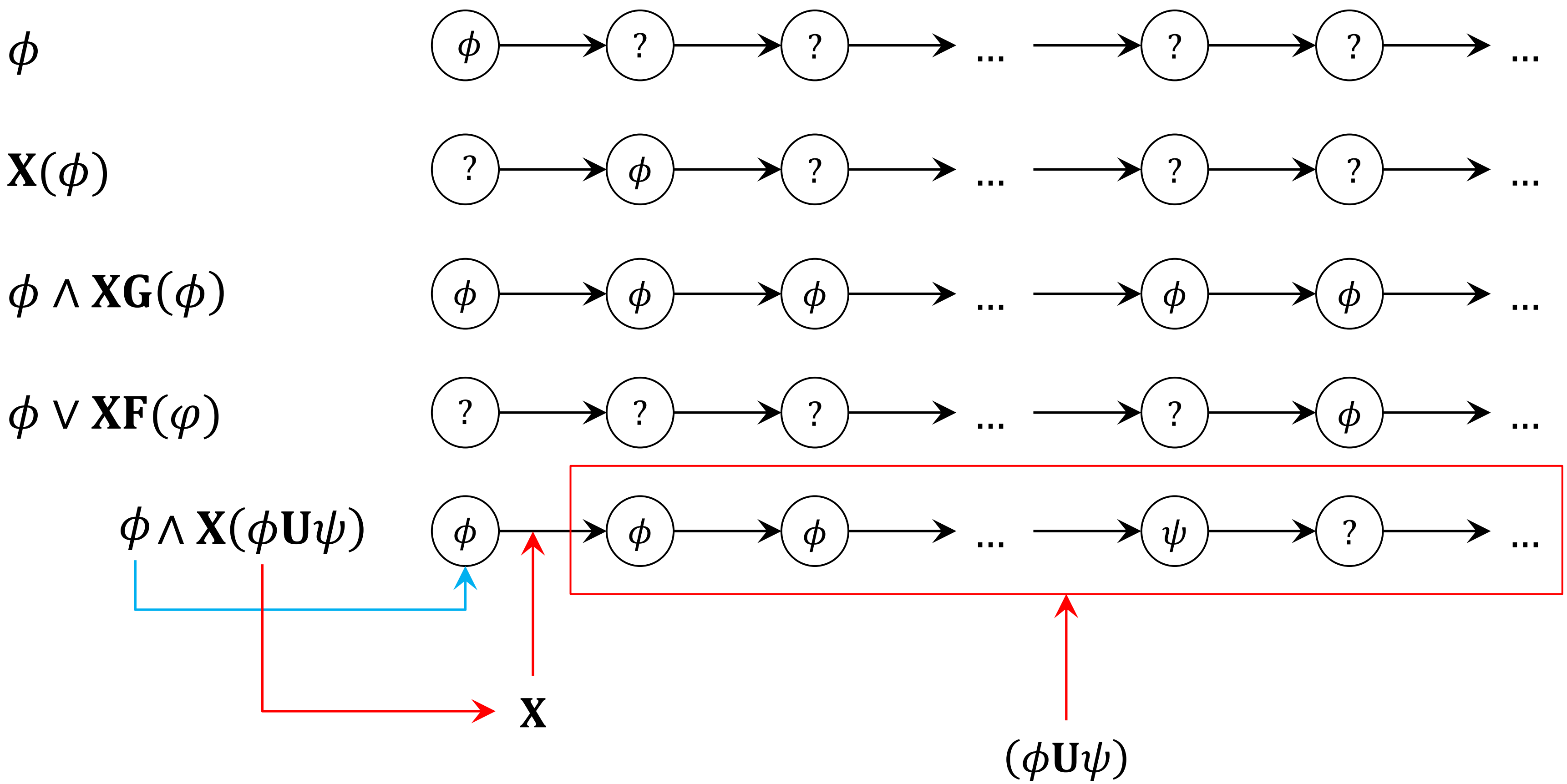
ϕ

Semantic



Temporal Expansion: Until

Formula **Semantic**



Temporal Expansion: Until

Formula **Semantic**

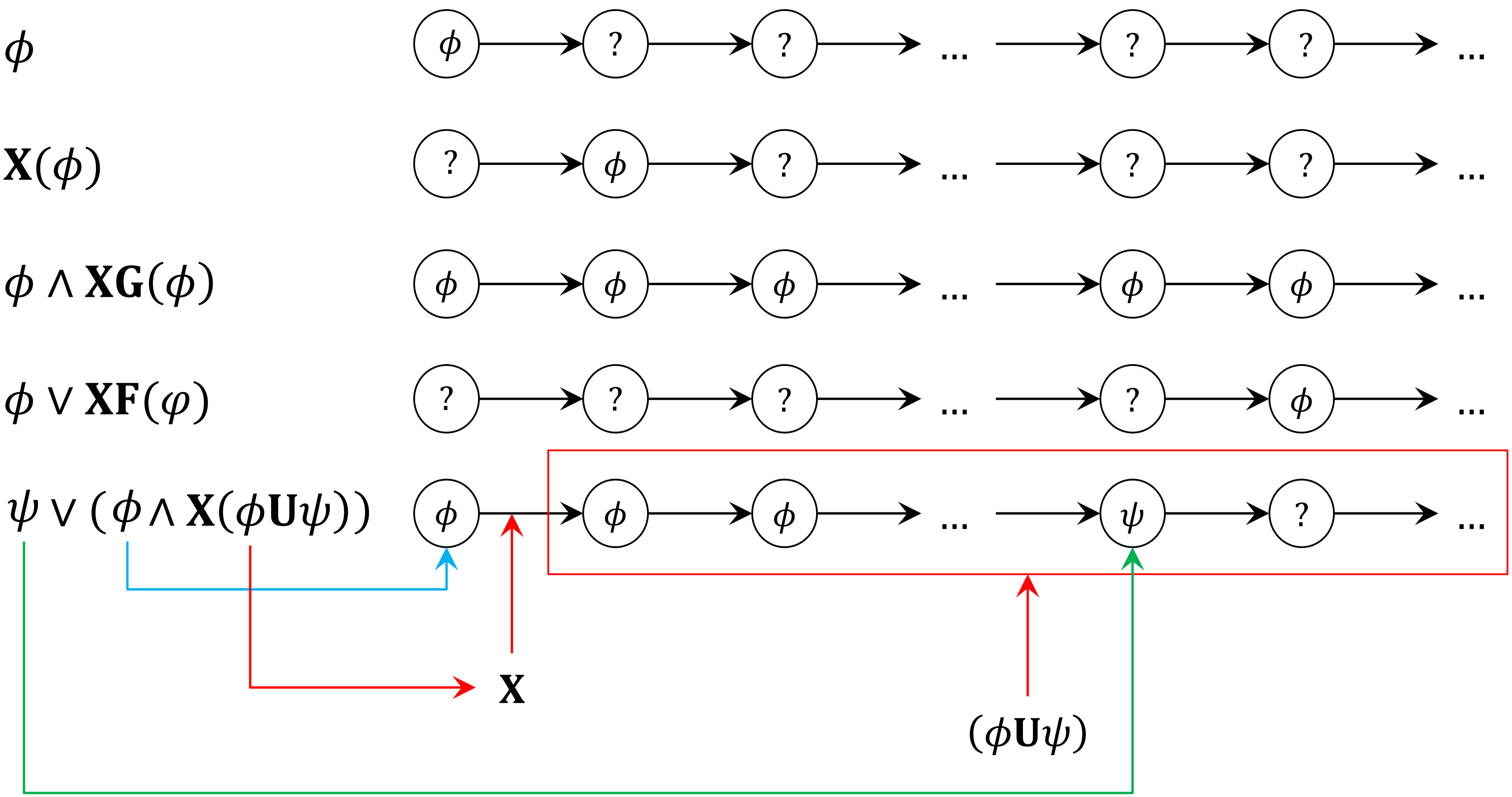
ϕ

$\mathbf{X}(\phi)$

$\phi \wedge \mathbf{XG}(\phi)$

$\phi \vee \mathbf{XF}(\varphi)$

$\psi \vee (\phi \wedge \mathbf{X}(\phi \mathbf{U} \psi))$



Temporal Expansion: Release

Formula

ϕ

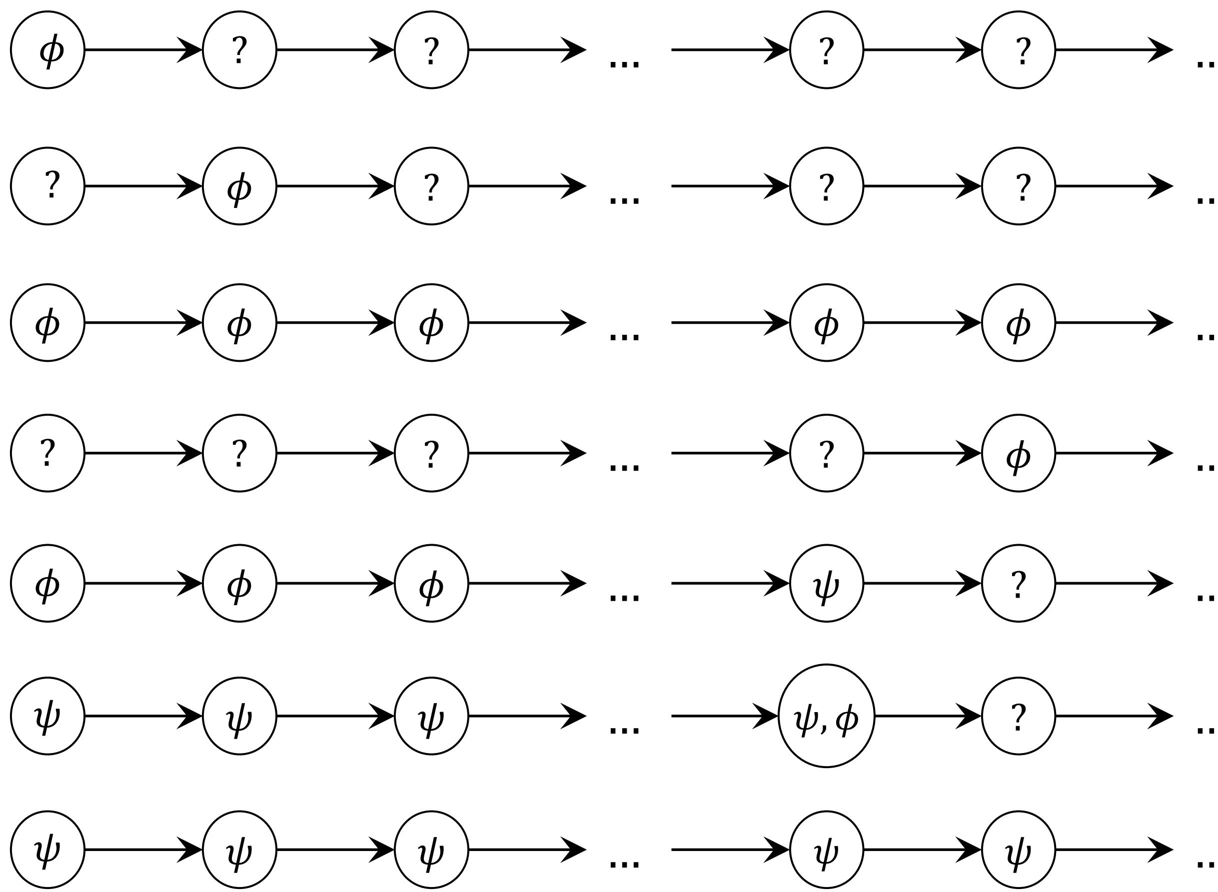
$\mathbf{X}(\phi)$

$\phi \wedge \mathbf{XG}(\phi)$

$\phi \vee \mathbf{XF}(\varphi)$

$\psi \vee \left(\phi \wedge \mathbf{X}(\phi\mathbf{U}\psi)\right)$

Semantic



Temporal Expansion: Release

Formula **Semantic**

ϕ

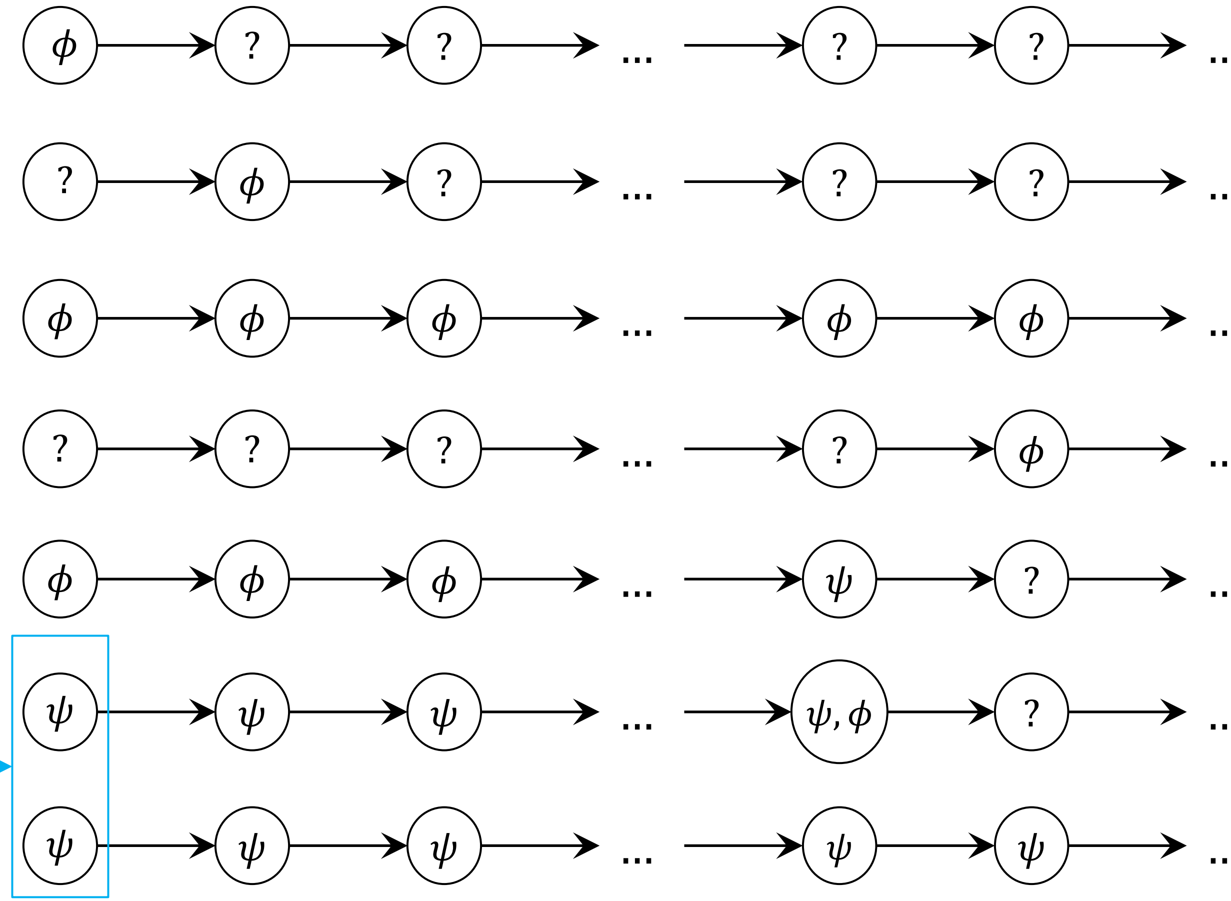
$\mathbf{X}(\phi)$

$\phi \wedge \mathbf{XG}(\phi)$

$\phi \vee \mathbf{XF}(\varphi)$

$\psi \vee \left(\phi \wedge \mathbf{X}(\phi \mathbf{U} \psi)\right)$

ψ



SCOS Secure & Correct Systems

Temporal Expansion: Release

Formula

ϕ

$\mathbf{X}(\phi)$

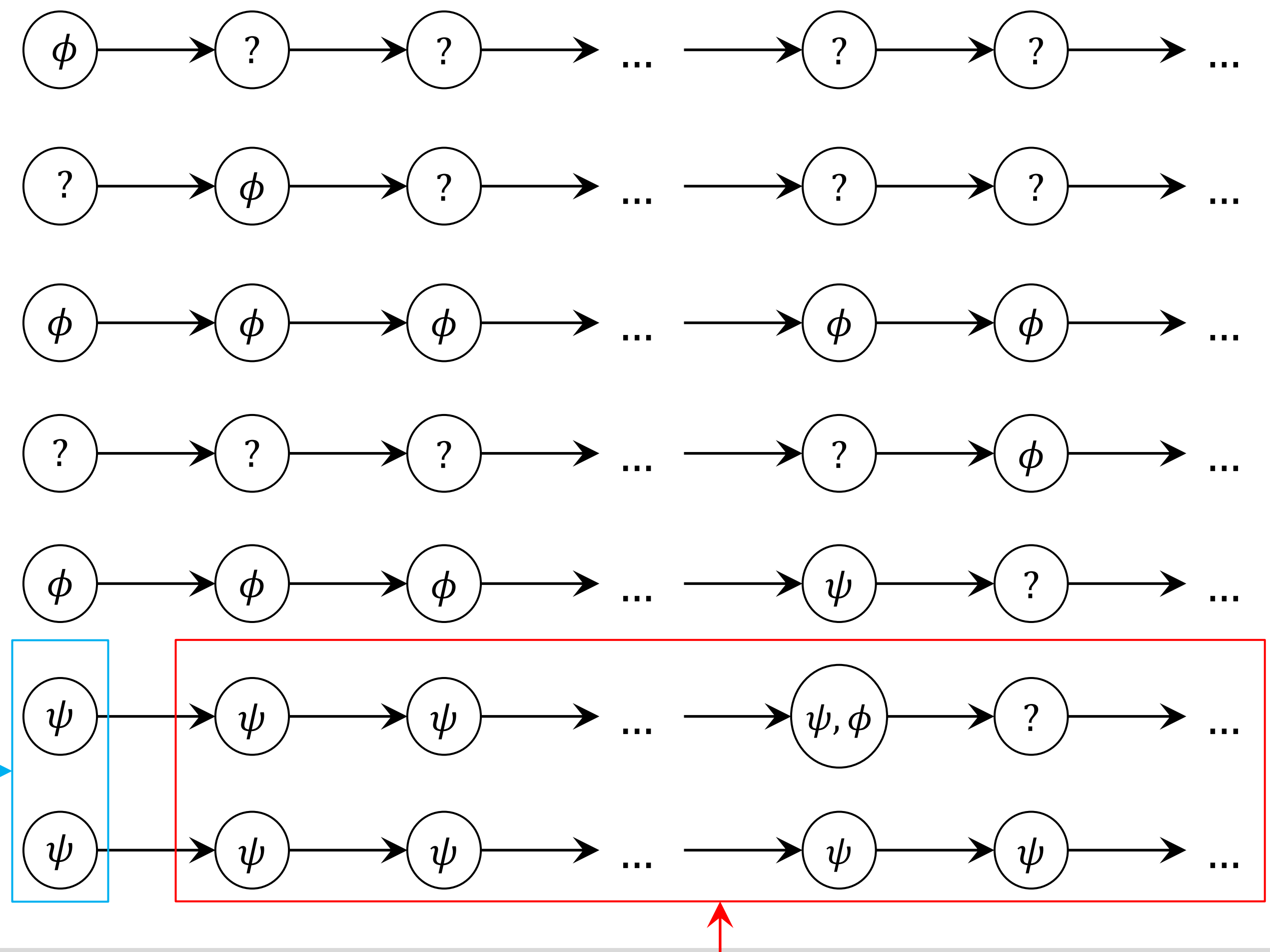
$\phi \land \mathbf{XG}(\phi)$

$\phi \lor \mathbf{XF}(\varphi)$

$\psi \lor (\phi \land \mathbf{X}(\phi \mathbf{U} \psi))$

$\psi \land (\quad \mathbf{X}(\phi \mathbf{R} \psi))$

Semantic



$(\phi \mathbf{R} \psi)$

Temporal Expansion: Release

Formula

ϕ

$\mathbf{X}(\phi)$

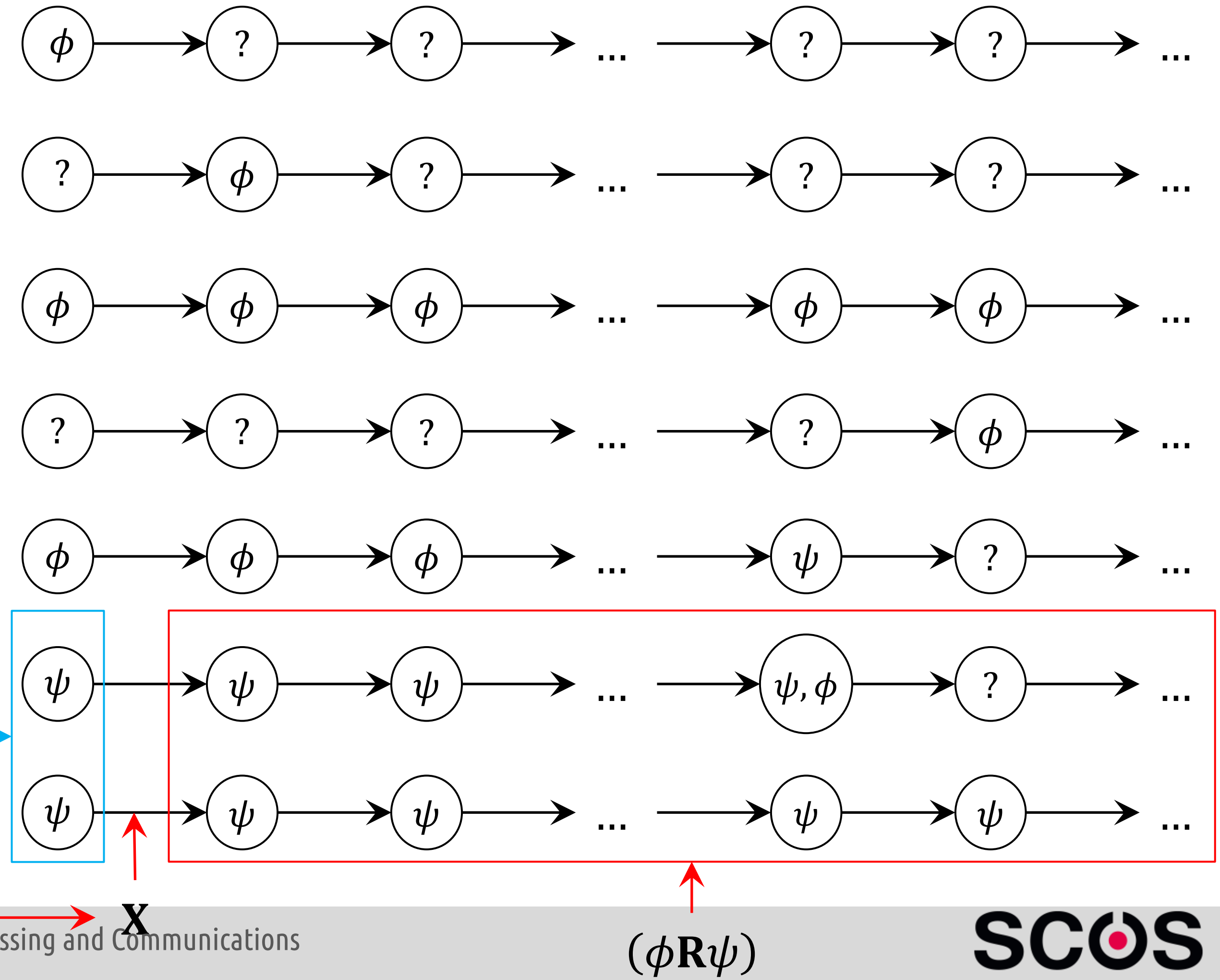
$\phi \wedge \mathbf{XG}(\phi)$

$\phi \vee \mathbf{XF}(\varphi)$

$\psi \vee (\phi \wedge \mathbf{X}(\phi \mathbf{U} \psi))$

$\psi \wedge (\quad \mathbf{X}(\phi \mathbf{R} \psi))$

Semantic



Temporal Expansion: Release

Formula

ϕ

$\mathbf{X}(\phi)$

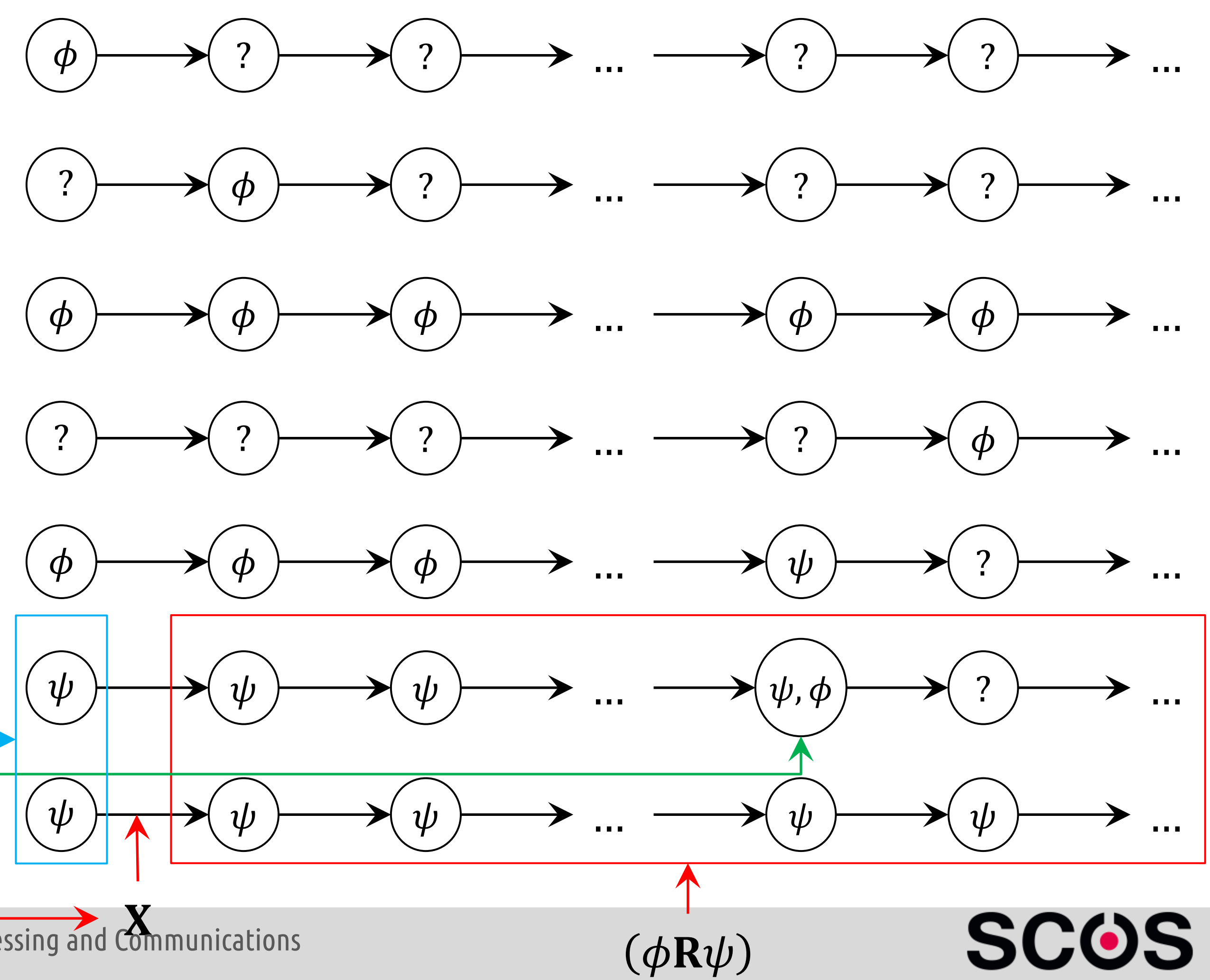
$\phi \wedge \mathbf{XG}(\phi)$

$\phi \vee \mathbf{XF}(\varphi)$

$\psi \vee \left(\phi \wedge \mathbf{X}(\phi \mathbf{U} \psi)\right)$

$\psi \wedge (\phi \vee \mathbf{X}(\phi \mathbf{R} \psi))$

Semantic



Temporal Expansions

Formula

ϕ

$\mathbf{X}(\phi)$

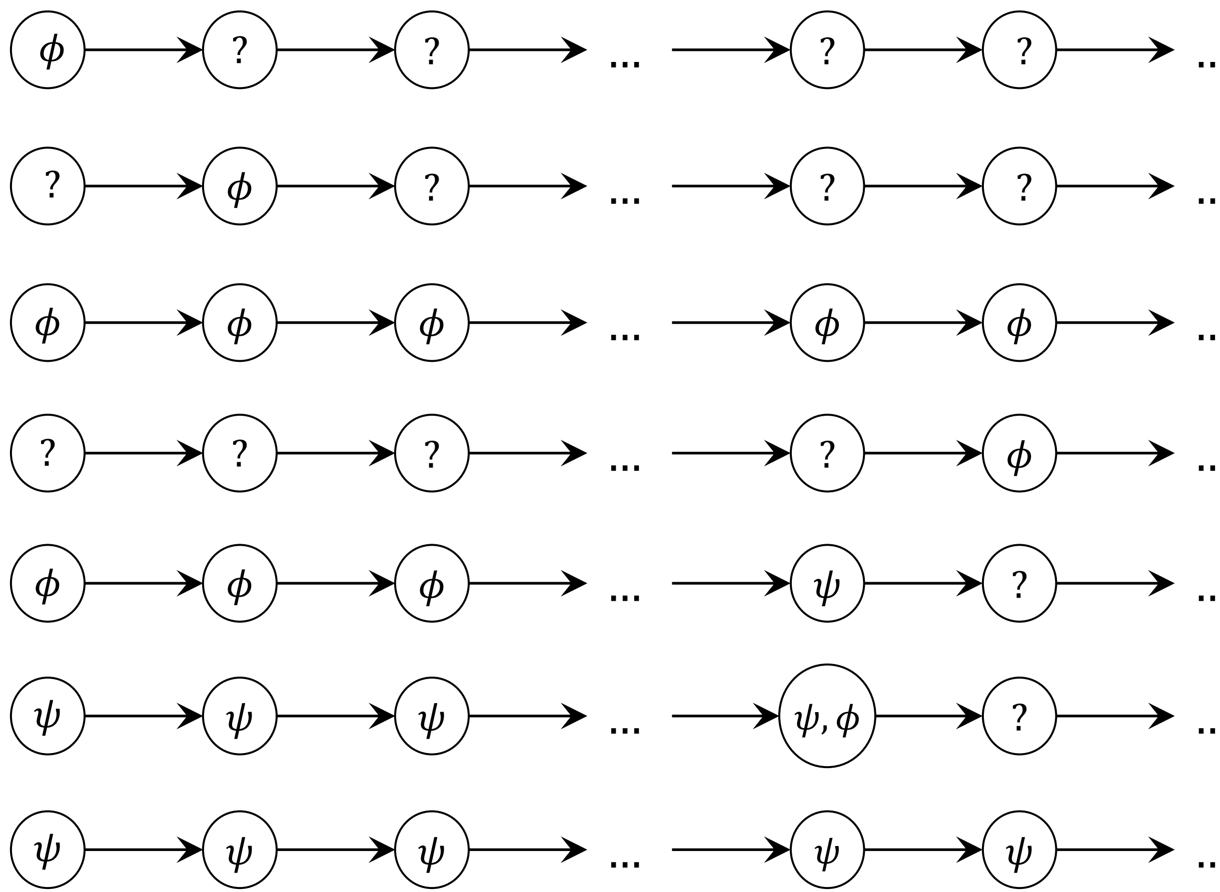
$\phi \land \mathbf{XG}(\phi)$

$\phi \lor \mathbf{XF}(\varphi)$

$\psi \lor \big(\phi \land \mathbf{X}(\phi \mathbf{U} \psi)\big)$

$\psi \land \big(\phi \lor \mathbf{X}(\phi \mathbf{U} \psi)\big)$

Semantic



LTL Identities

- $\mathbf{G}\phi = \phi \wedge \mathbf{XG}\phi$
- $\mathbf{F}\phi = \phi \vee \mathbf{XF}\phi$
- $\phi\ \mathbf{U}\ \psi = \psi \vee \big(\phi \wedge \mathbf{X}(\phi \mathbf{U} \psi)\big)$
- $\phi\ \mathbf{R}\ \psi = \psi \wedge \big(\phi \vee \mathbf{X}(\phi \mathbf{R} \psi)\big)$
- $\phi\ \mathbf{R}\ \psi = \neg(\neg\phi\ \mathbf{U}\ \neg\psi)$
- $\mathbf{G}\phi = \neg\mathbf{F}\neg\phi$
- $\mathbf{F}\phi = \neg\mathbf{G}\neg\phi$
- $\mathbf{F}\phi = \text{True}\ \mathbf{U}\ \phi$

- Homework
  - rewrite $\mathbf{G}(r \rightarrow \mathbf{F}g)$ only using Release.
  - rewrite $\mathbf{F}(r \rightarrow \mathbf{G}g)$ only using Until.

Two Kinds of Properties – What's the difference?

1. $\mathbf{G}(a \rightarrow \mathbf{X}b)$
2. $\mathbf{G}(a \rightarrow \mathbf{F}b)$

Safety and Liveness Properties

- Safety
  - nothing bad will happen
  - finite counterexamples
  - bad prefixes cannot be extended to good traces

- Liveness
  - something good will happen
  - infinite counterexamples
  - bad traces can be extended to good traces

Two Kinds of Properties – What's the difference?

1. $\mathbf{G}(a \rightarrow \mathbf{X}b)$

step	0	1	ω
a	1	0	?
b	0	0	?

2. $\mathbf{G}(a \rightarrow \mathbf{F}b)$

step	0	1	ω
a	1	0	?
b	0	0	?

HINT!

Two Kinds of Properties – Q5: What's the difference?

1. $\mathbf{G}(a \rightarrow \mathbf{X}b)$

2. $\mathbf{G}(a \rightarrow \mathbf{F}b)$

step	0	1	ω
a	1	0	?
b	0	0	?
$\mathbf{X}b$	0	?	?
$a \rightarrow \mathbf{X}b$	0	?	?
$\mathbf{G}(a \rightarrow \mathbf{X}b)$	0	¯_(ツ)_/¯	

step	0	1	ω
a	1	0	?
b	0	0	?
$\mathbf{F}b$	?	?	?
$a \rightarrow \mathbf{F}b$	?	?	?
$\mathbf{G}(a \rightarrow \mathbf{F}b)$	(⊙_⊙;)		

Two Kinds of Properties – Q5: What's the difference?

1. $\mathbf{G}\big(a \rightarrow \mathbf{X}(b)\big)$

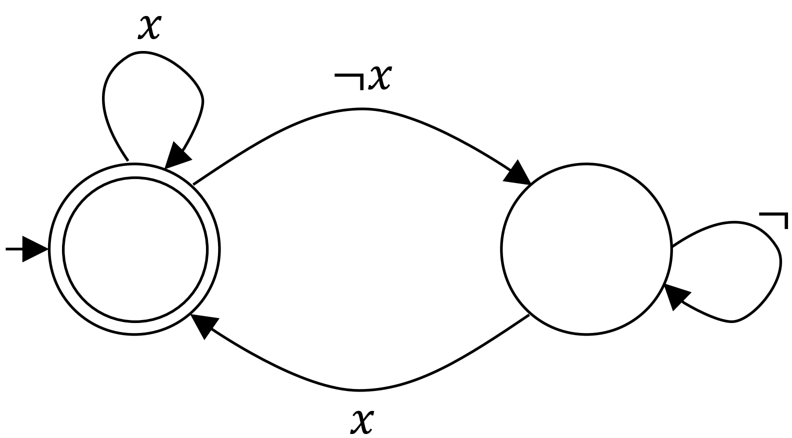
2. $\mathbf{G}\big(a \rightarrow \mathbf{F}(b)\big)$

step	0	1	ω
a	1	0	?
b	0	0	?
$\mathbf{X}(b)$	0	?	?
$a \rightarrow \mathbf{X}(b)$	0	?	?
$\mathbf{G}\big(a \rightarrow \mathbf{X}(b)\big)$	0	¯_(ツ)_/¯	

step	0	1	ω
a	1	0	0
b	0	0	0
$\mathbf{F}(b)$	0	0	0
$a \rightarrow \mathbf{F}(b)$	0	0	0
$\mathbf{G}\big(a \rightarrow \mathbf{F}(b)\big)$	**0**	**0**	0

Definition of Büchi Automata

- Set of States Q
- Initial state $q_o \in Q$
- Alphabet Σ
  - In our case: often $\Sigma = 2^P$
  - P: atomic propositions
- Labeled edges $E \subseteq Q \times \Sigma \times Q$
- Accepting States $F \subseteq Q$
  - Usually marked with double-circle



Runs of Büchi Automata

- Given: Input trace $\sigma = \sigma_0\sigma_1\sigma_2 \ldots \in \Sigma^\omega$
- Run: (Infinite) sequence of states $\rho = q_0q_1q_2 \ldots$
  - q_o is initial state
  - For all $i \geq 0$: $(q_i, \sigma_i, q_{i+1}) \in E$
- Accepting Run: $\inf(\rho) \cap F \neq \emptyset$

Büchi Automata vs. LTL Formulas

- Büchi Automaton $\mathcal{B}$
  - Language $L(\mathcal{B})$: Set of traces with accepting runs
- LTL Formula ϕ
  - Language $L(\phi)$: Set of traces that satisfy formula
- Expressiveness
  - LTL $\longrightarrow$ Büchi
  - Büchi $\not\rightarrow$ LTL

Notions of Büchi Automata

- Complete vs. Incomplete
  - In every state, (at least) one edge for every letter?
- Deterministic vs. Non-Deterministic
  - In every state, not more than one edge per letter?
- Generalized Büchi Automata
  - Accepting condition $\mathcal{F}$: set of sets of states
  - Accepting runs visit at least one state of every set of $\mathcal{F}$ infinitely often
  - As expressive as Büchi Automata

LTL to Büchi Automata

- Formula rewriting
  - Formula in Negated Normal Form (NNF)
  - Apply temporal expansions until “done?!”
- Core translation
  - LTL formula → Generalized Büchi Automaton
  - Automaton size is exponential in the size of formula
- Generalized Büchi Automaton → Büchi automaton
  - Degeneralization (beyond the scope of this lecture)

Core Translation: Formula Rewriting

- Given an LTL formula in NNF
  - Expand till no outer temporal operator except for neXt
  - Rewrite in Disjunctive Normal Form
    - *treat* $\mathbf{X}\phi$ *and literals the same*

Core Translation: Formula Rewriting

- Given an LTL formula in NNF
  - Expand till no outer temporal operator except for neXt
  - Rewrite in Disjunctive Normal Form
    - *treat* $\mathbf{X}\phi$ *and literals the same*

$$\mathbf{F}p = p \vee \mathbf{XF}p$$

Core Translation: Formula Rewriting

- Given an LTL formula in NNF
  - Expand till no outer temporal operator except for neXt
  - Rewrite in Disjunctive Normal Form
    - *treat $\mathbf{X}\phi$ and literals the same*

$$\mathbf{F}p = p \lor \mathbf{XF}p$$

- A disjunct reveals obligations for now and next

$$p = p \land \mathbf{X}\mathrm{true}$$
$$\mathbf{XF}p = \mathrm{true} \land \mathbf{XF}p$$
$$\mathrm{true} = \mathrm{true} \land \mathbf{X}\mathrm{true}$$

Core Translation: Automata Construction

Each disjunct represents a states and some edge:

$p \wedge \mathbf{X}\mathrm{true}$ $\mathrm{true} \wedge \mathbf{XF}p$

Core Translation: Automata Construction

- Each disjunct represents a states and some edge:

$p \wedge \mathbf{X}\text{true}$ $\text{true} \wedge \mathbf{XF}p$

$\mathbf{F}p$

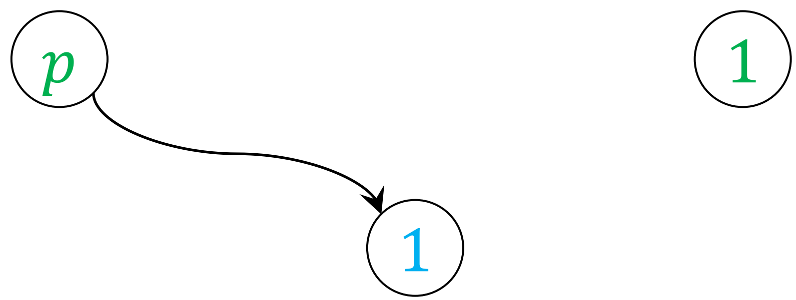
Core Translation: Automata Construction

- Each disjunct represents a states and some edge:

$p \wedge \mathbf{X}\text{true}$ $\qquad\qquad$ $\text{true} \wedge \mathbf{XF}p$

$\mathbf{F}p$

true



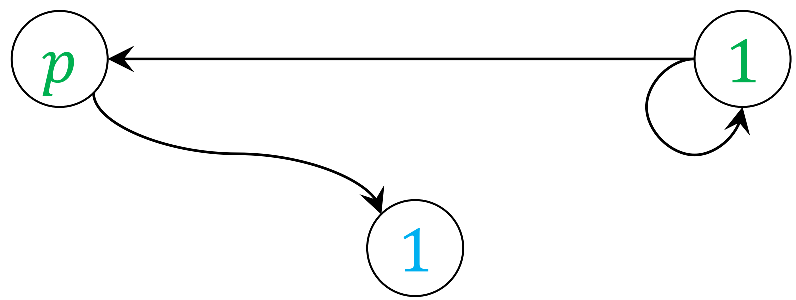
Core Translation: Automata Construction

- Each disjunct represents a states and some edge:

$p \land \mathbf{X}\text{true}$ $\quad\quad$ $\text{true} \land \mathbf{X}\mathbf{F}p$

$\mathbf{F}p$

true



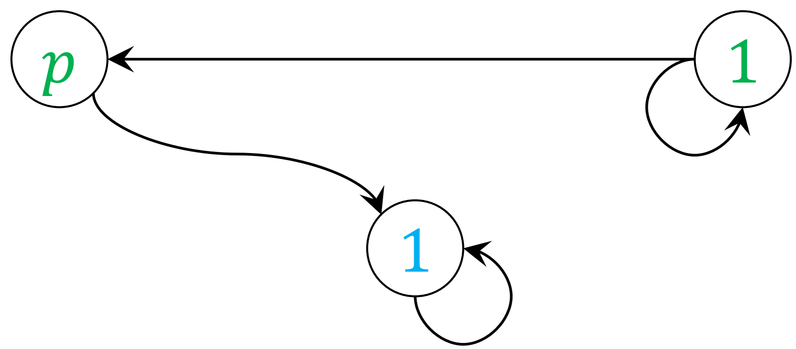
Core Translation: Automata Construction

- Each disjunct represents a states and some edge:

$p \wedge \mathbf{X}\mathrm{true}$ $\mathrm{true} \wedge \mathbf{XF}p$

$\mathbf{F}p$

true



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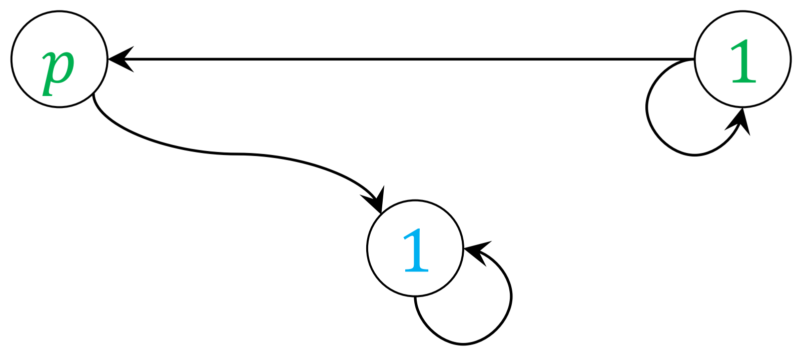

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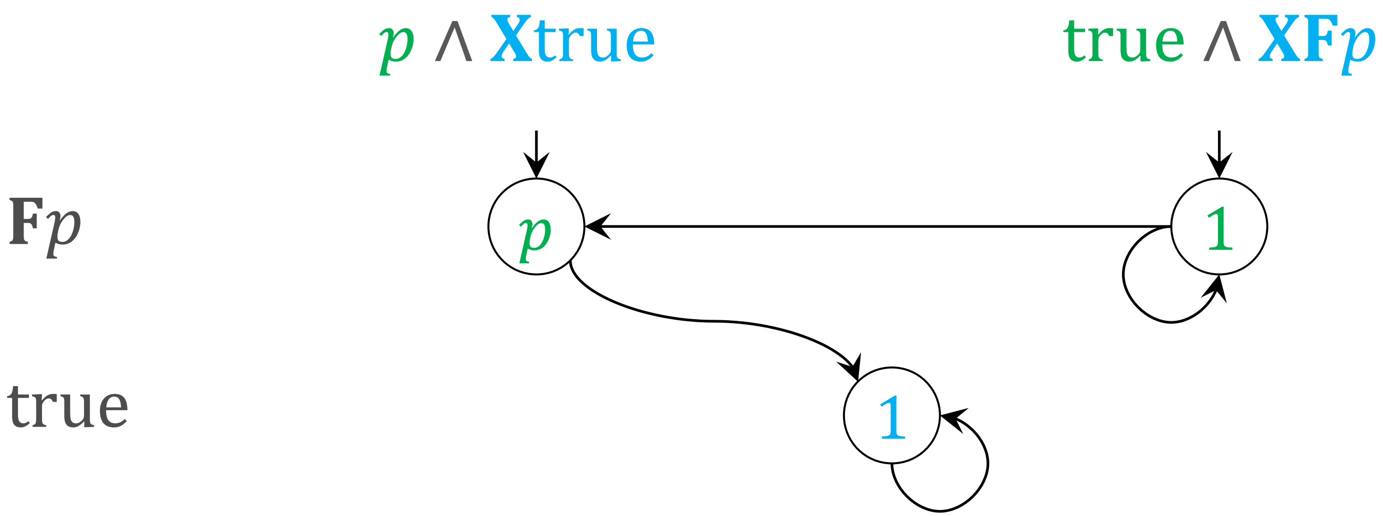

- initial states
  - all green states of the initial disjuncts

Core Translation: Accepting Condition

- $\mathcal{F}$: multiple accepting sets
- One for each Until sub-formula $(\phi\ \mathbf{U}\ \psi)$
  - a state is accepting if it doesn't have $\mathbf{X}(\phi\ \mathbf{U}\ \psi)$ in its conjunct, or it satisfies ψ.

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Remember LTL identities!

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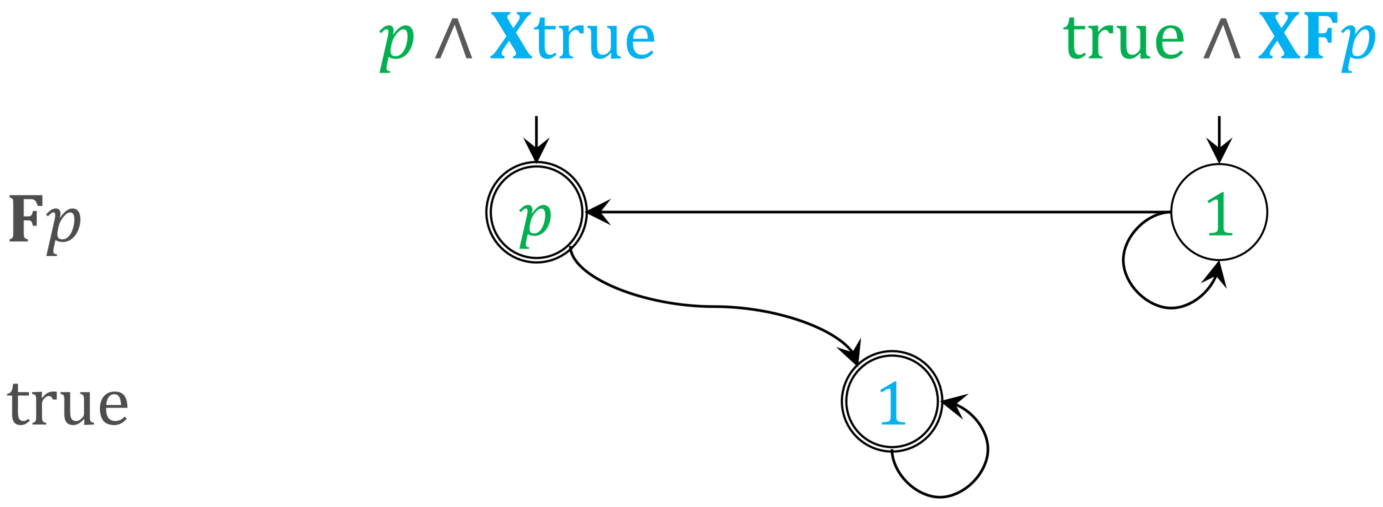
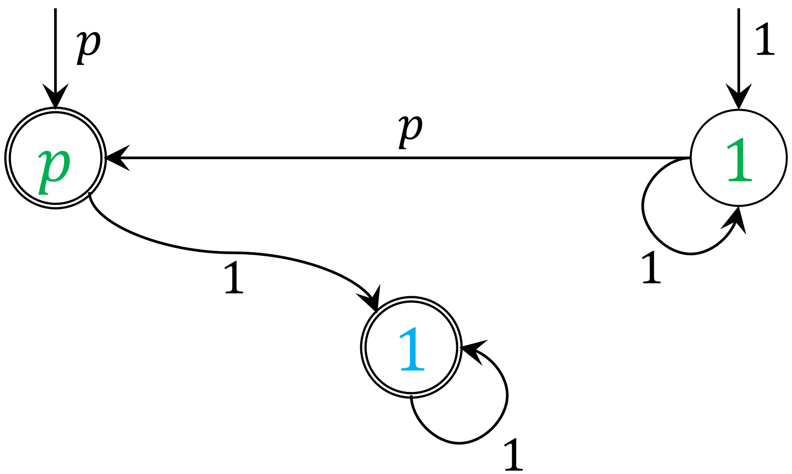


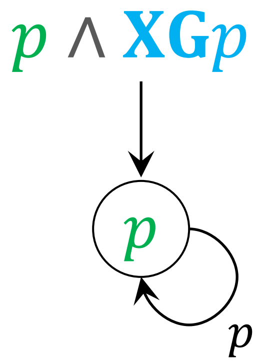
Core Translation: Labeling Edges

- State's incoming edges are labeled with literals they represent



Core Translation: Globally Automaton

- $\mathbf{G}p$
  - Rewrite: $p \wedge \mathbf{XG}p$
  - DNF: $p \wedge \mathbf{XG}p$
  - Automata:



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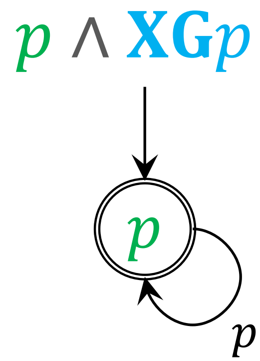

    - $\mathbf{G}p$

Core Translation: Until Automaton

- $p\ \mathbf{U}\ q$
  - Rewrite: $q \lor p \land \mathbf{X}(p\ \mathbf{U}\ q)$

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  - DNF: $q \land \mathbf{X}\mathrm{true} \lor p \land \mathbf{X}(p \mathbf{U} q)$
  - Automata:

$p \land \mathbf{X}(p \mathbf{U} q)$ $\quad$ $q \land \mathbf{X}\mathrm{true}$

- $p \mathbf{U} q$

Core Translation: Until Automaton

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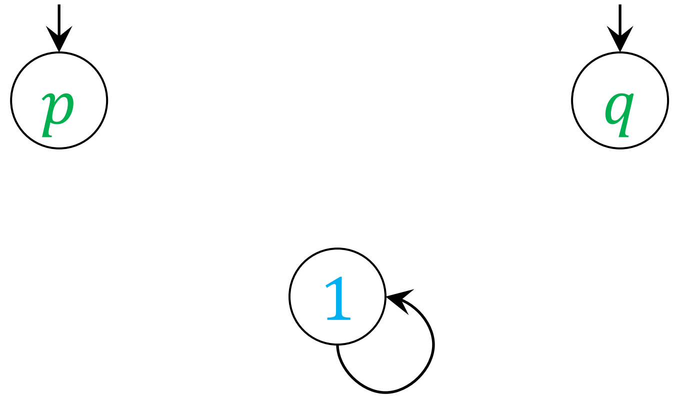
- $p\ \mathbf{U}\ q$
- true



Core Translation: Until Automaton

- $p \ \mathbf{U} \ q$
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  - DNF: $q \land \mathbf{X}\mathrm{true} \lor p \land \mathbf{X}(p \ \mathbf{U} \ q)$
  - Automata:

$p \land \mathbf{X}(p \ \mathbf{U} \ q)$ $\quad$ $q \land \mathbf{X}\mathrm{true}$

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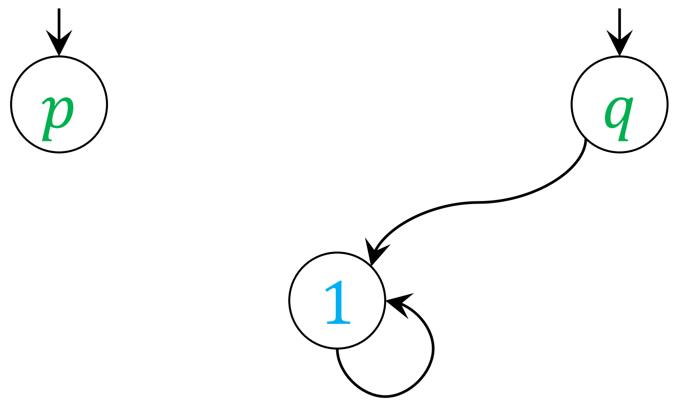

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  - Automata:

$p \land \mathbf{X}(p\ \mathbf{U}\ q)$ $\qquad$ $q \land \mathbf{X}\text{true}$

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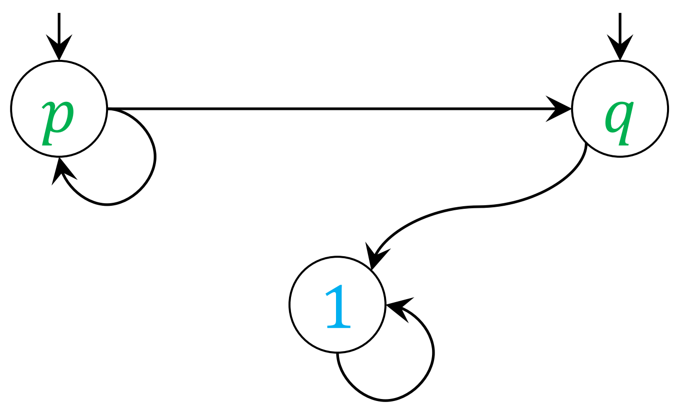

Core Translation: Until Automaton

- $p\ \mathbf{U}\ q$
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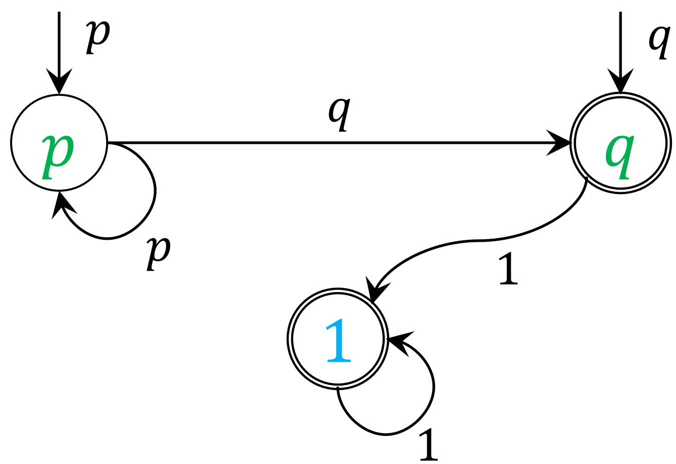
- $p\ \mathbf{U}\ q$
- true



Core Translation: Release Automaton

- $p\ \mathbf{R}\ q$
  - Rewrite: $q \wedge \left(p \vee \mathbf{X}(p\ \mathbf{R}\ q)\right)$

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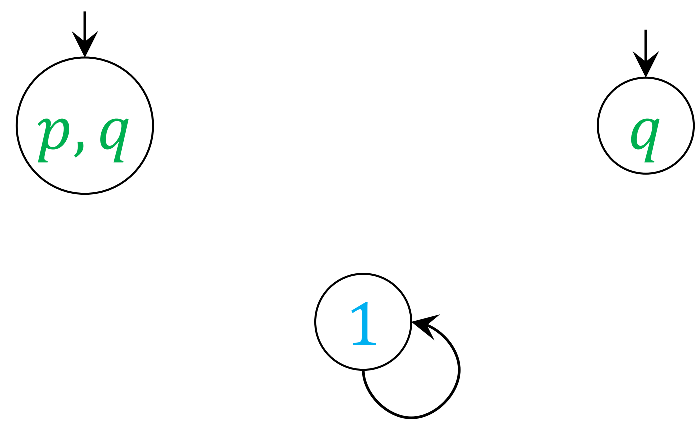
- $p\ \mathbf{R}\ q$
- true



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  - DNF: $q \wedge p \wedge \mathbf{X}\text{true} \vee q \wedge \mathbf{X}(p\ \mathbf{R}\ q)$
  - Automata:

$q \wedge p \wedge \mathbf{X}\text{true}$ $\qquad$ $q \wedge \mathbf{X}(p\ \mathbf{R}\ q)$

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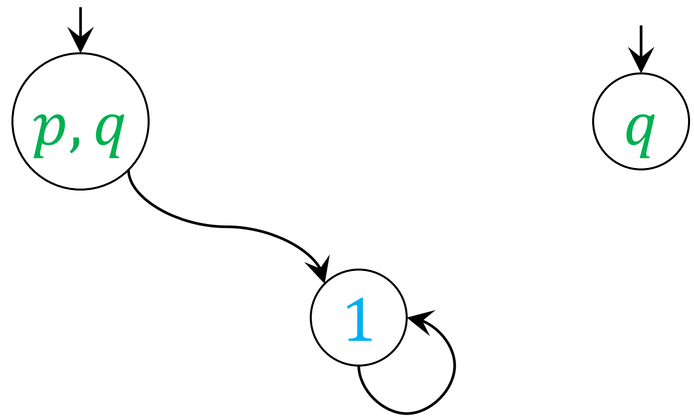
- true



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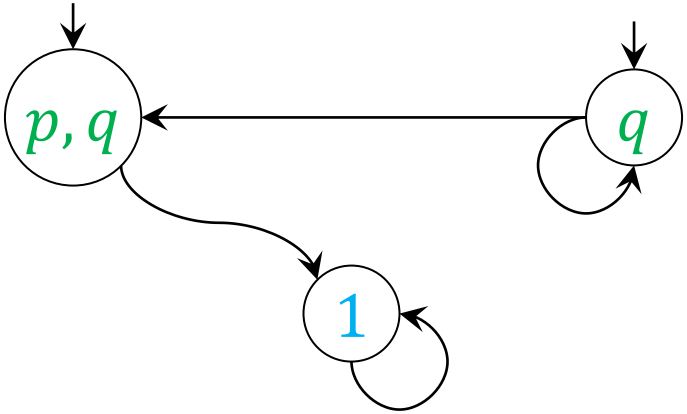
- true



Core Translation: Release Automaton

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  - DNF: $q \land p \land \mathbf{X}\text{true} \lor q \land \mathbf{X}(p\ \mathbf{R}\ q)$
  - Automata:

$q \land p \land \mathbf{X}\text{true}$ $\qquad$ $q \land \mathbf{X}(p\ \mathbf{R}\ q)$

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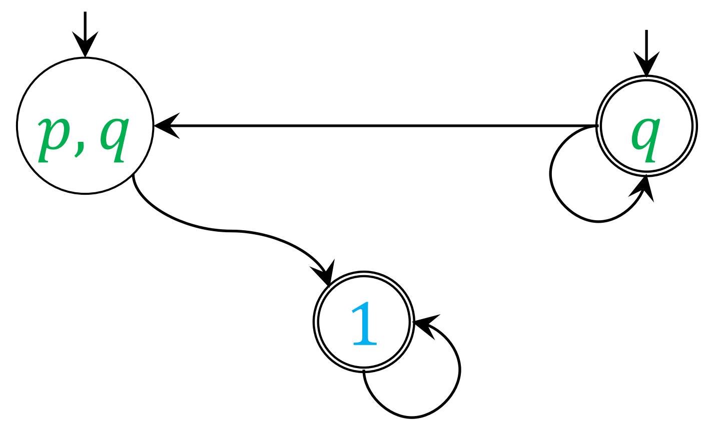

Core Translation: Release Automaton

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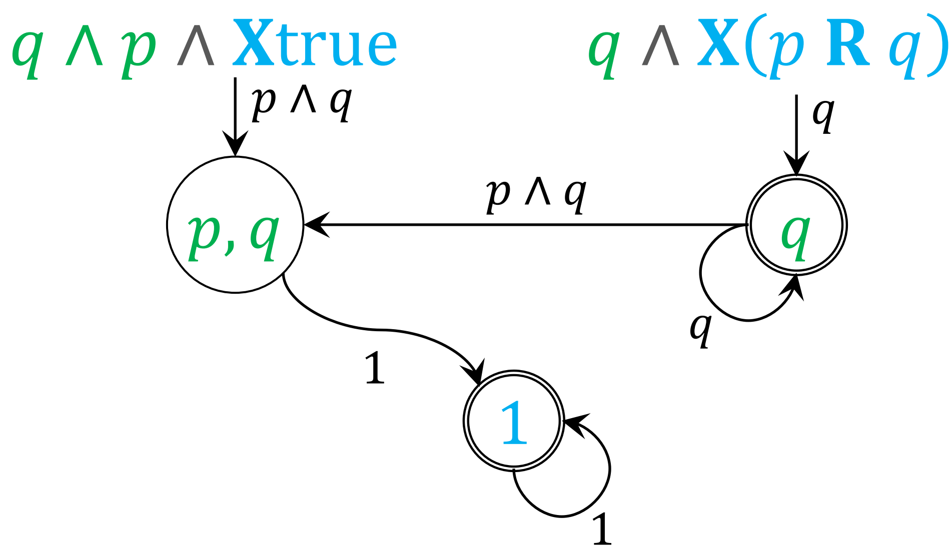
  - Rewrite: $q \wedge \big(p \vee \mathbf{X}(p \mathbf{R} q)\big)$
  - DNF: $q \wedge p \wedge \mathbf{X}\mathrm{true} \vee q \wedge \mathbf{X}(p \mathbf{R} q)$
  - Automata:
    - $p \mathbf{R} q$
    - true



Example: $\phi = \mathbf{F}(p \vee q)$

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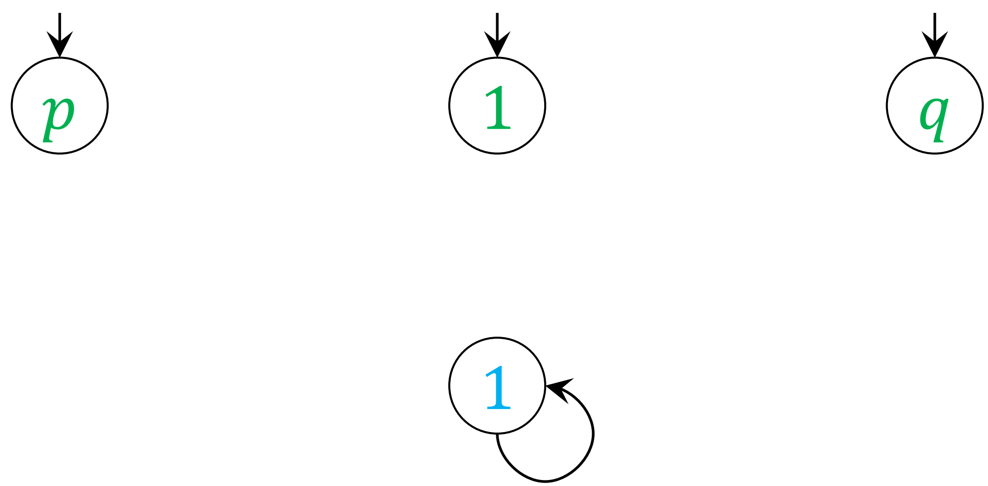
    - $\mathbf{F}(p \vee q)$
    - true



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- $\mathbf{F}(p \vee q)$
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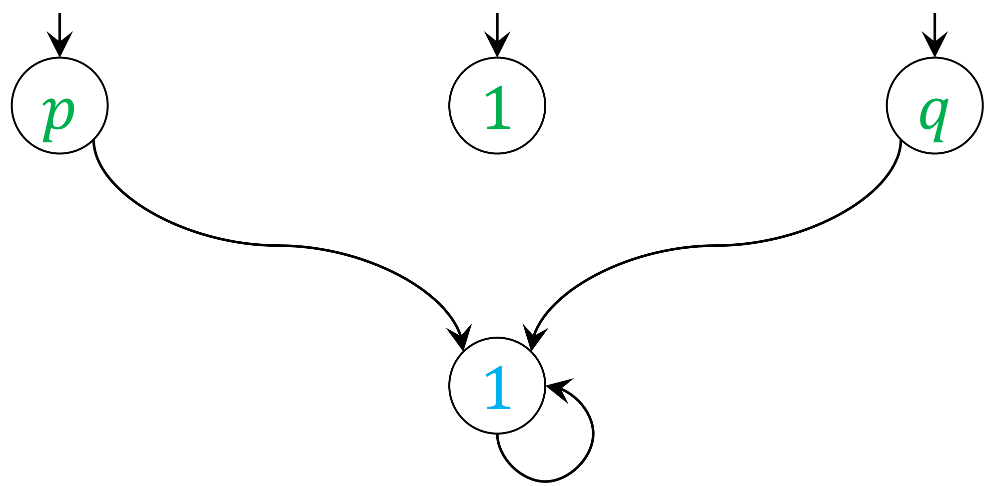

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- $\mathbf{F}(p \vee q)$
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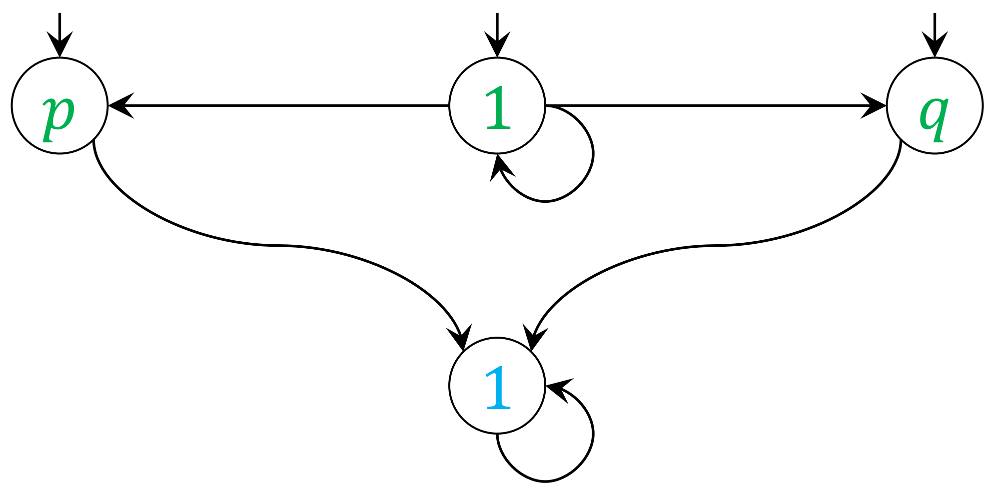

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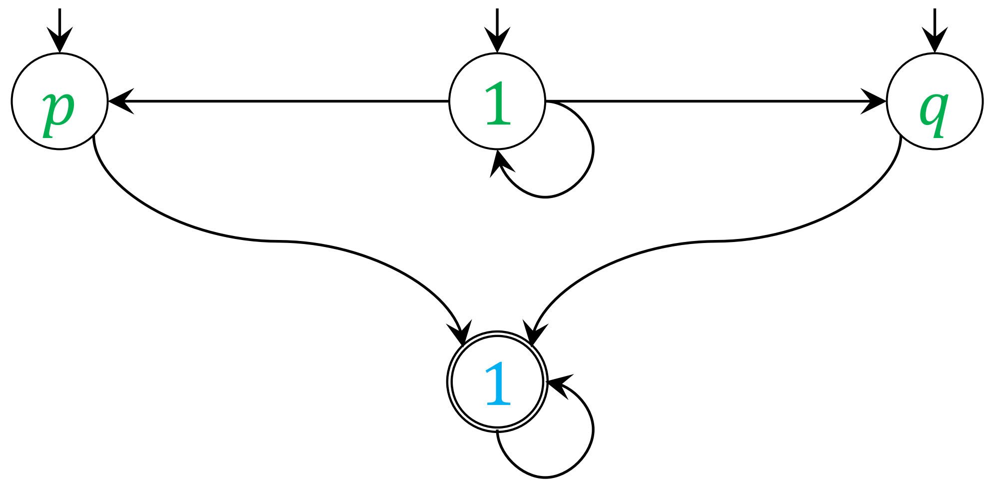

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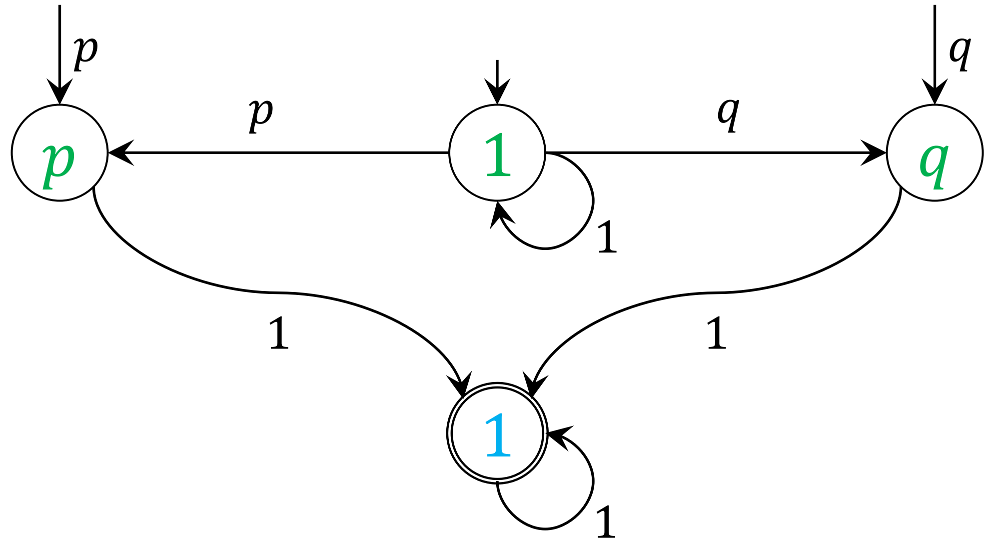
- $\mathbf{F}(p \vee q)$
- true



Homework: $\phi = \mathbf{GF}p$

- Translate $\mathbf{GF}p$ to Generalized Buchi Automaton